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KEYWORDS : : Reliability, Mixed series-parallel System, Interval valued fuzzy, Weighted Trapezoidal Fuzzy number, Non-linear Programming.

Optimal Reliability for Transmission Flow Mixed Series- Parallel System

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ABSTRACT

This paper considers a mixed system reliability problem. We are considering the mixed series and parallel configuration of the components which are connected in series and parallel. The objective is to maximize the reliability subject to the system cost. The paper differs from others because we have taken the transmission network in mixed system in fuzzy nature and the cost constraints are taken as interval valued weighted trapezoidal fuzzy number. Few previous models have been limited to the mixed series situation which is a certain case of this model. A numerical example is given to illustrate the model for non linear programming and to evaluate the mixed system reliability. 2010 Mathematics Subject Classification: 68M15, 03E72, 49M37, 60K10, 90C70.

1. Introduction

The reliability optimization plays a significant role inplanning,designof anyengineering product, operation and management of the industrial systems.Asystematic theory of reliability is based on probability theory. In conventional reliability, it is assumed that the precise probability and performance level of each component are given.However, with the improvement in recent industrial technologies, the product development cycle has become shorter and shorter while the lifetime of products has become longer and longer Huang et al [8]. In many highly reliable applications there may be only a few available observations. Therefore it is difficult to obtain sufficient data to estimate the exact values of these probabilities and performance levels in these systems. More-over, inaccuracy in system models that is caused by human error is difficult to deal with solely by means of conventional reliability theory Huang et al [7].

The fuzzy set theory providesan useful tool to complement conventional reliability theories. The concept of fuzzy reliability has been introduced and formulated in Cai[1] due to uncertainties and imprecision of data. Using fuzzy theoretic approach Singer [16], Utkinet al [18]and Guan et al [6] attempted to describe and evaluate system reliability in terms of fuzzy set theory and technique.Utkin [19] presented an redundancy optimization problem by fuzzy reliability and fuzzy cost performance.Using the fuzzy set theoretic approach Cai et al [2], [3], and [4] introduced the fuzzy state assumption and possibility assumption to replace binary state assumption and probability assumption.Chen[5] presented a method for fuzzy system reliability analysis using Simplified Fuzzy Arithmetic Operations of fuzzy numbers, in which the reliability of each component is represented by a triangular fuzzy number.

Almostthe problem of series, parallel or series-parallel system reliability may be formed as a typical non-linear programming with cost functions in fuzzy environment. Park [13] presented a two-component series system subject to a single constraint by fuzzy non-linear programming technique. Ruan and Sun [14] presented an exact method for cost minimization problem in series reliability system with multiple component choices. Sung et al [17] presented a reliability optimization problems for a series system with multiple-choice to maximize the system reliability subject to the system budget.Ezzatallah [10] presented an Evaluation ofsystem reliability using fuzzy lifetime distribution, in which theparametersare taken as trapezoidal fuzzy number.

SardarDonighi et al [15] presented a new approach in fuzzy reliability model for series-parallel system in which beta type distribution is used as its membership function.Liu C.M [9] presented redundancy – reliability allocation problems in multi-stage series parallel system under uncertain environment.Mahapatra and Roy [12] presented optimal redundancy allocation problem in series-parallel system using generalized fuzzy number to findout themaximum system reliability subject to available cost and weight.Ezzatallahet al[11] analyzing fuzzy system reliability ofa seriesand parallel system, using fuzzy confidence interval.

In this paper we haveconsidered the transmission system network for mixed series and parallel configuration. In General the transmission system design is series-parallel configuration or mixed series and parallel configuration. In series-parallel configuration, there are 'm'subsystem connected in series and those subsystems consisting of'n' components in parallel. In mixed series and parallel configuration the components are connected in series and parallel.This paper presents a solution for transmission system network in mixed series-parallel configuration to evaluate the maximum reliability subject to the system cost. The cost functionsare taken asthe intervalvalued for generalized trapezoidal fuzzy number.

