Research Paper

Volume : 3 | Issue : 10 | October 2014 • ISSN No 2277 - 8179

Construction of Partially Balanced Incomplete Block (Pbib) Designs Using Factorial Treatment Combinations

Statistics

KEYWORDS : Block designs, PBIB designs, factors, levels

Dr.Davinder Kumar Garg	Associate Professor and Head, Department of Statistics, Punjabi University, Patiala.
Syed Aadil Farooq	Research Scholar, Department of Statistics, Punjabi University, Patiala

ABSTRACT

In this paper, we have constructed two series of two and nth (n=2,3,...) associate class partially balanced incomplete block (PBIB) designs by establishing a link between PBIB designs and factorial treatment combinations. Some numerical illustrations of construction of such type of designs are also discussed in detail. Efficiencies of the newly constructed PBIB designs are also computed for the purpose of comparisons. During the study, it was found that some designs are new, and some newly constructed designs are more efficient as compared to the existing designs.

1. Introduction:

Factorial experiments have their origin in agriculture, but they have wide applications in many fields like biology, chemistry, animal husbandry, industry and so on. Factorial experiments have more precision on each factor than with a single factor. They are broadening the scope of an experiment (e.g. breeds of animals). They are good for work where we wish to find the most important factor or the optimal level of a factor (or combination of levels of more than one factor). In factorial experiments, it is possible to estimate interaction effect. It is well known that 'statistical designing' is indispensable because only data collected from a properly 'designed' experiment can lead to statistically valid conclusion about the objectives of the experiment. A factorial design experiment is an experiment whose designs consists of two or more factors, each with discrete possible "values" or "levels" and whose experimental units take on all possible combinations of these levels across all such factors. A factorial design may also be called a crossed design. Such an experiment allows the investigator to study the effect of each factor on the response variable, as well as the effects of interactions between factors on the response variable.

In this paper, using factorial treatment combinations Thannippara.A [2007] who introduced relatively easy methods for constructing hyper cubic designs from symmetrical experiments for $t=v^m$ treatments with v=2,3. Bose [1947] made pioneering contribution in several areas. His classical work in mathematical theory of symmetrical factorial experiments has revolutionized the field. He was the first to establish a link between the geometry of finite fields and combinatorial problems in the construction of designs. Mohan.*et.al* [2006a], Mohan.*et.al* [2006b] have constructed two and higher associate class PBIB designs, substituting even number by 1 and odd number by -1 using certain magnificent (M_n) matrices and its applications. Recently, Garg and Gurinder [2014] have constructed higher associate class partially balanced incomplete block (PBIB) designs by using Factorial combinations.

2. Some definitions:

2.1 Factor: A kind of treatment. Each factor will have two or more levels. Factors and their levels generate the treatment combinations.

2.2 Symmetrical Factorial Experiment:

Experiment in which number of levels of all factors are same i.e all s_i's are equal are called symmetrical factorial experiment and the experiments in which at least two of the s_i's are different are called as asymmetrical factorial experiments.

2.3 Incidence Matrix: Let $N = (n_{ij})_{v \times b}$ is the treatment-block incidence matrix, where $N = (n_{ij})$, (i=1,2,3,…,v; j=1,3,3,…,b) is the number of times the i^{th} treatment appears in the j^{th} block.

$$N = \begin{bmatrix} n_{11} & n_{12} & \dots & n_{1b} \\ n_{21} & n_{22} & \dots & n_{2b} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ n_{v1} & n_{v2} & \dots & n_{vb} \end{bmatrix}$$

Where n_{ij} = 1, if i^{th} treatment is present in the j^{th} block

= 0, otherwise

3. Propositions

3.1 Proposition: The existence of square matrix of order $s\times s$, where s is an even number $(s>4)$ implies the existence of PBIB design with parameters of first kind $v = s = b$, $k = (s+2)/2 = r$, $\lambda_1 = 2$, $\lambda_2 = s/2$ and parameters of second type $n_1 = s/2$, $n_2 = (s-2)/2$.

