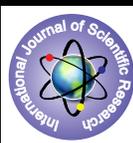


# Comparative Theoretical and Experimental Study on The Stresses and Strains in the Cardan Shaft



## Engineering

**KEYWORDS :** cardan shaft, Von Mises stresses, strains, experimental tests.

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### ABSTRACT

The paper presents a theoretical and experimental study on a component of the cardan transmission - the cardan shaft. The study presents the mathematical calculus method for this component, applied to the model of the cardan transmission from a Dacia car, as well as a study with finite elements, targeting the correlation of the results from the two methods. Further there is presented also an experimental study, realised using resistive electrical tensometry and tensometric transducers, needed for the confirmation of the results obtained in the theoretical study. The study represents a model that can be applied also to the other components of cardan transmissions, but also to other mechanical parts.

### INTRODUCTION

Cardan transmissions from cars and various industrial equipments are a part of the kinematic chain for transmitting the rotation motion from the engine to the motor wheels or to moving subassemblies [2].



Figure 1. Types of cardan transmissions made by the company Eurocardan [8]

A cardan transmission is an assembly of machine parts (joints, shafts, intermediate bearings etc.) used to transmit at a distance the mechanical energy, by means of a rotation motion without the amplification of the torque between the aggregates, having a variable or invariable position in space. A judicious design of these machine parts and of the manufacturing technology allows an increase of the reliability in exploitation and a reduced material consumption [2].

### CALCULUS OF THE CARDAN TRANSMISSION'S SHAFTS [1, 2, 3]

Considering that the loading regime of the car transmission which contains the cardan transmission has been determined, the initial stage of dimensioning the shafts is the determining of the forces specific for the considered transmission. The most complex loading case is encountered at the shafts of transmissions with cardan joints; the other transmission types can be considered as particular cases of the transmission with joints.

The cardan shaft is subjected to torsion stresses (the main stress), to bending due to the secondary bending couples, as well as to tensile-compression stresses due to the axial (friction) forces from the telescopic joint. In the case of long cardan shafts ( $r > 0.5D$ ), the axial force in the telescopic joint has a nonessen-

tial influence on the shaft's dimensioning, so it too can be neglected in the first calculus stage. However, the influence of this force is essential for the selection and assembling of the bearing on which it acts. For the above-mentioned reasons, the shaft's pre-dimensioning is done considering as dominant the torsion stress. For a given regime, in which the value of the torque  $M_t$  varies between a maximal value  $M_{tM}$  and a minimal value  $M_{tm}$ , the torsion stress increases with the increase of the angle formed by the shafts connected through the cardan joint. [4]

### CALCULUS OF THE CARDAN FORK

The cardan fork is subjected to the force  $F$  (which is perpendicular to the fork's plane - figure 2).

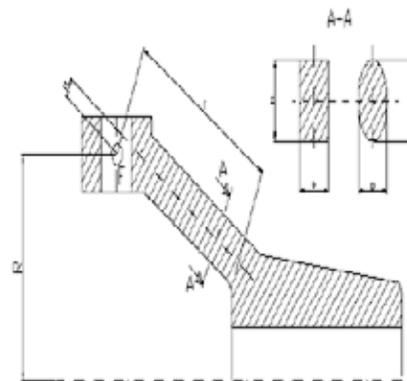


Figure 2. Calculus scheme of the cardan joint's fork

The dangerous cross-section A-A is subjected to bending and torsion. The force  $F$  that acts on each arm of the fork is given by the formula:

$$F = \frac{M_c}{2 \cdot R} = \frac{300 \cdot 10^3}{2 \cdot 27,7} = 5,42 \text{ kN}$$

where:  $M_c$  is the calculus moment of the cardan transmission,  $R$  - average radius at which the force  $F$  acts.

