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Research Paper

Properties of Bipolar Interval Valued Fuzzy Subgroups of a Group

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A.BALASUBRAMANIAN Department of Mathematics, Alagappa Government Arts College, Karaikudi–6300003, Tamilnadu, India.

K.L.MURUGANANTHA PRASAD Department of Mathematics, H.H.The Rajahs College, Pudukkottai – 622001, Tamilnadu, India.

K.ARJUNAN Department of Mathematics, H.H.The Rajahs College, Pudukkottai – 622001, Tamilnadu, India.

ABSTRACT

In this paper, we study some of the properties of bipolar interval valued fuzzy subgroup and prove some results on these.

INTRODUCTION: In 1965, Zadeh [14] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets etc [6]. Lee [8] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [8, 9]. Somasundara Moorthy.M.G & K.Arjunan[12] introduced the interval valued fuzzy subrings of a ring under homomorphism. In this paper we introduce the concept of bipolar interval valued fuzzy subgroup and established some results.

1.PRELIMINARIES:

1.1 Definition: A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form $A = \{ < x, A^{+}(x), A^{-}(x) > / x \in X\}$, where $A^{+} : X \to [0, 1]$ and $A^{-} : X \to [-1, 0]$. The positive membership degree $A^{+}(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^{-}(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A. If $A^{+}(x) \neq 0$ and $A^{-}(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for A and if $A^{+}(x) = 0$ and $A^{-}(x) \neq 0$, it is the situation that x does not satisfy the property of A, but somewhat satisfies the counter property of A. It is possible for an element x to be such that $A^{+}(x) \neq 0$ and $A^{-}(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X.

1.2 Example: $A = \{ < a, 0.5, -0.3 >, < b, 0.1, -0.7 >, < c, 0.5, -0.4 >\}$ is a bipolar valued fuzzy subset of X= {a, b, c }.

1.3 Definition: A bipolar interval valued fuzzy set (BIVFS) [A] in X is defined as an object of the form $[A] = \{ < x, [A]^{+}(x), [A]^{-}(x) >/ x \in X\}$, where $[A]^{+} : X \to D[0, 1]$

and $[A]^{-}$: X→ D[−1, 0]. The positive membership degree $[A]^{+}(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar interval valued fuzzy set [A] and the negative membership degree $[A]^{-}(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar interval valued fuzzy set [A]. If $[A]^{+}(x) \neq [0, 0]$ and $[A]^{-}(x) = [0, 0]$, it is the situation that x is regarded as having only positive satisfaction for [A] and if $[A]^{+}(x) = [0, 0]$ and $[A]^{-}(x) \neq [0, 0]$, it is the situation that x does not satisfy the property of [A], but somewhat satisfies the counter property of [A]. It is possible for an element x to be such that $[A]^{+}(x) \neq [0, 0]$ and $[A]^{-}(x) \neq [0, 0]$ when the membership function of the property overlaps that of its counter property over some portion of X.

1.4 Example: [A] = { < a, [0.5, 0.6], [−0.6, −0.4] >, < b, [0.1, 0.4], [−0.7, −0.5] >, < c, [0.5, 0.6], [−0.6, −0.4] >} is a bipolar interval valued fuzzy subset of X = {a, b, c }.

1.5 Definition: Let G be a group. A bipolar interval valued fuzzy subset [A] of G is said to be a bipolar interval valued fuzzy subgroup of G if the following conditions are satisfied,

(i) $[A]^{+}(xy) \geq \text{rmin}\{ [A]^{+}(x), [A]^{+}(y) \}$

(ii) $[A]^{+}(x^{-1}) \geq [A]^{+}(x)$

(iii) $[A]^{-}(xy) \leq \text{rmax}\{ [A]^{-}(x), [A]^{-}(y) \}$

(iv) $[A]^{-}(x^{-1}) \leq [A]^{-}(x)$ for all x and y in G.

1.6 Example: Let G = { 1, −1, i, −i } be a group with respect to the ordinary multiplication. Then [A] = { < 1, [0.5, 0.5], [−0.6, − 0.6] >, < −1, [0.4, 0.4], [−0.5, − 0.5] >, < i, [0.2, 0.2], [−0.4, −0.4] >, < −i, [0.2, 0.2], [−0.4, −0.4] > } is a bipolar interval valued fuzzy subgroup of G.

1.7 Definition: Let $[A] = \langle [A]^{+}, [A]^{-} \rangle$ and $[B] = \langle [B]^{+}, [B]^{-} \rangle$ be any two bipolar interval valued fuzzy subsets of sets G and H respectively. The product of [A] and [B], denoted by [A]×[B], is defined as $[A]\times[B] = \{ \langle (x, y), ([A]\times[B])^{+}(x, y), ([A]\times[B])^{-}(x, y) \rangle$ / for all x in G and y in H }, where $([A]\times[B])^{+}(x, y) = \text{rmin}\{ [A]^{+}(x), [B]^{+}(y) \}$ and $([A]\times[B])^{-}(x, y) = \text{rmax}\{ [A]^{-}(x), [B]^{-}(y) \}$ for all x in G and y in H.

