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Research Paper

Comparitive Study of Various Methods for Solving Transportation Problem

Mathematics

KEYWORDS : Transportation Problem, Optimal Solution, Degeneracy, Sources, Destinations, Reliable

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ABSTRACT

In this paper 'Various method are compared for finding an optimal solution. The methods are NWC, LCM, VAM, NMD and Zero Suffix Method all gives an optimal solution. In this paper the best optimality condition has been checked. Numerical examples are provided to illustrate the condition.. The Simplex method is not suitable for the Transportation Problem especially for large Scale transportation problem due to its special Structure of the model in 1954 Charnes and Cooper was developed Stepping Stone method for the efficiency reason. An Initial Basic Feasible Solution(IBFS) for the transportation problem can be obtained by using the north-west corner rule row minima, Coiumn minima, Matrix minima and Vogel's Approximation Method [Reinfeld and Vogel 1958,Goyal"s version of VAM ..Kirca and Stair developed a heuristic method to obtain an efficient initial basic feasible solution. Sudhakar etal proposed zero suffix method for finding an optimal solution for transportation problem directly in 2012

INTRODUCTION:

Transportation problem is a special class of Linear Programming Problem which deals with the distribution of single commodity from various Sources of supply to various destination of demand in such manner that the total transportation cost is minimized..Usually for the initial allocation in the case of a transportation problem methods such as North-West corner Method (NWCM) , Least Cost Method (LCM) , Vogel's Approximation Method (VAM), NMD and Zero Suffix Method are used. Finally the purpose for optimality MODI check is carried out. This Problem was first presented in 1941 by Hitchcock and it was further developed Koopmans (1949) and Dantzig (1951). The Simplex method is not suitable for the Transportation Problem especially for large Scale transportation problem due to its special Structure of the model in 1954 Charnes and Cooper was developed Stepping Stone method for the efficiency reason. An Initial Basic Feasible Solution(IBFS) for the transportation problem can be obtained by using the north-west corner rule row minima, Coiumn minima, Matrix minima and Vogel's Approximation Method [Reinfeld and Vogel 1958,Goyal"s version of VAM ..Kirca and Stair developed a heuristic method to obtain an efficient initial basic feasible solution. Sudhakar etal proposed zero suffix method for finding an optimal solution for transportation problem directly in 2012.This paper presents a new simple approach to solve the transportation problem. Even in the above mentioned method needs more iteration to arrive optimal solution. The algorithm of the approach is detailed in the paper and finally numerical examples are given to illustrate the approach and minimized cost comparison table is given. Paper is organized as follows: Mathematical Representation and degeneracy in section first, Various methods is summarized in the second section .In third section, comparison has been done with Numerical examples of the transportation problem and results are clarified in fourth section. Finally the beat optimality is discussed. In the last section conclusion is discussed.

1. Mathematical Representation

Let there be m origins oi having $ai(i=1,2,.....,m)$ units of Source respectively which are to be transported to n destinations $Dj's$ with $bj(j=1,2,.....,n)$ units of demand respectively .Let Cij be the cost of source one unit of commodity from origin i to destination j. If xij represents the units source from origin i to destination j then problem is to determine the transportation Schedule so as to minimize the total transportation Cost satisfying supply and demand condition.

Mathematically the problem can be stated as

Minimize $z= C_{ij} n_j=1 \ \ m_i=1 x_{ij}$

Subject to $xijnj=1=ai \ for \ i=1,2,.....,m$ (supply constraints and $xijmi=1=bj \ forj=1,2,.....,n$(demand constraints) and

$xi\,j \geq 0 \ for \ all \ i \ and \ j.$

A transportation problem is said to be balanced if the total supply from all sources equals the total demand in all destinations $aimi=1= bjnj=1$ otherwise it is called Unbalanced.