This paper is segmented asgiven in the below mentioned sections. Section 2 gives the notations formixedseries-parallel systems and fuzzy mathematics prerequisites.Section 3 gives the reliability optimization model incrisp and fuzzy environment. Section 4 gives the mathematical analysis of given model. Section 5gives the solution procedure for reliability optimization problem. Section 6 illustrates the construction of Transmission flow network for mixed series parallel system. In section 7,a conclusion has been discussed.

2. NOTATIONS

In order to formulate the problem in Series, Parallel and Mixed series-parallel system configurationthe following notations has been developed.

R_i- Reliability of series system i, for i=1,2 ... n

R_j- Reliability of parallel system j, for j=1,2 ... n

r_i^L -Left interval valueof reliability in subsystem i, for i=1,2 ... n

r_i^R -Right interval valueof reliability in subsystem i, for i=1,2 ... n

c_i^L - Left interval cost value of component in subsystem i, for i=1,2 ...n

c_i^R - Right interval cost value of component in subsystem i, for i=1,2 ...n

$R_S(R_1,R_2,...R_n)$ - System reliability of series system with each reliability R_i, for i=1,2 ... n

$R_P(R_1,R_2,...R_n)$ - System reliability of parallel system with each reliability R_j, for j=1,2 ... m

$C_S(R_1,R_2 ...R_n)$ - System cost of series system with each reliability R_i, for i=1,2 ...n

$C_P(R_1,R_2 ...R_n)$ - System cost of parallel system with each reliability R_j, for j=1,2 ...m

2.1Fuzzy Mathematics Prerequisites

2.1.1 Fuzzy set

Fuzzy Set theory was first introduced by Zadeh [20] in 1965. A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}}(x) : X \to [0,1]$ is a mapping called the membership function of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is called the membership value or degree of membership of $x \in X$ in the fuzzy set $\tilde{A}$.

2.1.2 α-cut of a Fuzzy set or Interval of confidence at level α:

An Interval of confidence at level α or α-cut of a fuzzy set $\tilde{A}$ of X is a crisp set A_α that contains all the elements of X that have membership value in $\tilde{A}$ greater than or equal to α. i.e. $A_\alpha = \{x : \mu_{\tilde{A}}(x) \ge \alpha, x \in X, \alpha \in [0,1]\}$.

2.1.3 GeneralizedTrapezoidal Fuzzy Number:

Let $\tilde{A}$ be generalized trapezoidalFuzzy number (GTFN), $\tilde{A} = (a, b, c, d : \omega)$, where $0 \le \omega \le 1$ and a, b, c and d $(a \le b \le c \le d)$ are real numbers. The generalizedtrapezoidal fuzzy number $\tilde{A}$ is a fuzzy set of real line R, whose membership function $\mu_{\tilde{A}}(x) : X \to [0,1]$ is defined as follows.

$$\mu_{\tilde{A}} = \begin{cases} \omega\dfrac{x-a}{b-a} & for \quad a \le x \le b \\ \omega & for \quad b \le x \le c \\ \omega\dfrac{d-x}{d-c} & for \quad c \le x \le d \\ 0 & otherwise \end{cases}$$

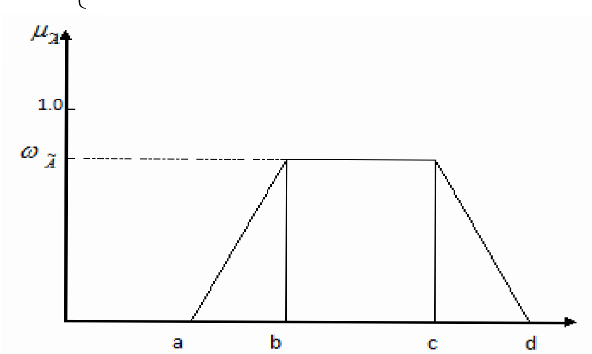

Figure 1: Generalized Trapezoidal Fuzzy Number

3. Reliability Optimization Model

Reliability optimization is described as the selection of components and a system design to maximize the reliability. The reliability of the components measured for the system is generally not known in practice and must be predictable from data. Therefore there is some uncertainty associated with the reliability estimate of the component.In practical, the problem of series, parallel or its combination system reliability may be formed as a usual non-linear programming with cost functions in fuzzy environment.