1.8 Definition: Let $[A] = \langle [A]^{+}, [A]^{-} \rangle$ be a bipolar interval valued fuzzy subset in a set S, the strongest bipolar interval valued fuzzy relation on S, that is a bipolar interval valued fuzzy relation on [A] is $[V] = \{ \langle (x, y), [V]^{+}(x, y), [V]^{-}(x, y) \rangle$ / x and y in S} given by $[V]^{+}(x, y) = \text{rmin}\{ [A]^{+}(x), [A]^{+}(y) \}$ and $[V]^{-}(x, y) = \text{rmax}\{ [A]^{-}(x), [A]^{-}(y) \}$ for all x and y in S.

2. PROPERTIES:

2.1 Theorem: Let $[A] = \langle [A]^{+}, [A]^{-} \rangle$ be a bipolar interval valued fuzzy subgroup of G. Then $[A]^{+}(x^{-1}) = [A]^{+}(x)$ and $[A]^{-}(x^{-1}) = [A]^{-}(x)$, $[A]^{+}(x) \leq [A]^{+}(e)$ and $[A]^{-}(x) \geq [A]^{-}(e)$ for all x in G and the identity element e in G.

Proof: Let x be in G. Now $[A]^{+}(x) = [A]^{+}((x^{-1})^{-1}) \geq [A]^{+}(x^{-1}) \geq [A]^{+}(x)$. Therefore $[A]^{+}(x) = [A]^{+}(x^{-1})$ for all x in G. And $[A]^{-}(x) = [A]^{-}((x^{-1})^{-1}) \leq [A]^{-}(x^{-1}) \leq [A]^{-}(x)$. Therefore $[A]^{-}(x^{-1}) = [A]^{-}(x)$ for all x in G. Now $[A]^{+}(e) = [A]^{+}(xx^{-1}) \geq \text{rmin}\{ [A]^{+}(x), [A]^{+}(x^{-1}) \} = [A]^{+}(x)$. Therefore $[A]^{+}(e) \geq [A]^{+}(x)$ for all x in G. And $[A]^{-}(e) = [A]^{-}(xx^{-1}) \leq \text{rmax}\{ [A]^{-}(x), [A]^{-}(x^{-1}) \} = [A]^{-}(x)$. Therefore $[A]^{-}(e) \leq [A]^{-}(x)$ for all x in G.

2.2 Theorem: Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar interval valued fuzzy subgroup of G. Then (i) $[A]^+(xy^{-1}) = [A]^+(e)$ implies that $[A]^+(x) = [A]^+(y)$ for x and y in G.
(ii) $[A]^-(xy^{-1}) = [A]^-(e)$ implies that $[A]^-(x) = [A]^-(y)$ for x and y in G.

Proof: Now $[A]^+(x) = [A]^+(xy^{-1}y) \geq \text{rmin}\{ [A]^+(xy^{-1}), [A]^+(y) \} = \text{rmin}\{ [A]^+(e), [A]^+(y) \} = [A]^+(y) = [A]^+(yx^{-1}x) \geq \text{rmin}\{ [A]^+(yx^{-1}), [A]^+(x) \} = \text{rmin}\{ [A]^+(e), [A]^+(x) \} = [A]^+(x)$. Therefore $[A]^+(x) = [A]^+(y)$ for x and y in G. And $[A]^-(x) = [A]^-(xy^{-1}y) \leq \text{rmax}\{ [A]^-(xy^{-1}), [A]^-(y) \} = \text{rmax}\{ [A]^-(e), [A]^-(y) \} = [A]^-(y) = [A]^-(yx^{-1}x) \leq \text{rmax}\{ [A]^-(yx^{-1}), [A]^-(x) \} = \text{rmax}\{ [A]^-(e), [A]^-(x) \} = [A]^-(x)$. Therefore $[A]^-(x) = [A]^-(y)$ for x and y in G.

2.3 Theorem: Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar interval valued fuzzy subgroup of a group G. (i) If $[A]^+(xy^{-1}) = [1, 1]$, then $[A]^+(x) = [A]^+(y)$ for x and y in G.
(ii) If $[A]^-(xy^{-1}) = [-1, -1]$, then $[A]^-(x) = [A]^-(y)$ for x and y in G.