Table 1.1: Transportation table

Origins (i)	Destination(j)				SUPPLY(ai)
	1	2	....	n	
1	X_{11} C11	X_{12} C12	....	C1n	a1
2	C21	C22	....	C2n	a2
3	C31	C32	....	C3n	a3
....	....	....	....	....	....
M	Cm1	Cm2	....	Cmn	am
Demand (bj)	b1	b2	....	b n	Σai = Σb j

1.2 Basic Definitions The following terms are to be defined with reference to the transportation problem.

A) Feasible Solution (F.S)

A set of non negative allocation $xij \geq 0$ which satisfies the row and column restriction is known as Feasible Solution

B) Basic Feasible Solution (BFS)

A feasible solution to a $m-$ origins and $n-$destination problem is said to be Basic Feasible Solution if the number of positive allocation are $(m+n-1)$. If the number of allocations in basic feasible solution are less than$(m+n-1)$, it is called Degenerate Basic Feasible Solution (DBFS) otherwise non-degenerate.

C) Degenerate Basic Feasible Solution

A basic feasible solution that contains less than $m+n$-1 non-negative allocation.

D) Optimal Solution

A feasible solution (not necessarily basic) is said to be Optimal if minimizes the total transportation cost.

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E) Balanced and Unbalanced Transportation Problem

A transportation problem is said to be balanced if the total supply from all sources equals the total demand in the destinations and is called unbalanced otherwise. Thus for a balanced problem, $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ and for unbalanced problem, $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$

F) Optimality test

Optimality test can be performed if "the number of allocation cells in an initial basic feasible solution = m+n-1 (no. of rows + no. of columns -1)".Otherwise the optimality test cannot be performed.

2. VOGEL'S APPROXIMATION METHOD (VAM):

The Vogel's approximation method is an iterative procedure for computing a basic feasible solution of the transportation problem.

Step 1: Identify the boxes having minimum and next to minimum transportation cost in each row and write the difference (penalty) along the side of the table against the corresponding row.

Step 2: Identify the boxes having minimum and next to minimum transportation cost in each column and write the difference (penalty) along the side of the table against the corresponding column

Step 3: Identify the minimum penalty. If it is along the side of the table make maximum allotment to the box having minimum cost of transportation in that row. If it is below the table, make maximum allotment to the box having minimum cost of transportation in that column.

Step 4: If the penalties corresponding to two or more rows or columns are equal . Select the top most row and the extreme left column.

3. ZERO SUFFIX METHOD:

We, now introduce a new method called the zero suffix method for finding an optimal solution to a transportation problem .The zero suffix method proceeds as follows.

Step 1: Construct the transportation table for the given TP

Step 2: Subtract each row entries of the transportation table from the row minimum and then subtract each column entries of the resulting transportation table after using the Step 1 from the column minimum.

Step 3: In the reduced cost matrix there will be at least one zero in each row and column, then find the suffix value of all the zeros in the reduced cost matrix by following simplification, the suffix value is denoted by S, Therefore S={Add the costs of nearest adjacent sides of zeros/ No. of costs added}

Step 4: Choose the maximum of S, if it has one maximum value then first supply to that demand corresponding to the cell. If it has more equal values then select {ai,bj} and supply to that demand maximum possible.

Step 5: After the above step, the exhausted demands (column) or supplies (row) are to be trimmed. The resultant matrix must possess at least one zero is each row and column, else repeat step2

Step 6: Repeat Step 3 to Step 5 until the optimal cost is obtained.

4. NMD METHOD:

Algorithm for solving Transportation problem

Step 1: Construct the Transportation matrix from given transportation problem

Step 2: Select minimum odd cost from all cost in the matrix

Step 3: Subtract selected least odd cost only from odd cost in matrix. Now there will be at least one zero and remaining all cost become even

Step 4: Multiply by 12 each column (*i.e* 12*Cij*) or To get minimum cost 1 in any column ,if possible by dividing cost in that column.

Step 5: Again select minimum odd cost in the remaining column except zeros in the column.

Step 6: Go to Step 3. Now there will be at least one zero and remaining all cost will become even.

Step 7: Repeat step 4 and 5 , for remaining sources and destinations till (m+n−1) cells are allocated.

Step 8: Start the allocation from minimum of supply and demand. Allocate this minimum of supply/demand at the place of 0 first and then 1.

Step 9: Finally total minimum cost is calculated as sum of the product of cost and corresponding allocated value of supply/ demand.

