Research Paper

Volume : 3 | Issue : 9 | September 2014 • ISSN No 2277 - 8179

The Effective Use of Resource for Bus Scheduling Using Linear Programming

Mathematics

KEYWORDS : Linear Programming Problem (LPP), Bus scheduling, Constraints, Objective function.

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ABSTRACT

In this paper, we investigate the application of constraints programming bus scheduling problem with hour limitation patterns and formulated the mathematical model using a Linear Programming Problem, it is a tool of Operations Research. A numerical illustration example of scheduling bus driver for 8 hour shift is presented and the optimum solution is solved by Excel solver

1. Introduction

1.1 Operations Research

Operations Research is a Science designed to provide quantitative tools to decision-making procedure OR represents the study of optimal resource allocation. The goal of OR is to provide rational bases for decision making by seeking to understand and structure complex situations, and to utilize this understanding to predict system behavior and improve system performance. Much of the actual work is conducted by using analytical and numerical techniques to develop and manipulate mathematical models of organizational systems that are composed of people, machines, and procedures. The methodology of the operations research is employed to problems that concern how to conduct and coordinate the operations within organizations. The nature of organization is essentially immaterial, OR has been applied extensively in such diverse areas as

- Manufacturing
- Transportation
- Telecommunication
- Public services
- Health care
- The military

1.2 Linear Programming Problem

The Linear Programming (LP) started with the work of George Dantzig in 1947. However, it must be said that many other scientists have also made seminal contributions to the subject, and some would argue that the origins of LP predate Dantzig's contribution. It is matter open to debate. However, what is not open to debate is Dantzig's key contribution to LP computation. In contrast to the economists of his time, Dantzig viewed LP not just as a qualitative tool in the analysis of economic phenomena, but as a method that could be used to compute actual answers to specific real-world problems. A major proportion of all the scientific computation on computer is devoted to the use of Linear Programming.

Linear programming uses a mathematical model to describe the problem of concern. The adjective *linear* means that all the mathematical functions in this model are required to be *linear* functions. The word *Programming* does not refer to computer programming; rather, it is essentially a synonym for planning. Thus, *Linear Programming* involves the *planning of activities* to obtain an optimal result. Dozens of text books have been written about linear programming and published articles describing important applications now number in the hundreds. Part of the effort was to build computer implementations of the simplex algorithm, and Orchard-Hays were assigned the task of working with Dantzig. The result was four-year collaboration at RAND that laid the foundation for the computational development of the subject.

In this work, Bus driver scheduling involves to finding the most efficient way of providing drivers for a given set of bus movements, including dead running (journeys with no passengers). There are several restrictions on efficient provision, imposed by legal and logistical considerations as well as trade union agreements. For example, a driver may only legally drive a certain number of consecutive hours. The criterion is usually that the schedule should have the minimum number of shifts and lowest total hours of work. The progress city is studying the feasibility of introducing a mass-transmit bus system that will alleviate the smog problem by reducing in city-driving. The study seeks the minimum number of buses that can handle the transportation needs. After gathering necessary information, the city engineer noticed that the minimum number of buses needed fluctuated with the time of the day and that the required number of buses could be approximated by constant values over the successive 4 hours intervals. To carry out the required daily maintenance, each bus can operate twelve successive hours a day only.

2. Structure of Linear Programming model.

The general structure of the Linear Programming model essentially consists of three components.

- The activities and their relationships
- The objective function
- The constraints

The activities are represented by + +......+these are known as Decision variables.

The objective function of an LPP (Linear Programming Problem) is a mathematical representation of the objective in terms a measurable quantity such as profit, cost, revenue, etc.

Optimize (Maximize or Minimize) Z=+ +......+

Where Z is the measure of performance variable and, , , ,.......,are the decision variables and 12n are the parameters that give contribution to decision variables. The constraints are the set of linear inequalities and/or equalities which impose restriction of the limited resources

3. General Mathematical Model of an LPP

Optimize (Maximize or Minimize) Z=+ +......+Subject to constraints,

$$a_{11}x_1 + a_{21}x_2 + \cdots \ldots \ldots \ldots \ldots + a_{1n}x_n \ (<,=,>) \ b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots \ldots \ldots \ldots \ldots + a_{2n}x_n \ (<,=,>) \ b_2$$
$$a_{31}x_1 + a_{32}x_2 + \cdots \ldots \ldots \ldots \ldots + a_{3n}x_n \ (<,=,>) \ b_3$$
$$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$
$$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$
$$a_{n1}x_1 + a_{11}x_2 + \cdots \ldots \ldots \ldots \ldots + a_{mn}x_n \ (<,=,>) \ b_m$$
$$and \ x_1, x_2 \ldots . x_n > 0$$

3.1 Characteristic for formulating linear programming model

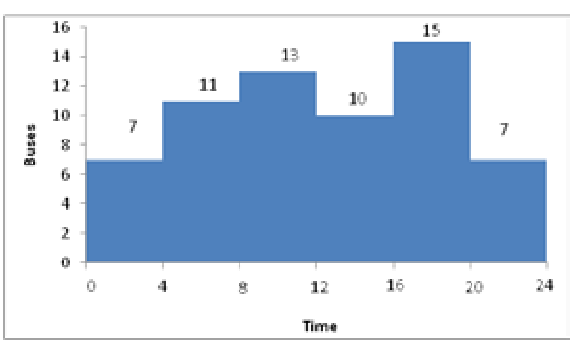
- Identify and define the decision variable of the problem
- Define the objective function
- State the constraints to which the objective function should be optimized (*i.e.* Maximization or Minimization)
- Add the non-negative constraints from the consideration that the negative values of the decision variables do not have any valid physical interpretation

4. A Numerical Example

Consider a bus company scheduling drivers for its buses. The requirement for buses varies from hour to hour because of customer demand as shown in the figure. Time 0 on the figure represents midnight, and times are shown with a 24 hour clock starting at midnight. For example, seven buses must run from midnight to 4 a.m., while eleven buses must run from 4 a.m. until 8 a.m. We assume that the bus requirements are the same every day.