Proof: Now $[A]^+(x) = [A]^+(xy^{-1}y) \geq \text{rmin}\{ [A]^+(xy^{-1}), [A]^+(y) \} = \text{rmin}\{ [1, 1], [A]^+(y) \} = [A]^+(y) = [A]^+(y^{-1}) = [A]^+(x^{-1}xy^{-1}) \geq \text{rmin}\{ [A]^+(x^{-1}), [A]^+(xy^{-1}) \} = \text{rmin}\{ [A]^+(x^{-1}), [1, 1] \} = [A]^+(x^{-1}) = [A]^+(x)$. Therefore $[A]^+(x) = [A]^+(y)$ for x and y in G. Hence (i) is proved. Also $[A]^-(x) = [A]^-(xy^{-1}y) \leq \text{rmax}\{[A]^-(xy^{-1}), [A]^-(y) \} = \text{rmax}\{[-1, -1], [A]^-(y) \} = [A]^-(y) = [A]^-(y^{-1}) = [A]^-(x^{-1}xy^{-1}) \leq \text{rmax}\{ [A]^-(x^{-1}), [A]^-(xy^{-1}) \} = \text{rmax}\{ [A]^-(x^{-1}), [-1, -1] \} = [A]^-(x^{-1}) = [A]^-(x)$. Therefore $[A]^-(x) = [A]^-(y)$ for x and y in G. Hence (ii) is proved.

2.4 Theorem: Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar interval valued fuzzy subgroup of a group G. (i) If $[A]^+(xy^{-1}) = [0, 0]$, then either $[A]^+(x) = [0, 0]$ or $[A]^+(y) = [0, 0]$ for x and y in G. (ii) If $[A]^-(xy^{-1}) = [0, 0]$, then either $[A]^-(x) = [0, 0]$ or $[A]^-(y) = [0, 0]$ for x and y in G.

Proof: Let x and y in G. (i) By the definition $[A]^+(xy^{-1}) \geq \text{rmin}\{ [A]^+(x), [A]^+(y) \}$, which implies that $[0, 0] \geq \text{rmin}\{ [A]^+(x), [A]^+(y) \}$. Therefore either $[A]^+(x) = [0, 0]$ or $[A]^+(y) = [0, 0]$. (ii) By the definition $[A]^-(xy^{-1}) \leq \text{rmax}\{ [A]^-(x), [A]^-(y) \}$ which implies that $[0, 0] \leq \text{rmax}\{ [A]^-(x), [A]^-(y) \}$. Therefore either $[A]^-(x) = [0, 0]$ or $[A]^-(y) = [0, 0]$.

2.5 Theorem: If $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar interval valued fuzzy subgroup of G, then (i) $[A]^+(xy) = [A]^+(yx)$ if and only if $[A]^+(x) = [A]^+(y^{-1}xy)$ for x and y in G.
(ii) $[A]^-(xy) = [A]^-(yx)$ if and only if $[A]^-(x) = [A]^-(y^{-1}xy)$ for x and y in G.

Proof: Let x and y be in G. Assume that $[A]^+(xy) = [A]^+(yx)$, so $[A]^+(y^{-1}xy) = [A]^+(y^{-1}yx) = [A]^+(ex) = [A]^+(x)$. Therefore $[A]^+(x) = [A]^+(y^{-1}xy)$ for x and y in G. Conversely assume that $[A]^+(x) = [A]^+(y^{-1}xy)$, we get $[A]^+(xy) = [A]^+(xyxx^{-1}) = [A]^+(yx)$. Therefore $[A]^+(xy) = [A]^+(xy)$ for x and y in G. Hence $[A]^+(xy) = [A]^+(yx)$ if and only if $[A]^+(x) = [A]^+(y^{-1}xy)$ for x and y in G. Also assume that $[A]^-(xy) = [A]^-(yx)$, we get $[A]^-(y^{-1}xy) = [A]^-(y^{-1}yx) = [A]^-(ex) = [A]^-(x)$. Therefore $[A]^-(x) = [A]^-(y^{-1}xy)$ for x and y in G. Conversely, assume that $[A]^-(x) = [A]^-(y^{-1}xy)$, so $[A]^-(xy) = [A]^-(xyxx^{-1}) = [A]^-(yx)$. Therefore $[A]^-(xy) = [A]^-(xy)$ for x and y in G. Hence $[A]^-(xy) = [A]^-(yx)$ if and only if $[A]^-(x) = [A]^-(y^{-1}xy)$ for x and y in G.

2.6 Theorem: If $[A] = \langle [A]^+, [A]^- \rangle$ is a bipolar interval valued fuzzy subgroup of a group G, then $H = \{ x \in G \mid [A]^+(x) = [1, 1], [A]^-(x) = [-1, -1] \}$ is either empty or a subgroup of G.

Proof: If no element satisfies this condition, then H is empty. If x and y in H, then $[A]^+(xy^{-1}) \geq \text{rmin}\{ [A]^+(x), [A]^+(y) \} = \text{rmin}\{ [1, 1], [1, 1] \} = [1, 1]$. Therefore $[A]^+(xy^{-1}) = [1, 1]$. And $[A]^-(xy^{-1}) \leq \text{rmax}\{ [A]^-(x), [A]^-(y) \} = \text{rmax}\{ [-1, -1],$

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$[-1, -1] \} = [-1, -1]$. Therefore $[A]^{-}(xy^{-1}) = [-1, -1]$. That is $xy^{-1} \in H$. Hence H is a subgroup of G. Hence H is either empty or a subgroup of G.