3.1 Formulation of System Reliability

In series system reliabilitythe‘n’subsystem is connected in series as in figure 2 ,in parallel system reliabilitythe ‘m’subsystem connected in parallel as in figure 3 and mixed system is combination of series and parallel system.

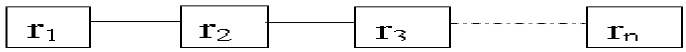

Figure 2: Configuration of Series System Reliability

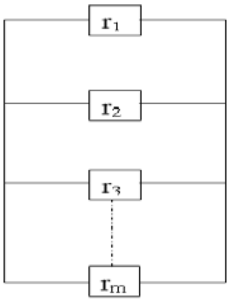

Figure3: Configuration of Parallel System Reliability

Then, the system reliability in the form of reliability of subsystemis defined as

$$R = \begin{cases} \prod_{i=1}^{n} R_i & \text{for series system} \\ 1 - \prod_{j=1}^{m} (1 - R_i) & \text{for parallel system} \\ \text{or combination of series and parall system} & \end{cases} \qquad \ldots\ldots\ldots\ldots 1$$

In reliability optimization problems, we have to find the maximization of reliability subject to several constraints. Forinstance designer is required to minimize the system cost while simultaneously maximizing the system reliability.

Consider the series system model with n components, then the maximization of reliability $R_S(R_1, R_2, R_3 \ldots R_n)$ having subject to the limited available cost c_i i=1,2,…,n and thecost constraint C_S is

$$\text{Max} \quad R_S(R_1, R_2, R_3 \ldots R_n) = \prod_{i=1}^{n} R_i$$

$$\text{Subject to} \quad C_S(R_1, R_2, R_3 \ldots R_n) = \sum_{i=1}^{n} C_i \, R_i \le C_S \qquad \ldots\ldots 2$$

Consider the parallel system model with m components, then the maximization of reliability $R_P(R_1, R_2, R_3.....R_m)$ having subject to the limited available cost c_j, j=1,2,…,m and thecost constraint C_P is

$$\text{Max} \qquad R_P(R_1, R_2, R_3.....R_m) = 1-\prod_{j=1}^{m}(1-R_j)$$

$$\text{Subject to} \qquad C_P(R_1, R_2, R_3.....R_m) = \sum_{j=1}^{m} C_j R_j \le C_P \qquad ......3$$

To perform the objective of mixed system, both series and parallel system is required in action together.

3.2 System Reliability Optimization in Fuzzy Environment

To analyze the fuzzy system reliability, the system reliability of cost component and cost constraints can be involved in uncertain factorsandis represented as fuzzy number.So the constraint of system reliability becomes fuzzy in a reliability optimization problem.Therefore it can be represented as fuzzy non-linear programmingwith fuzzy number.

Then maximization of series system reliability $R_S(R_1, R_2, R_3.....R_n)$ is

$$\text{Max} \qquad R_S(R_1, R_2, R_3.....R_n) = \prod_{i=1}^{n} R_i$$

$$\text{Subject to } C_S(R_1, R_2, R_3.....R_n) = \sum_{i=1}^{n} \tilde{C}_i R_i \le \tilde{C}_S \quad , 0 \le R_i \le 1, \tilde{C}_i \text{ and } \tilde{C} \ge 0 \qquad .... 4$$

The maximization of parallel system reliability $R_P(R_1, R_2, R_3.....R_m)$ is

$$\text{Max} \qquad R_P(R_1, R_2, R_3.....R_m) = 1-\prod_{j=1}^{m}(1-R_j)$$

$$\text{Subject to } C_P(R_1, R_2, R_3.....R_m) = \sum_{i=1}^{m} \tilde{C}_j R_j \le \tilde{C}_P \; 0 \le R_j \le 1, \tilde{C}_j \text{ and } \tilde{C} \ge 0 \qquad ......5$$