The normal bending stress in the cross-section A-A is:

$$\sigma_i = \frac{M_i}{w_i} = \frac{F \cdot l}{w_i} = \frac{5,42 \cdot 10^3 \cdot 32}{3528} = 49,13 \text{ MPa}$$

where, for an elliptical cross-section:

$$w_i = \frac{b \cdot h^2}{10} = \frac{20 \cdot 42^2}{10} = 3528 \text{ mm}^3$$

The fork's arm, in the cross-section A-A, is subjected to the force  $F$ , to torsion:

$$\tau_i = \frac{M_t}{w_i} = \frac{5,42 \cdot 10^3 \cdot 7}{3360} = 11,28 \text{ MPa}$$

where  $w_t = \frac{\pi b^2 h}{16} \approx 0,2 \cdot 20^2 \cdot 42 = 3360 \text{ mm}^3$

for an elliptical cross-section.

**STUDY THROUGH THE FINITE ELEMENT METHOD – STATICAL ANALYSIS APPLIED TO THE CARDAN SHAFT**

Currently, the problem of geometrical modelling can be tackled using computer-aided design software packages or modules incorporated in software for analysis with finite elements for computer-aided design. Such a software for analysis through the finite element method is Cosmos, a product incorporated in the Solidworks package, that, due to the facilities and the accuracy of results it offers, is often used in the study of the static and dynamic behaviour of component elements of technological systems. In fact, the Solidworks software is used for the geometrical modelling of component elements of cardan transmissions and for their assembling, while Cosmos, based on the geometry taken from Solidworks, generates the finite elements network, idealises the contacts between the component parts and allows the application of constraints and loads. The software Cosmos has a set of modules dedicated to certain domains, such as: the structures analysis in general, fluid mechanics, thermal analysis. Each module, in turn, has a complete set of analyses for linear or nonlinear, static or dynamic problems, which help to unfold a complete research.

For the current research, there has been chosen the cardan transmission of the cardan transmission of a Romanian-manufactured Dacia car, the researches being easily transposable to cars with other typodimensions of cardan transmissions as well.

As mentioned, the geometrical modelling was realised using the Solidworks software. This solution was chosen due to the problems that arise when transferring a model, saved in IGS format, from other computer-aided design softwares into the Cosmos software.



**Figure 3 Assembly of the cardan transmission modelled with Solidworks**

In order to obtain a model with a behaviour as close as possible to the real one and to achieve a processing time that is as short as possible, insignificant details (fillets with small radiuses or niches) were eliminated and the nonhomogeneous areas on the structure were approximated with homogeneous finite elements. This allowed the obtaining of an uniform finite elements network as shown in figure 4.

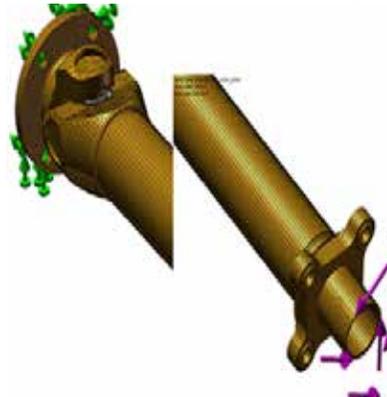
The static analysis aims to determine the stresses and strains state at the model's loading in static regime. This analysis was applied to the parametrised model of the assembly of the cardan transmission of the Dacia car, for two cases: construction with

closed grooved transmission and construction with open grooved transmission, respectively. The static behaviour was studied in both cases, in order to compare the results from the finite elements analysis for the structural elements of cardan transmissions.



**Figure 4. The discretised model of the cardan transmission**

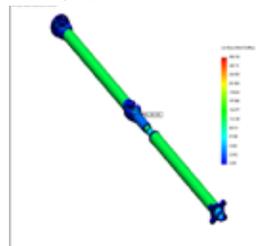
For the unfolding of the static analysis on the end towards the differential, on the flanged fork there are applied constraints by cancelling the degrees of liberty, so that this end is fixed. At the end located in the actuation area, there is applied a constant moment with a value of 300 Nm. The positioning of constraints and loads



can be seen in figures 5.

**Figure 5. Application of the constraints on the cardan transmission**

In the following there are presented the variation graphs of the equivalent von Mises stress, of the principal stresses  $\sigma_1, \sigma_2, \sigma_3$ , the variation graphs of the equivalent strain, of the principal strains  $\epsilon_1, \epsilon_2, \epsilon_3$ , the graphs of the resulting nodal displacements and the graph of the safety coefficient for the cardan shaft.