2.7 Theorem: If $[A] = \langle [A]^{+}, [A]^{-} \rangle$ is a bipolar interval valued fuzzy subgroup of G, then $H = \{ x \in G \mid [A]^{+}(x) = [A]^{+}(e)$ and $[A]^{-}(x) = [A]^{-}(e) \}$ is a subgroup of G.

Proof: Here $H = \{ x \in G \mid [A]^{+}(x) = [A]^{+}(e)$ and $[A]^{-}(x) = [A]^{-}(e) \}$, by Theorem 2.1, $[A]^{+}(x^{-1}) = [A]^{+}(x) = [A]^{+}(e)$ and $[A]^{-}(x^{-1}) = [A]^{-}(x) = [A]^{-}(e)$. Therefore $x^{-1} \in H$. Now $[A]^{+}(xy^{-1}) \geq \text{rmin} \{ [A]^{+}(x), [A]^{+}(y) \} = \text{rmin} \{ [A]^{+}(e), [A]^{+}(e) \} = [A]^{+}(e)$ and $[A]^{+}(e) = [A]^{+}((xy^{-1})(xy^{-1})^{-1}) \geq \text{rmin} \{ [A]^{+}(xy^{-1}), [A]^{+}(xy^{-1}) \} = [A]^{+}(xy^{-1})$. Hence $[A]^{+}(e) = [A]^{+}(xy^{-1})$. Also $[A]^{-}(xy^{-1}) \leq \text{rmax} \{ [A]^{-}(x), [A]^{-}(y) \} = \text{rmax} \{ [A]^{-}(e), [A]^{-}(e) \} = [A]^{-}(e)$ and $[A]^{-}(e) = [A]^{-}((xy^{-1})(xy^{-1})^{-1}) \leq \text{rmax} \{ [A]^{-}(xy^{-1}), [A]^{-}(xy^{-1}) \} = [A]^{-}(xy^{-1})$. Therefore $[A]^{-}(e) = [A]^{-}(xy^{-1})$. Hence $[A]^{+}(e) = [A]^{+}(xy^{-1})$ and $[A]^{-}(e) = [A]^{-}(xy^{-1})$. Therefore $xy^{-1} \in H$. Hence H is a subgroup of G.

2.8 Theorem: Let G be a group. If $[A] = \langle [A]^{+}, [A]^{-} \rangle$ is a bipolar interval valued fuzzy subgroup of G, then $[A]^{+}(xy) = \text{rmin} \{ [A]^{+}(x), [A]^{+}(y) \}$ and $[A]^{+}(xy) = \text{rmax} \{ [A]^{-}(x), [A]^{-}(y) \}$ for each x, y in G with $[A]^{+}(x) \neq [A]^{+}(y)$ and $[A]^{-}(x) \neq [A]^{-}(y)$.

Proof: Assume that $[A]^{+}(x) > [A]^{+}(y)$ and $[A]^{-}(x) < [A]^{-}(y)$. Then $[A]^{+}(y) = [A]^{+}(x^{-1}xy) \geq \text{rmin} \{ [A]^{+}(x^{-1}), [A]^{+}(xy) \} = \text{rmin} \{ [A]^{+}(x), [A]^{+}(xy) \} = [A]^{+}(xy) \geq \text{rmin} \{ [A]^{+}(x), [A]^{+}(y) \} = [A]^{+}(y)$. Therefore $[A]^{+}(xy) = [A]^{+}(y) = \text{rmin} \{ [A]^{+}(x), [A]^{+}(y) \}$. And $[A]^{-}(y) = [A]^{-}(x^{-1}xy) \leq \text{rmax} \{ [A]^{-}(x^{-1}), [A]^{-}(xy) \} = \text{rmax} \{ [A]^{-}(x), [A]^{-}(xy) \} = [A]^{-}(xy) \leq \text{rmax} \{ [A]^{-}(x), [A]^{-}(y) \} = [A]^{-}(y)$. Therefore $[A]^{-}(xy) = [A]^{-}(y) = \text{rmax} \{ [A]^{-}(x), [A]^{-}(y) \}$.

2.9 Theorem: If $[A] = \langle [A]^{+}, [A]^{-} \rangle$ and $[B] = \langle [B]^{+}, [B]^{-} \rangle$ are two bipolar interval valued fuzzy subgroups of a group G, then their intersection $A \cap B$ is a bipolar interval valued fuzzy subgroup of G.