5. NUMERICAL EXAMPLES:

We illustrate the proposed method by the following balanced transportation problem having 3 sources and 4 destinations

Example 1:

Source	Destination				Availability
	1	2	3	4	
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Requirement	60	40	30	110	240

By applying VAM method the allocations are obtained as follows:

Source	Destination				Availability
	1	2	3	4	
1	20	22 (40)	17	4 (80)	120
2	24 (10)	37	9 (30)	7 (30)	70
3	32 (50)	37	20	15	50
Requirement	60	40	30	110	240

The minimum Transportation Cost associated with this solution is :

Z = $ ((20x4)+(4x80)+(24x10)+(7x30)+(32x50)+(9x30))

= $ 3,520

By applying Zero Suffix Method the allocation are obtained as follows:

Source	Destination 1	2	3	4	Availability
1	20 [60]	22 [40]	17	4 [20]	120
2	24	37	9 [30]	7 [40]	70
3	32	37	20	15 [50]	50
Requirement	60	40	30	110	240

The minimum Transportation Cost associated with this solution is :

Z = $ ((20x60)+(22x40)+(4x20)+(9x30)+(7x40)+(15x50))
= $ 3,460

By applying NMD method the allocation are obtained as follows:

Source	Destination 1	2	3	4	Availability
1	20 [10]	22 [40]	17 [30]	4 [40]	120
2	24	37	9	7 [70]	70
3	32 [50]	37	20	15	50
Requirement	60	40	30	110	240

The minimum Transportation Cost associated with this solution is :

Z = $ ((20x10)+(22x40)+(4x40)+(17x30)+(7x40)+(32x50)+(7x70))
= $ 3,840

Example 2: Unbalanced Transportation Problem having 4 sources and 4 destinations:

Source	Destination 1	2	3	4	Availability
1	4	6	8	13	500
2	13	11	10	8	700
3	14	4	10	13	300
4	9	11	13	3	500
Requirement	250	350	1050	200	

By applying VAM the allocations are obtained as follows:

Source	Destination 1	2	3	4		Availability
1	4 [250]	6 [50]	8 [200]	13	0	500
2	13	11	10 [50]	8	0 [150]	700
3	14	4 [300]	10	13	0	300
4	9	11	13 [300]	3 [200]	0	500
Requirement	250	350	1050	200	150	2000

The minimum Transportation Cost associated with this solution is :

Z = $ ((4x250)+(6x50)+(8x200)+(10x550)+(0x150)+(4x300)+(13x300)+(3x200))
= $ 14,100

By applying Zero Suffix Method the allocation are obtained as follows:

Source	Destination 1	2	3	4		Availability
1	4 [250]	6 [50]	8 [50]	13	0 [150]	500
2	13	11	10 [700]	8	0	700
3	14	4 [300]	10	13	0	300
4	9	11	13 [300]	3 [200]	0	500
Requirement	250	350	1050	200	150	2000

The minimum Transportation Cost associated with this solution is :

Z = $ ((4x250)+(6x50)+(8x50)+(10x700)+(0x150)+(4x300)+(13x300)+(3x200))
= $ 14,400

By applying NMD Method the allocation are obtained as follows:

Source	Destination 1	2	3	4		Availability
1	4 [250]	6 [250]	8	13	0	500
2	13	11 [100]	10 [600]	8	0	700
3	14	4	10 [300]	13	0	300
4	9	11	13 [50]	3 [200]	0 [150]	500
Requirement	250	350	1050	200	150	2000

The minimum Transportation Cost associated with this solution is :

Z = $ ((4x250)+(6x250)+(11x100)+(10x600)+(10x300)+(13x50)+(0x150)+(3x200))
= $ 13,850

5. COMPARISON OF TOTAL COST OF TRANSPORTATION PROBLEM FROM VAM AND ZERO SUFFIX METHOD:

Problem	Problem Size	NWCM	LCM	VAM	Zero Suffix Method	NMD
1	3x4	3680	3720	3520	3460	3840
2	4x4	13850	14400	14100	14400	13850

CONCLUSION:

In this paper, The optimal solution obtained in Zero Suffix Method is optimum in problem 1 and in problem 2 NWCM and NMD methods are optimum . So, the user cannot fix to one particular method as best optimum method. Depends on the problem the reliability condition may differ.

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