3.2 Proposition: The existence of square matrix of order $t\times t$, where t is an odd number $(t>3)$, implies the existence of PBIB design with parameters of first kind $v = t = b$, $k = (t+1)/2 = r$, λ_i vary from 1 to $(t-1)/2$ and parameters of second type n_i's and p^k_{ij} cannot be obtained theoretically as it is difficult, but can be evaluated in numerical problems.

4. Association scheme of new and n^{th} associate class partially balanced incomplete block designs: In this section, association scheme of new and n^{th} associate class partially balanced incomplete block designs are defined as follows:

4.1 Association scheme: In this association scheme, we have total number of treatments and blocks as v = s and b = s respectively and every treatment repeats exactly $r=(s+2)/2$ times in the blocks. Let us consider those blocks which have exactly one treatment in common namely as Θ. On these blocks, we define a two associate class association scheme as follows: Take any two treatments say Θ and Φ from these blocks then, these treatments are first associates of each other, if they occur exactly λ_1 = 2 times, and if these treatments occurs exactly $\lambda_1 = s/2$ times, they are termed as second associates of each other. The parameters of two associate class association schemes are as follows:

$v = s = b$, $r = (s+2/2) = k$, $n_1 = s/2$, $n_2 = (s-2)/2$, $\lambda_1 = 2$, $\lambda_2 = s/2$ and the P- matrices are

$$P_1 = \begin{bmatrix} 0 & (s-2)/2 \\ (s-2)/2 & 0 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} s/2 & 0 \\ 0 & (s-4)/2 \end{bmatrix}$$

Where s > 4

4.2 Association scheme: In this association scheme, we have total number of treatments and blocks as v = t and b = t respectively and every treatment repeats exactly $r=$

Volume : 3 | Issue : 10 | October 2014 • ISSN No 2277 - 8179

(t+1)/2 times in the blocks. When we consider matrix 'M' as an incidence matrix then it becomes a PBIBD design in which there are 't' rows and 't' columns .We consider t columns/rows as blocks of the new designs.

The association scheme of the newly constructed designs will be as follows:

If two treatments occurs together 1, 2, . . . , n times in the blocks of the new design, then they are said to be Ist, 2st, 3rd,. . ., nth associates respectively.

These parameters satisfy all the conditions which are necessary for the existence of an association scheme.

5. Construction Method of two and nth associate class PBIB Designs

In this section we construct the following two associate and nth associate class PBIB designs in even and odd cases:

5.1 Even cases

For the construction of two associate class PBIB designs, we consider a factorial experimental design s^n, s is an even number *(s>4)* and $n=2$ are the factors, each varied at's' levels. It has s^n treatment combinations and is called an s^n experiment. The power*' n'* represents the number of factors and base represents the level of each factors. The levels of s^n factorial design represents treatments *i.e.* $v=s$. Firstly, a square matrix of order $s\times s$. The levels are represented by 1,2 3. . . *s*. In this, method of construction is based on grouping of treatment combinations of 's' levels, take all possible combinations of treatment (1,2,3. . . s) with rest of the treatments, then putting in the ist, 2nd ,…sth rows respectively of the $s\times s$ square matrix, which occurs only once in each group G_i (i=1,2,3…*s*). Replace levels with same treatment combinations by 1 *i.e 11, 22, 33. . . ss* . If sum of treatments is even, replace it by 0. If their sum is odd, then replace it by 1. The positions in the matrix so obtained, in which we get 1 are considered as blocks in the respective rows/columns. Let us name this matrix N as incidence matrix which yields a two associate class PBIB designs with parameters

$v=s=b$, $r = (s+2)/2 = k$, $\lambda_1 = 2$, $\lambda_2 = s/2$, $n_1 = s/2$, $n_2 = (s-2)/2$ which follows association scheme defined in section 4.1.

The number of elements in the blocks represents the block size, $k = (s+2)/2$. Also we see that treatment replicated in 'r' blocks and $r = (s+2)/2$. In the resulting incidence matrix, it is also observed that this is an $s\times s$ symmetric matrix.