Proof: Let $[A] = \{ < x, [A]^{+}(x), [A]^{-}(x) > / x \in G \}$, $[B] = \{ < x, [B]^{+}(x), [B]^{-}(x) > / x \in G \}$. Let $[C] = [A] \cap [B]$ and $[C] = \{ < x, [C]^{+}(x), [C]^{-}(x) > / x \in G \}$. Now $[C]^{+}(xy^{-1}) = \text{rmin} \{ [A]^{+}(xy^{-1}), [B]^{+}(xy^{-1}) \} \geq \text{rmin} \{ \text{rmin} \{ [A]^{+}(x), [A]^{+}(y) \}, \text{rmin} \{ [B]^{+}(x), [B]^{+}(y) \} \} \geq \text{rmin} \{ \text{rmin} \{ [A]^{+}(x), [B]^{+}(x) \}, \text{rmin} \{ [A]^{+}(y), [B]^{+}(y) \} \} = \text{rmin} \{ [C]^{+}(x), [C]^{+}(y) \}$. Therefore $[C]^{+}(xy^{-1}) \geq \text{rmin} \{ [C]^{+}(x), [C]^{+}(y) \}$. Also $[C]^{-}(xy^{-1}) = \text{rmax} \{ [A]^{-}(xy^{-1}), [B]^{-}(xy^{-1}) \} \leq \text{rmax} \{ \text{rmax} \{ [A]^{-}(x), [A]^{-}(y) \}, \text{rmax} \{ [B]^{-}(x), [B]^{-}(y) \} \} \leq \text{rmax} \{ \text{rmax} \{ [A]^{-}(x), [B]^{-}(x) \}, \text{rmax} \{ [A]^{-}(y), [B]^{-}(y) \} \} = \text{rmax} \{ [C]^{-}(x), [C]^{-}(y) \}$. Therefore $[C]^{-}(xy^{-1}) \leq \text{rmax} \{ [C]^{-}(x), [C]^{-}(y) \}$. Hence $A \cap B$ is a bipolar interval valued fuzzy subgroup of G.

2.10 Theorem: The intersection of a family of bipolar interval valued fuzzy subgroups of a group G is a bipolar interval valued fuzzy subgroup of G.

Proof: Let $\{ [V]_i : i \in I \}$ be a family of bipolar interval valued fuzzy subgroups of a group G and let $[A] = \bigcap_{i \in I} [V]_i$. Let x and y in G. Now $[A]^{+}(xy^{-1}) = \inf_{i \in I} [V]_i^{+}(xy^{-1}) \geq \inf_{i \in I} \text{rmin} \{ [V]_i^{+}(x), [V]_i^{+}(y) \} = \text{rmin} \{ \inf_{i \in I} [V]_i^{+}(x), \inf_{i \in I} [V]_i^{+}(y) \} = \text{rmin} \{ [A]^{+}(x), [A]^{+}(y) \}$. Therefore $[A]^{+}(xy^{-1}) \geq \text{rmin} \{ [A]^{+}(x), [A]^{+}(y) \}$ for all x and y in G. And $[A]^{-}(xy^{-1}) = \sup_{i \in I} [V]_i^{-}(xy^{-1}) \leq \sup_{i \in I} \text{rmax} \{ [V]_i^{-}(x), [V]_i^{-}(y) \} = \text{rmax} \{ \sup_{i \in I} [V]_i^{-}(x), \sup_{i \in I} [V]_i^{-}(y) \} = \text{rmax} \{ [A]^{-}(x), [A]^{-}(y) \}$. Therefore $[A]^{-}(xy^{-1}) \leq \text{rmax} \{ [A]^{-}(x), [A]^{-}(y) \}$ for all x and y in G. Hence the intersection of a family of bipolar interval valued fuzzy subgroups of a group G is a bipolar interval valued fuzzy subgroup of G.

2.11 Theorem: If $[A] = \langle [A]^{+}, [A]^{-} \rangle$ and $[B] = \langle [B]^{+}, [B]^{-} \rangle$ are any two bipolar interval valued fuzzy subgroups of the groups G_1 and G_2 respectively, then $[A]\times[B] = \langle ([A]\times[B])^{+}, ([A]\times[B])^{-} \rangle$ is a bipolar interval valued fuzzy subgroup of $G_1\times G_2$.