**Figure 6. Variation graphs of the equivalent von Mises stress – closed transmission**

From the analysis of the maps of the equivalent von Mises stress for the whole assembly of the cardan transmission, it can be noticed that the equivalent von Mises stress in the shaft is 141 MPa.

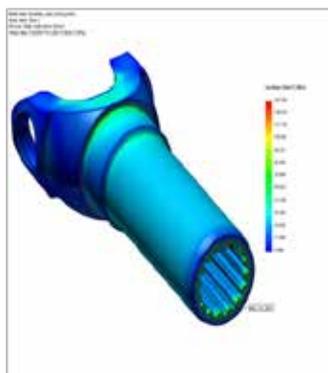


Figure 7. Variation graphs of the equivalent von Mises stress – fork with shaft ( $\sigma_{VM}=141.28$  MPa)

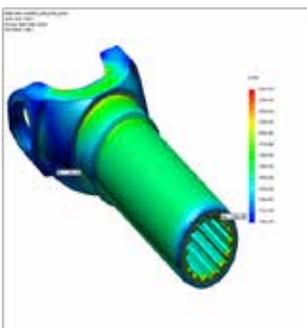


Figure 8. Variation graph for the equivalent specific strain - fork with shaft, closed transmission ( $\epsilon_{VM}=3.094 \cdot 10^{-4}$ )

**VERIFYING THE SPECIFIC STRAINS THROUGH RESISTIVE ELECTRIC TENSOMETRY (RET) OF THE CARDAN SHAFT**

The determination of the stress and strain state in a point on the surface of a structure with the help of tensometric techniques is based, usually, on the transformation of the variation of the specific strain in that point in the variation of an electric parameter (the voltage), by means of a circuit element called a transducer.

The tensometric techniques using capacitive transducers and semiconductors allow the determining of even very small strains, with a high degree of accuracy. However, these transducers are very expensive and the apparatus needed for this technique is sophisticated, so that it is used only rarely. Inductive electric tensometry is used generally for measuring the displacements of strength elements.

The resistive electric tensometry is the most used technique for determining the strain state in a point. It allows the measuring of specific strains of the order of 10-6 μm/m, with a high accuracy. The transducers used for this technique are simple and rather cheap, compared to the other, above-mentioned transducers, while the measurement apparatus is not very complicated. The electro-resistive transducer consists of a thin wire or foil with a diameter or thickness of 0.015...0.02 mm, made of materials with the electrical resistivity of  $R = 50 \dots 1000 \Omega$ . This wire or foil is placed as a grid incorporated into a support made of special paper or into a foil made of a phenolic resin.[3]

**APPLYING THE RESISTIVE ELECTRICAL TRANSDUCERS**

The area on the structure’s surface where the resistive electrical transducers will be applied has to be cleaned thoroughly with a wire brush or with abrasive paper until obtaining a shiny

surface. In order to remove possible traces of rust, fats etc., the surfaces are also cleaned with acetone and then with alcohol, so that the cleaning motion is in a single direction, after which they are wiped with a neutralising solution. The points in which the transducers are to be applied are marked by two perpendicular axes. A thin layer of adhesive is applied to the transducer, then this is placed on the structure and pressed for several minutes until the transducer is perfectly fastened.

It has been determined experimentally that the relative variation of the resistivity is proportional with the variation of the specific lengthening, i.e.:

$$\frac{\Delta R}{R} = k \cdot \frac{\Delta L}{L} = k \cdot \epsilon \tag{1}$$

**The tensometric bridge**

The most important element for measuring specific strains using the method of electrical resistive tensometry is the Wheatstone bridge. In the classical configuration, this bridge has four arms on which resistors are fastened. On the arm AB there is fastened the measuring transducer (active transducer) with resistance R1, on each of the arms AD and CD there is fastened a calibrated resistance R2 and R3, respectively, while on the arm BC there is fastened a resistor R4 with variable resistance (potentiometer). The bridge is supplied with power on the diagonal, between the nodes A and C from a direct current source.