5.2 Odd cases

In this we consider a new factorial experimental design t^n, t is an odd number (>3) and n = 2 are the factors, each varied at't' levels. It has t^n treatment combinations and is called an t^n experiment. The power' n' represents the number of factors and base represents the level of each factors. The levels of t^n factorial designs represents treatments v = t. The levels are represented by (1, 2, 3. . . t). Firstly, a square matrix of order t×t, take all possible combinations of treatment 1 with rest of the treatments, then put in the Ist row of the $t\times t$ square matrix. Second possible combinations of treatment 2 with rest of the treatments, then put in the 2nd row of the $t\times t$ square matrix respectively and so on. By doing this, then we take sum of treatment combinations individually, in every row and every column of $t\times t$ matrix, if reduce it by 't'. In resulting matrix, those elements which get divided by t and are odd, sum of treatment is greater than ‘t’ then take them as 1, those which are even take them as 0. Then the positions in the matrix so obtained in which we get 1 are considered as blocks in the respective rows/columns. Let us name this matrix M as incidence matrix. Treating v =‘t’ levels as treatments and rows/columns as blocks which yields PBIB designs with parameters

$v = t = b$, $k = (t+1)/2 = r$, λ_i vary from 1 to $(t-1)/2$ and is a positive integer. Also parameters of second type n_i's and p^k_{ij} cannot be obtained theoretically as it is difficult to find them, but can be evaluated in numerical problems along with P-matrices which follows association scheme defined in section 4.2. The number of elements in the blocks represents the block size and $k = (t+1)/2$. In the resulting incidence matrix, it is also observed that this is a $t \times t$ symmetric matrix.

6 Illustrations:

Illustration 6.1: As per our construction methodology let us consider 6^2 Factorial experiments with levels S = {1, 2, 3, 4, 5, 6}. Let the six groups be G_1 = [11, 12, 13, 14, 15, 16], G_2 = [21, 22, 23, 24, 25, 26], G_3 = [31, 32, 33, 34, 35, 36], G_4 = [41, 42, 43, 44, 45, 46], G_5 = [51, 52, 53, 54, 55, 56], G_6 = [61, 62, 63, 64, 65, 66]. Then, placing these groups in the i^{st}, 2^{nd}…6^{th} row of the 6×6 square matrix. Replace levels with same treatment combinations by 1. If sum of treatments is even, replace it by 0. If their sum is odd, then replace it by 1. Then the positions in which we get 1 in the respective row/column are considered as blocks. Treating these rows/columns as blocks, then we see that treatment replicated once in a block. We get matrix say N is as follows:

$$N = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Consider the above matrix N as incidence matrix then it will generate a two associate class PBIB design with parameters v= 6 = b, r = 4 = k, $\lambda_1 = 2$, $\lambda_2 = 3$, $n_1 = 3$, $n_2 = 2$ and follows the association scheme defined as ***4.1.***

P-matrices of the new association scheme are given by

$$P_1 = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$

Illustration 6.2: As per our construction let us consider 5^2 Factorial experiments with levels S = {1, 2, 3, 4, 5}. Let the five groups be G_1 = [11, 12, 13, 14, 15], G2 = [21, 22, 23, 24, 25], G3 = [31, 32, 33, 34, 35], G_4 =

Volume : 3 | Issue : 10 | October 2014 • ISSN No 2277 - 8179

[41, 42, 43, 44, 45], G5 = [51, 52, 53, 54,

$$M = \begin{bmatrix} 11 & 12 & 13 & 14 & 15 \\ 21 & 22 & 23 & 24 & 25 \\ 31 & 32 & 33 & 34 & 35 \\ 41 & 42 & 43 & 44 & 45 \\ 51 & 52 & 53 & 54 & 55 \end{bmatrix}$$