Proof: Let [A] and [B] be two bipolar interval valued fuzzy subgroups of the groups G_1 and G_2 respectively. Let x_1 and x_2 be in G_1, y_1 and y_2 be in G_2. Then (x_1, y_1) and (x_2, y_2) are in $G_1\times G_2$. Now $([A]\times[B])^{+}[(x_1, y_1)(x_2, y_2)^{-1}] = ([A]\times[B])^{+}(x_1x_2^{-1}, y_1y_2^{-1})$ $=$ rmin $\{ [A]^{+}(x_1x_2^{-1}), [B]^{+}(y_1y_2^{-1}) \}\geq$ rmin $\{$ rmin $\{ [A]^{+}(x_1), [A]^{+}(x_2) \}$, rmin $\{ [B]^{+}(y_1), [B]^{+}(y_2)\}\}$ = rmin {rmin $\{ [A]^{+}(x_1), [B]^{+}(y_1) \}$, rmin $\{ [A]^{+}(x_2), [B]^{+}(y_2)\}$ = rmin$\{([A]\times[B])^{+} (x_1, y_1), ([A]\times[B])^{+} (x_2, y_2)\}$. Therefore $([A]\times[B])^{+}[(x_1, y_1)(x_2, y_2)^{-1}]$ $\geq$ rmin$\{ ([A]\times[B])^{+}(x_1, y_1), ([A]\times[B])^{+}(x_2, y_2) \}$. Also $([A]\times[B])^{-} [(x_1, y_1)(x_2, y_2)^{-1}] =$ $([A]\times[B])^{-}(x_1x_2^{-1}, y_1y_2^{-1})$ = rmax $\{ [A]^{-}(x_1x_2^{-1}), [B]^{-}(y_1y_2^{-1}) \} \leq$ rmax $\{$ rmax $\{ [A]^{-}(x_1), [A]^{-}(x_2) \}$, rmax $\{ [B]^{-}(y_1), [B]^{-}(y_2) \} \}$ = rmax $\{$ rmax $\{ [A]^{-}(x_1), [B]^{-}(y_1)\}$, rmax $\{ [A]^{-}(x_2), [B]^{-}(y_2) \} \}$ = rmax $\{ ([A]\times[B])^{-}(x_1, y_1), ([A]\times[B])^{-}(x_2, y_2) \}$. Therefore $([A]\times[B])^{-}[(x_1, y_1)(x_2, y_2)^{-1}] \leq$ rmax$\{([A]\times[B])^{-}(x_1, y_1), ([A]\times[B])^{-}(x_2, y_2)\}$. Hence $[A]\times[B]$ is a bipolar interval valued fuzzy subgroup of $G_1\times G_2$.

2.12 Theorem: Let $[A] = \langle [A]^{+}, [A]^{-} \rangle$ and $[B] = \langle [B]^{+}, [B]^{-} \rangle$ be any two bipolar interval valued fuzzy subsets of the groups G and H respectively. Suppose that e and e^{l} are the identity elements of G and H respectively. If $[A]\times[B]$ is a bipolar interval valued fuzzy subgroup of $G\times H$, then at least one of the following two statements must hold.

(i) $[B]^{+}(e^{l}) \geq [A]^{+}(x)$, for all x in G and $[B]^{-}(e^{l}) \leq [A]^{-}(x)$, for all x in G,

(ii) $[A]^{+}(e) \geq [B]^{+}(y)$, for all y in H and $[A]^{-}(e) \leq [B]^{-}(y)$, for all y in H.

Proof: Let $[A]\times[B]$ is a bipolar interval valued fuzzy subgroup of $G\times H$. By contraposition, suppose that none of the statements (i) and (ii) holds. Then we can find a in G and b in H such that $[A]^{+}(a) > [B]^{+}(e^{l})$, $[A]^{-}(a)< [B]^{-}(e^{l})$ and $[B]^{+}(b) >$ $[A]^{+}(e)$, $[B]^{-}(b) < [A]^{-}(e)$. We have $([A]\times[B])^{+}(a, b)$ = rmin $\{ [A]^{+}(a), [B]^{+}(b) \}>$ rmin $\{ [A]^{+}(e), [B]^{+}(e^{l}) \} = ([A]\times[B])^{+}(e, e^{l})$. Also $([A]\times[B])^{-}(a, b)$ = rmax $\{ [A]^{-}(a), [B]^{-}(b) \} <$ rmax $\{ [A]^{-}(e), [B]^{-}(e^{l}) \} = ([A]\times[B])^{-}(e, e^{l})$. Thus $[A]\times[B]$ is not a bipolar interval valued fuzzy subgroup of $G\times H$. Hence either $[B]^{+}(e^{l}) \geq [A]^{+}(x)$ for all x in G and $[B]^{-}(e^{l}) \leq [A]^{-}(x)$, for all x in G or $[A]^{+}(e) \geq [B]^{+}(y)$ for all y in H and $[A]^{-}(e) \leq [B]^{-}(y)$ for all y in H.

2.13 Theorem: Let $[A] = \langle [A]^{+}, [A]^{-} \rangle$ and $[B] = \langle [B]^{+}, [B]^{-} \rangle$ be any two bipolar interval valued fuzzy subsets of the groups G and H respectively and $[A]\times[B]$ is a bipolar interval valued fuzzy subgroup of $G\times H$. Then the following are true:

(i) if $[A]^{+}(x) \leq [B]^{+}(e^{l})$ for all x in G and $[A]^{-}(x) \geq [B]^{-}(e^{l})$ for all x in G, then [A] is a bipolar interval valued fuzzy subgroup of G, where e^{l} is identity element of H.

(ii) if $[B]^{+}(x) \leq [A]^{+}(e)$ for all x in H and $[B]^{-}(x) \geq [A]^{-}(e)$ for all x in H, then [B] is a bipolar interval valued fuzzy subgroup of H, where e is identity element of G.

(iii) either [A] is a bipolar interval valued fuzzy subgroup of G or [B] is a bipolar interval valued fuzzy subgroup of H, where e and e^{l} are the identity elements of G and H respectively.