The specific strain is determined using the relation:

$$\epsilon = \frac{2}{k} \cdot \frac{\Delta i}{S} \tag{2}$$

where:

- k – transducer constant (indicated by the manufacturer);
  - $\Delta i = i_1 - i_0$  – difference between the values indicated by the bridge instrument for the charge values  $P=P_1$  and  $P=P_0$ ;
  - S – sensitivity factor of the bridge.
- If using tensometric bridges that are calibrated by the manufacturer for  $k=2$ , the relation becomes:

$$\epsilon = \frac{\Delta i}{S} \tag{3}$$

The measured value,  $i$ , is read in μm/m (10-6 mm/mm), so in calculations it has to be multiplied by 10-6 to obtain the strain in m/m.

**THE DATA ACQUISITION AND PROCESSING SYSTEM**

In order to improve the knowledge related to the static and dynamic behavior of cardan transmissions in general and especially of cardan shafts, in the following there are presented some experimental researches carried out by the author. The researches aimed to validate experimentally earlier theoretical researches carried out on finite element models of the cardan transmission assembly of a Dacia car. [6]

The objectives of the experimental researches were selected based on the current state of the knowledge in this domain and on some practical considerations related to the applicability of the research results. Thus, the main objective for the presented researches is the analysis of the static behaviour of the cardan cross from the assembly of the cardan transmission.

The research method used for the experimental researches was the method of electrical resistive tensometry.

**REALISING OF THE EXPERIMENTAL TESTS**

The experiments were carried out in the laboratories of the Faculty of Engineering of Sibiu, using an experimental setup installed on an INSTRON 4303 tensile, compression and buckling

testing machine (figure 9). This testing machine allows both a control of the load applied on the cardan cross and the control of the displacement (maximal flexure).

The planning and management of the experimental researches was done observing the currently valid standards. The methods used for the acquisition and statistical processing of the experimental data are those consecrated through Romanian and international standards.

The experimental setup consists of the force introduction system, the measurement transducers and the data acquisition system (figure 10).

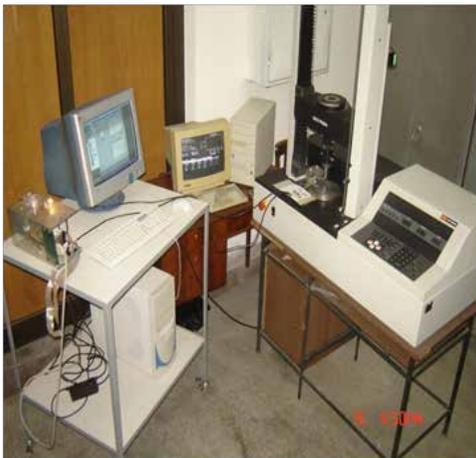


Figure 9. Overview on the experimental setup installed on the INSTRON 4303 machine

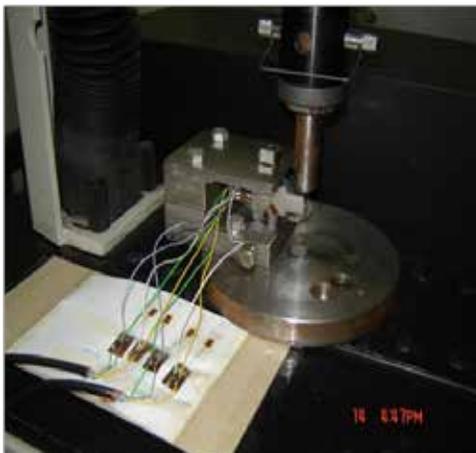


Figure 9. Detail of the experimental setup installed on the INSTRON 4303 machine



Figure 10. The cardan shaft before the experimental analysis

The elaborated experimental research methodology aimed at validating the theoretical results obtained by numerical analysis using the finite element method, both with regard to the static analysis and with regard to the dynamic one.

**THE DATA ACQUISITION AND PROCESSING SYSTEM**

The data acquisition system, presented in figure 11 consists of four modules: the transducers attached to the analysed directions of the cardan cross, the signal conditioning modules (MB-38 produced by Keithley Instruments Inc.), the analog-digital conversion device (KPCI 3108, Keithley Instruments Inc.) and a software package that controls the acquisition system and processes the acquired data. The two transducers are connected and supplied with stabilized electrical power from the analog-digital conversion device, by means of the conditioning modules. These modules provide the selection of the signal amplification factor, of the measurement domain and the filtering of the acquired signal. [7]

The analog-digital conversion device has an acquisition capacity on 16 channels with a cumulated sampling frequency of 100 ksamples/sec. The maximal transfer rate is obtained in the case of the connection on the PCI bus and is of 132 MB/sec. The obtained signals are amplified, processed and then transmitted on the data bus to the computer motherboard.