We take sum of treatment combinations individually, in every row and every column of 5×5 matrix, if sum is greater than 5 then reduce it by 5 we get

$$M = \begin{bmatrix} 2 & 3 & 4 & 5 & 1 \\ 3 & 4 & 5 & 1 & 2 \\ 4 & 5 & 1 & 2 & 3 \\ 5 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

Those elements in the 5×5, which get divided by 5 and are odd, take them as 1, those which are even take them as 0. We get matrix say M as follows

$$M = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

P-matrices of the new association scheme are given by

Then the positions in which we get 1 of M matrix are considered as blocks in the respective rows/columns. Consider the above M matrix as incidence matrix then it will generate a two associate class PBIB design with parameters

$v = 5 = b$, $r = 3 = k$, $\lambda_1 = 1$, $\lambda_2 = 2$, $n_1 = 2$, $n_2 = 2$ and follows the association scheme defined as ***4.2.***

7. Conclusion:

In this paper, we have constructed two new series by establishing a link between PBIB designs and factorial treatment combinations and as a result, we get new PBIB designs with two and n^{th} associate class PBIB designs. Efficiencies of the new designs are also computed for the purpose of comparison. Some designs are new and some newly constructed designs are more efficient, as compared to the existing designs with same parameters as listed in the tables of Clatworthy [1973].

$$P_1 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

8. List of designs:

Here, we enlisted some new designs for different values of parameters along with their efficiencies which are given in table 8.1 and 8.2

Table-8.1

Table of two associate class PBIB designs (s>4) for even cases

S .no	s=v	b	r	k	λ_1	λ_2	n_1	n_2	E_1	E_2	E
1	6	6	4	4	2	3	3	2	0.8654	0.9375	0.8929
2	8	8	5	5	2	4	4	3	0.8533	0.9600	0.9111
3	10	10	6	6	2	5	5	4	0.8454	0.9722	0.8974
4	12	12	7	7	2	6	6	5	0.8396	0.9795	0.8979
5	14	14	8	8	2	7	7	6	0.8085	0.9843	0.8812
6	16	16	9	9	2	8	8	7	0.8317	o.9876	0.8978
7	18	18	10	10	2	9	9	8	0.8288	0.9900	0.8976

Volume : 3 | Issue : 10 | October 2014 • ISSN No 2277 - 8179

Table-8.2

Table of series of n associate class PBIB designs (s>3) for odd cases and λ_i is a positive integer and vary from 1 to (t-1)/2

Sr.No.	v	b	r	k	λ_1	λ_2	λ_3	λ_4	λ_5,	.	.	.. λ_t
1.	5	5	3	3	1	2						
2.	7	7	4	4	1	2	3					
3.	9	9	5	5	1	2	3	4				
4.	11	11	6	6	1	2	3	4	5			
5.	13	13	7	7	1	2	3	4	5	6		
.												
.												
.												
n.	t	t	*(t+1)/2*	*(t+1)/2*	1	2	3	4	5	6,	. .	. , *(t-1)/2*

REFERENCE

Thannippara.A, A et.al (2007). Hyper cubic designs and applications. Statistical papers | 48, 503-508. | [2] Masumi, B. (2003). The Mathematics of Symmetrical Factorial Designs. Resonance 8(9) | 14- 19. | [3] Mohan, R.N., & Kageyama, S., Moon Ho Lee, & Yang, G.(2006a). Certain new M | matrices and their properties and applications. Submitted to Linear Algebra and | Applications released as e print, arXiv:Math cs/ 0604035 Dt. April 6,2009 | [4] Mohan, R.N., Kageyama, S., Moon Ho Lee, & Yang,G.(2006b) .Certain new M matrices | and their properties and applications. Submitted to Linear Algebra and Applications | released as e print,arXiv:Math cs/ 0604035 Dt. June 6,2009. | [5] Garg and Gurinder (2014).Construction of Higher associate class partially balanced | incomplete block (PBIB) designs using Factorial Combinations. Accepted for publication in | Journal of Probability and Statistical Science (JPSS).Manuscript # 073113. |