Proof: Let $[A]\times[B]$ be a bipolar interval valued fuzzy subgroup of $G\times H$ and x and y in G. Then (x, e^{l}) and (y, e^{l}) are in $G\times H$. Now using the property if $[A]^{+}(x) \leq [B]^{+}(e^{l})$, for all x in G and $[A]^{-}(x) \geq [B]^{-}(e^{l})$ for all x in G, where e^{l} is identity element of H, we get $[A]^{+}(xy^{-1})$ = rmin $\{ [A]^{+}(xy^{-1}), [B]^{+}(e^{l}e^{l}) \}= ([A]\times[B])^{+}((xy^{-1}), (e^{l}e^{l})) =$ $([A]\times[B])^{+}[(x, e^{l})(y^{-1}, e^{l})] \geq$ rmin $\{ ([A]\times[B])^{+}(x, e^{l}), ([A]\times[B])^{+}(y^{-1}, e^{l}) \}$ = rmin $\{$ rmin $\{ [A]^{+}(x), [B]^{+}(e^{l}) \}$, rmin $\{ [A]^{+}(y^{-1}), [B]^{+}(e^{l}) \} \}$ = rmin $\{ [A]^{+}(x), [A]^{+}(y^{-1}) \}$

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$\geq$ rmin $\{ [A]^{+}(x), [A]^{+}(y) \}$. Therefore $[A]^{+}(xy^{-1}) \geq$ rmin $\{ [A]^{+}(x), [A]^{+}(y) \}$ for all x and y in G. Also $[A]^{-}(xy^{-1})=$ rmax $\{ [A]^{-}(xy^{-1}), [B]^{-}(e^{|}e^{|}) \} = ([A]\times[B])^{-}((xy^{-1}), (e^{|}e^{|}))$ $= ([A]\times[B])^{-} [(x, e^{|})(y^{-1}, e^{|})] \leq$ rmax $\{ ([A]\times[B])^{-}(x, e^{|}), ([A]\times[B])^{-}(y^{-1}, e^{|}) \}$ = rmax $\{ [A]^{-}(x), [B]^{-}(e^{|}) \}$, rmax $\{ [A]^{-}(y^{-1}), [B]^{-}(e^{|}) \}\}=$ rmax $\{ [A]^{-}(x), [A]^{-}(y^{-1}) \}$ $\leq$ rmax $\{ [A]^{-}(x), [A]^{-}(y) \}$. Therefore $[A]^{-}(xy^{-1}) \leq$ rmax $\{ [A]^{-}(x), [A]^{-}(y) \}$ for all x and y in G. Hence [A] is a bipolar interval valued fuzzy subgroup of G. Thus (i) is proved. Now using the property $[B]^{+}(x) \leq [A]^{+}(e)$ for all x in H and $[B]^{-}(x) \geq [A]^{-}(e)$, for all x in H, we get $[B]^{+}(xy^{-1})$ = rmin $\{ [B]^{+}(xy^{-1}), [A]^{+}(e.e) \}= ([A]\times[B])^{+}((e.e), (xy^{-1})) = ([A]\times[B])^{+} [(e, x)(e, y^{-1})] \geq$ rmin $\{ ([A]\times[B])^{+}(e, x), ([A]\times[B])^{+}(e, y^{-1}) \}$ = rmin $\{$ rmin $\{ [A]^{+}(e), [B]^{+}(x) \}$, rmin $\{ [A]^{+}(e), [B]^{+}(y^{-1}) \} \}$ = rmin $\{ [B]^{+}(x), [B]^{+}(y^{-1}) \} \geq$ rmin $\{ [B]^{+}(x), [B]^{+}(y) \}$. Therefore $[B]^{+}(xy^{-1}) \geq$ rmin $\{[B]^{+}(x), [B]^{+}(y) \}$, for all x and y in H. Also $[B]^{-}(xy^{-1})$ = rmax $\{ [B]^{-}(xy^{-1}), [A]^{-}(ee)\}= ([A]\times[B])^{-}((ee), (xy^{-1})) = ([A]\times[B])^{-} [(e, x)(e, y^{-1})] \leq$ rmax $\{ ([A]\times[B])^{-}(e, x), ([A]\times[B])^{-}(e, y^{-1}) \}$ = rmax $\{$ rmax $\{ [A]^{-}(e), [B]^{-}(x) \}$, rmax $\{ [A]^{-}(e), [B]^{-}(y^{-1}) \} \}$ = rmax $\{ [B]^{-}(x), [B]^{-}(y^{-1}) \} \leq$ rmax $\{ [B]^{-}(x), [B]^{-}(y)\}$. Therefore $[B]^{-}(xy^{-1}) \leq$ rmax $\{[B]^{-}(x), [B]^{-}(y) \}$ for all x and y in H. Hence [B] is a bipolar interval valued fuzzy subgroup of H. Thus (ii) is proved. Hence (iii) is clear.