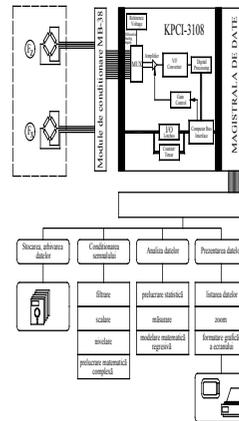


Figure 11. Block scheme of the experimental data acquisition system

The data are processed, archived, analysed and presented using the specialized software package TestPoint, produced by Keithley Instruments Inc. Beneath the archiving of the collected data, the software also realises a conditioning of the signal (filtering, scaling), an analysis of the data (statistical processing, measurement) as well as a graphical presentation of the data on a display or on paper using a printer. The collected data are stored on magnetic memory devices.

The wide-band input module for tensometric stamps acquires data from bridges and halfbridges with resistances comprised between 300 Ω and 10 kΩ. The module provides an excitation voltage of +10 V and at the output a voltage comprised between -5 V and 5 V. The frequency band of this module is 10 kHz.

The galvanic separation is done by means of a transformer that uses an own modulation technology in order to provide a linear and stable behaviour. At the output of the transformer, a demodulating circuit restores the initial signal that is then filtered and rectified in order to provide a clean, low impedance output signal. A serial output switch is also included in the module's structure in order to eliminate the need for an external multiplexing circuit (for applications in which data acquisition is done on several acquisition channels simultaneously). This switch is con-

trolled by an active-low type input signal (a impulse train voltage signal, the switch's activation is done on the area 0 logic of the voltage impulse, this corresponding usually to a voltage level comprised between 0-2.5 V). If the switch is not used, its control signal must be connected to the common grounding of the power lines. The +5 V voltage feeds a clock-type oscillator which controls the functioning of the transformer.

In order to measure the specific strains of the cardan shaft, there have been applied two HBM 350XY11 transducers on each side of the shaft.

The place for applying the tensometric stamps was determined following the observations made, based on the results of the static and dynamic analyses through the finite element method. The application position of the transducers was determined so as to correspond with the position of two component elements of the finite element model.

The experimental determination of the specific strain the two directions, was done by applying a specific formula function of the employed circuitry (figure 12), namely for a full bridge structure; (1)

$$\epsilon = \frac{-4V_r}{E [(1 + \nu) - V_r(\nu - 1)]}$$

The signal acquired from the analog-digital device KPCI 3108 is processed, filtered and saved by means of two virtual instruments created by the author in the software TestPoint, software that accompanies the acquisition device and is dedicated for data acquisition.

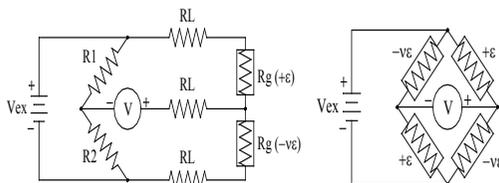


Figure 12. The fastening of tensometric transducers HBM 350XY11

The instruments contain blocks provided to the used by the software, allowing the modifying of the number of channels on which the acquisition is done, of the acquisition rate and of the total duration of the acquisition, as well as the filtering of data in order to eliminate the “noise” inherent to any acquisition and the saving of the data as text files.

One instrument allows the acquisition and visualization of the data, while the other one allows the filtering and saving of data as text (ASCII) files.