The problem is to determine how many drivers to schedule at each starting time to cover the requirements for buses. Drivers work eight hour shifts that start at times: 0, 4, 8, 12, 16 or 20. For example, a driver starting at time 0 can drive a bus from time 0 to 8. A driver scheduled to start at time 20 works for the final four hours of the day and the first four hours of the next day. The goal is to minimize the number of drivers used. Note that although a driver can be hired for an eight hour period, there is no requirement that he drive a bus for the entire period. He might be idle for a four hour interval within the period.

One feasible solution to this problem is to schedule 11 drivers at time 0, 13 drivers at time 8, and 15 drivers at time 16. This solution will cover all the requirements and use a total of 39 drivers. The problem is to find the smallest number of drivers.



Requirements for buses in a 24 hour period

5. Mathematical model of Bus Scheduling:

5.1 Decision variable

$x(t)$: Number of drivers scheduled at time t, t = 0, 4, 8, 12, 16, 20

We assume that this problem continues over an indefinite number of days with the same bus requirements and that $x(t)$ is the number used in every day at time t.

$x(0)$ = Number of buses starting at the time 12.00A.M

$x(4)$ = Number of buses starting at the time 4.00 A.M.

$x(8)$ = Number of buses starting at the time 8.00 A.M.

$x(12)$=Number of buses starting at the time 12.00 P.M

$x(16)$=Number of buses starting at the time 16.00 P.M.

$x(20)$=Number of buses starting at the time 20.00 P.M.

5.2 Objective function

The objective is to Minimize $Z = x(0) + x(4) + x(8) + x(12) + x(16) + x(20)$

5.3 Constraints

We need constraints that assure that the drivers scheduled at the times that cover the requirements of a particular interval sum to the number required.

$x(0) + x(20) \geq 7,$

$x(0) + x(4) \geq 11,$

$x(4) + x(8) \geq 13,$

$x(8) + x(12) \geq 10,$

$x(12) + x(16) \geq 15,$

$x(16) + x(20) \geq 7$

5.4 Model in Matrix Format

It is useful to express the model in matrix format. The relevant matrices are

$$c = (1,1,1,1,1,1)$$

$$X = \begin{bmatrix} x_0 \\ x_4 \\ x_8 \\ x_{12} \\ x_{16} \\ x_{20} \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 7 \\ 11 \\ 13 \\ 10 \\ 15 \\ 7 \end{bmatrix}, \ \mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

5.5 Expressed in matrices model

$$Minimize \ Z = CX$$
$$Subject \ to \ the \ constraints:$$
$$AX = b$$
$$X \geq 0$$

The matrix **A** has the interesting characteristic in that it is composed entirely of 0's and 1's. Each row represents a demand interval and each column represents a decision. The 1's in each column appear as a sequence, starting at one row and ending at another. This is typical of scheduling problems of this kind.

It is reasonable to place the requirement of integrality on the decision variables **x** because clearly it is impractical to schedule fractions of drivers. It is not necessary in this case because the matrix A possesses the important characteristic of *total* unimodularity. When the constraint coefficient matrix has this characteristic, every basic solution of the linear programming model is entirely integer. As a result the optimum solution is also integer. Although adding the integrality requirement would not change the solution, it is usually not better to require integrality when it is not necessary.

5.6 Solution

The solution found by the linear programming algorithm (Excel-shown below) uses the minimum number of 24 drivers to meet the schedule.

Optimum Solution

x_0	x_4	x_8	x_{12}	x_{16}	x_{20}	Z
7	4	9	1	7	0	28

Volume : 3 | Issue : 9 | September 2014 • ISSN No 2277 - 8179

5.7 Excel Model and solution

BUS SCHEDULLING										
INPUT DATA										
		NUMBER OF BUSES STARTING AT								
		12. A.M	4.00 A.M	8.00 A.M	12.00 P.M	16.00 P.M	20.00 P.M			
								Totals		
	Objective	1	1	1	1	1	1	28		Limits
TIME	(12.00 A.M-4.00A.M)	1	0	0	0	0	1	7	>=	7
	(4.00A.M-8.00A.M)	1	1	0	0	0	0	11	>=	11
	(8.00A.M-12.00P.M)	0	1	1	0	0	0	13	>=	13
	(12.00P.M-4.00P.M)	0	0	1	1	0	0	10	>=	10
	(4.00P.M-8.00A.M)	0	0	0	1	1	0	8	>=	15
	(8.00A.M-.12.00A.M)	0	0	0	0	1	1	7	>=	7

OUTPUT RESULT

	x(0)	x(4)	x(8)	x(12)	x(16)	x(20)	Z
Solution	7	4	9	1	7	0	28

6. Conclusion

This paper gives an overview of the planning and bus scheduling problem, this seeks the minimum number of buses can handle the transportation needs. Although we have describe the constraints satisfaction system in terms of shift and piece of works. Finally the bus scheduling is discussed with the help of mathematical formulation, numerical example and produces the optimum solution by using Ms Excel solver. The effectiveness of the solution is obtained for the bus scheduling problem can greatly be reduced the number of buses to satisfy the demand.

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