2.14 Theorem: Let $[A] = \langle [A]^{+}, [A]^{-} \rangle$ be a bipolar interval valued fuzzy subset of a group (G, .) and $[V] = \langle [V]^{+}, [V]^{-} \rangle$ be the strongest bipolar interval valued fuzzy relation of G. Then [A] is a bipolar interval valued fuzzy subgroup of G if and only if [V] is a bipolar interval valued fuzzy subgroup of G×G.

Proof: Suppose that [A] is a bipolar interval valued fuzzy subgroup of G. Then for any $x =(x_1, x_2)$ and $y =(y_1, y_2)$ are in G×G. We have $[V]^{+}(xy^{-1})= [V]^{+}[(x_1, x_2)(y_1, y_2)^{-1}]$ $= [V]^{+}(x_1y_1^{-1}, x_2y_2^{-1})$ = rmin $\{ [A]^{+}(x_1y_1^{-1}), [A]^{+}(x_2y_2^{-1}) \}\geq$ rmin $\{$ rmin $\{ [A]^{+}(x_1), [A]^{+}(y_1) \}$, rmin $\{ [A]^{+}(x_2), [A]^{+}(y_2) \}\}$ = rmin $\{$ rmin $\{ [A]^{+}(x_1), [A]^{+}(x_2) \}$, rmin $\{[A]^{+}(y_1), [A]^{+}(y_2) \} \}$ = rmin $\{ [V]^{+}(x_1, x_2), [V]^{+}(y_1, y_2) \}$ = rmin $\{ [V]^{+}(x), [V]^{+}(y) \}$. Therefore $[V]^{+}(xy^{-1}) \geq$ rmin $\{ [V]^{+}(x), [V]^{+}(y) \}$ for all x and y in G×G. Also we have $[V]^{-}(xy^{-1}) = [V]^{-}[(x_1, x_2)(y_1, y_2)^{-1}] = [V]^{-}(x_1y_1^{-1}, x_2y_2^{-1})$ = rmax $\{ [A]^{-}(x_1y_1^{-1}), [A]^{-}(x_2y_2^{-1}) \} \leq$ rmax $\{$ rmax $\{ [A]^{-}(x_1), [A]^{-}(y_1) \}$, rmax $\{ [A]^{-}(x_2), [A]^{-}(y_2) \} \}$ = rmax $\{$ rmax $\{ [A]^{-}(x_1), [A]^{-}(x_2) \}$, rmax $\{ [A]^{-}(y_1), [A]^{-}(y_2) \}\}$ = rmax $\{ [V]^{-}(x_1, x_2), [V]^{-}(y_1, y_2) \}=$ rmax $\{ [V]^{-}(x), [V]^{-}(y) \}$. Therefore $[V]^{-}(xy^{-1}) \leq$ rmax $\{[V]^{-}(x), [V]^{-}(y) \}$ for all x and y in G×G. This proves that [V] is a bipolar interval valued fuzzy subgroup of G×G. Conversely assume that [V] is a bipolar interval valued fuzzy subgroup of G×G, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in G×G, we have rmin $\{ [A]^{+}(x_1y_1^{-1}), [A]^{+}(x_2y_2^{-1}) \} = [V]^{+}(x_1y_1^{-1}, x_2y_2^{-1}) = [V]^{+}[(x_1, x_2)(y_1, y_2)^{-1}] = [V]^{+}(xy^{-1}) \geq$ rmin $\{ [V]^{+}(x), [V]^{+}(y) \}=$ rmin $\{ [V]^{+}(x_1, x_2), [V]^{+}(y_1, y_2) \}$ = rmin $\{$ rmin $\{ [A]^{+}(x_1), [A]^{+}(x_2) \}$, rmin $\{ [A]^{+}(y_1), [A]^{+}(y_2) \}\}$. If we put $x_2 = y_2 = e$, we get $[A]^{+}(x_1y_1^{-1}) \geq$ rmin $\{ [A]^{+}(x_1), [A]^{+}(y_1) \}$ for all x_1 and y_1 in G. Also we have rmax $\{[A]^{-}(x_1y_1^{-1}), [A]^{-}(x_2y_2^{-1}) \} = [V]^{-} (x_1y_1^{-1}, x_2y_2^{-1}) = [V]^{-}[(x_1, x_2)(y_1, y_2)^{-1}] = [V]^{-}(xy^{-1})$ $\leq$ rmax $\{ [V]^{-}(x), [V]^{-}(y) \}$ = rmax $\{ [V]^{-}(x_1, x_2), [V]^{-}(y_1, y_2) \}$ = rmax $\{$ rmax $\{ [A]^{-}(x_1), [A]^{-}(x_2) \}$, rmax $\{ [A]^{-}(y_1), [A]^{-}(y_2) \} \}$. If we put $x_2 = y_2 = e$, we get $[A]^{-}(x_1y_1^{-1}) \leq$ rmax $\{ [A]^{-}(x_1), [A]^{-}(y_1) \}$ for all x_1 and y_1 in G. Hence [A] is a bipolar interval valued fuzzy subgroup of G.

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