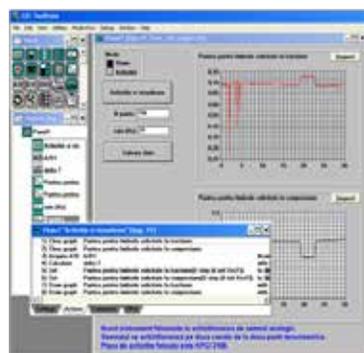


Figure 13. Virtual instrument used for the acquisition of the specific strains on two directions

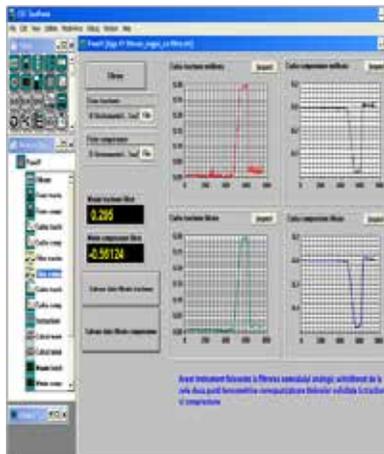


Figure 14. Virtual instrument used for filtering the electric signal

Figure 15 presents the loading curves at the INSTRON 4303 tensile, compression and buckling testing machine for the specific case of the cardan shaft and for the maximal value of 20 kN.

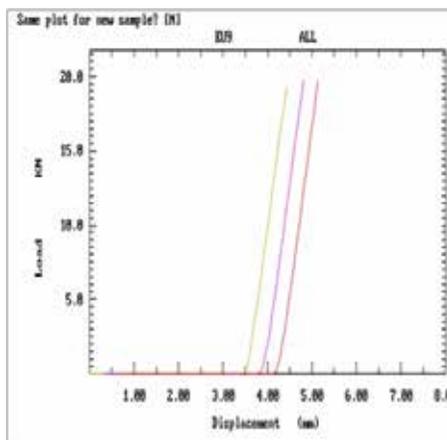


Figure 15. Loading curve for the cardan shaft for maximal loads of 20 kN

In order to determine the values of the two considered characteristics in each point of the experimental programme, there have been used the readings of the values of the maximal forces developed during the active run.

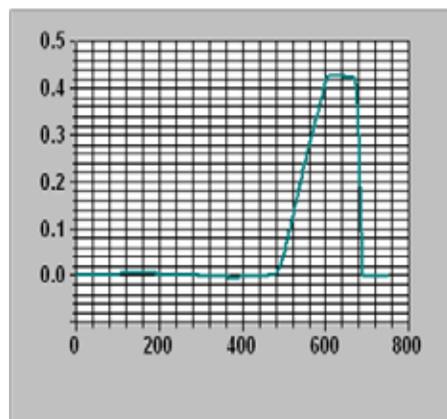


Figure 16. The variation graph for the tensile strain for the cardan shaft for a load of 20 kN [mV/V]

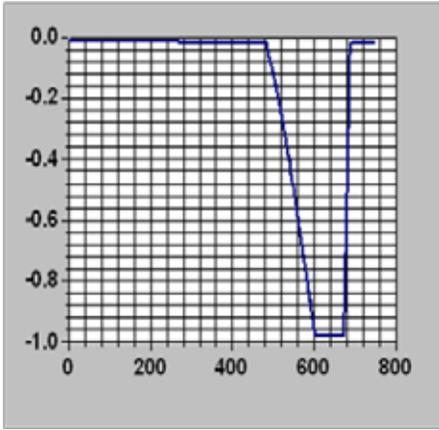


Figure 17. The variation graph for the compression strain for the cardan shaft for a load of 20 kN[mV/V]

TABLE – 1  
Transformation of the results from mV/V into strains

Curr. no.	F[kN]	$\epsilon_x$ [mm/mm]	$\epsilon_y$ [mm/mm]
1.	20.1980198	0.017385938	-0.0172138
2.	20.5049505	0.017650136	-0.017475382
3.	20.65346535	0.017777974	-0.017601954
Average	20.45215	0.017604683	-0.01743019

**RAPID PROTOTYPING OF THE CARDAN SHAFT OPTIMISED BASED ON THE MINIMAL STRESSES**

Until the emergence of the technologies described in this chapter, the most efficient way to physically create objects was the numerical control of technological processes, which brings the potential of using directly, during the CAM phase, the geometrical information generated in the CAD phase. It should be noted, however, that this technology cannot provide a higher degree of integrating the CAD and CAM phases due to the need to go through intermediate stages for obtaining the technological information and the information regarding the involved tools.

The new technologies represent another way for the geometric data generated in the CAD phase to be used directly in the manufacturing phase. These technologies are based on the availability of 3D modelling using solids and information on the geometries realised during the CAD phase in order to physically obtain the desired objects in an easy and highly efficient manner. These technologies are called Rapid Prototyping (RP).

**REALISING THE CARDAN SHAFT USING THE ZPRINTER 450 EQUIPMENT**

The concept of Rapid Prototyping refers to a type of processing technology which allows the direct obtaining of a physical model created using CAD – Computer Aided Design.

The 3D realising of the model is an excellent visual support for the communication between members of the product team or between the product team and customers. The model is realised fast and with relatively few materials, being a great help in testing, checking and accepting a prototype prior to launching the object into production.



Figure 18. ZPrinter 450 Plus

The RP technology for constructing the 3D model is basically a 3D printing technology: in a first step, the roll in the printer head makes a left-right motion in order to collect the powder from a container. Then, in step 2, this powder is deposited in a layer in another container, in the model creation area. In step 3, the excess powder is removed to a third container. In the next step, the printer head brings the binding material onto the powder's surface and realises the material's hardening in the model's geometry area. Then in step 5, the piston in the material container is raised so that the roll can bring in a new layer in the work area, while the piston in the model container descends by a layer.

This technology allows the realising of 3D models with complex geometrical configurations, models from various activity domains: prototypes for the industry, models for medicine, consumer goods, automotive parts, in education, architecture etc. Figure 20-21 presents other models realising in SmartLab.

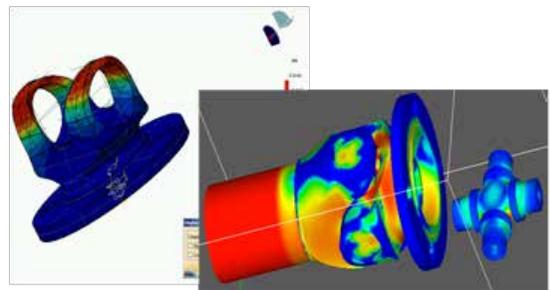


Figure 19. Image of the cardan transmission transferred into the machine's specific software

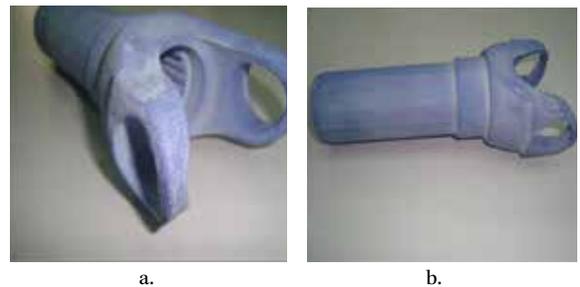


Figure 20. Realising the model of the cardan fork with shaft



Figure 21. Realising the subassembly of the cardan joint

The advantages of rapid prototyping are as follows:

- it allows the physical realising of a model, regardless of its shape (however, it does not allow the realising of large models, due to the dimensions of the work area);
- it allows the fast realising of models obtained through optimisation with the finite elements method;
- it allows the "freezing" of the stresses and strains state and transferring it on physical models.

## CONCLUSIONS

From the variation graphs of the strains on the two directions there can be noticed on the one hand a linearity of the experimental results and on the other hand a good agreement with the results of the simulations using the finite element method. Thus, in the least favourable case, the error percentage compared to the numerical simulations is around %.

The specific equivalent strain obtained experimentally after applying a torque of 300 Nm is of 0.0066 mm, while the specific strain obtained using FEM in the same study area is of 0.0073 mm.

The modern computer-aided design software packages offer very good results if the input data are accurate, for this the characteristic data for each material having to be determined experimentally.

## PROPOSALS

Recommendations from the author for any designer of cardan transmissions would include to take into account the optimal dimensional values obtained for decreasing the stresses and strains in the main component elements of the cardan transmission. Since the optimised values are not functional, the connections between the component elements of cardan transmissions do not have to be verified.

The manufacturer of cardan transmissions should take into account the modified part drawings from the designer and should modify the dies and tools according to these drawings.

Also, it is important for the maintenance personnel to take into consideration the maintenance schedule for the cardan transmissions.

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