4. Mathematical Analysis

Consider a non-linear programming problem having one inequality constraint of the type

Maximize $Z = f(x_1, x_2 \ldots x_n)$

Subject to $h(x) \le 0$ where $h(x) = g(x_1, x_2 \ldots x_n) - b, h(x) \ge 0$ …....6

By Kuhn-Tucker condition the necessary conditions for the maximization in non-linear programming problem can be summarized as

$$\frac{\partial f}{\partial x_j} - \lambda \frac{\partial h}{\partial x_j} = 0 \quad \text{…...7}$$
$$\lambda\, h(x) = 0, where\ h(x)\ and\ \lambda \geq 0$$

The objective function and constraint (5) into fuzzy non-linear programming problem is as follows

Max z = f($x_1, x_2 \ldots x_n$) Sub to $\tilde{h}(x) \leq 0$ ...8

5.Solution procedure for Reliability Optimization Problems

Step:1

Let the cost parameter be $\tilde{c}_i = (c_{i1}, c_{i2}, c_{i3}, c_{i4} : \omega)$ and the cost constraint be $\tilde{C}_i = (C_{i1}, C_{i2}, C_{i3}, C_{i4} : \omega)$ i = 1, 2...n, are taken asweighted Trapezoidal fuzzy number.

Step : 2

Using α cut membership function of cost parameters and cost constraints are given by $\tilde{c}_i = [c_{i1} + \frac{\alpha(c_{i2} - c_{i1})}{\omega}, c_{i4} - \frac{\alpha(c_{i4} - c_{i3})}{\omega}], \quad i = 1,2,3,...,n$ and

$\tilde{C}_i = [C_{i1} + \frac{\alpha(C_{i2} - C_{i1})}{\omega}, C_{i4} - \frac{\alpha(C_{i4} - C_{i3})}{\omega}], \ i = 1,2,3,....,n$ respectively.

Step : 3

Applying the Kuhn - Tucker condition in a fuzzy non-linear programming problem for given modelsfor i = 1, 2... n of left and right interval α-cutis expressed as

Max $R_S(R_i^L) = 1 - \prod_{i=1}^{n}(1 - r_i^L)$

Subject to $\sum_{i=1}^{n} \tilde{c}_i^L \ r_i - \tilde{C}_i^L \leq 0 \quad where \quad 0 < r_i \leq 1, C_i^L \geq 0$ and

Max $R_S(R_i^R) = 1 - \prod_{i=1}^{m}(1 - r_i^R)$

Subject to $\sum_{i=1}^{m} \tilde{c}_i^R \ r_i - \tilde{C}_i^R \leq 0 \quad where \quad 0 < r_i \leq 1, C_i^R \geq 0$

Step : 4

To find the optimal solution of $R_S(R_i^L)$ and $R_S(R_i^L)$ i = 1, 2... n for each membership value of αfrom step-2,3.

Step 5:

To calculate the system reliability $R_S^L = \prod_{i=1}^{n} R_S(R_i^L)$ and $R_S^R = \prod_{i=1}^{n} R_S(R_i^R)$ i=1,2,…,n for series system reliability.

Step: 6

Apply the above same procedure calculate the system reliability $R_P^L = 1 - \prod_{j=1}^{m}(1 - R_S(R_j^L))$ and $R_P^R = 1 - \prod_{j=1}^{m}(1 - R_S(R_j^R))$, j=1, 2… m for parallel system reliability.

Step:7

Using series and parallel system reliability, find the maximum reliability of mixed system for each membership valuesof α.

6.IllustrativeExamples:

For numerical explanation here, we have considered the transmission system network for mixed system reliability,which can be maximized for system reliability subject to available cost. The design configuration of transmission system network is presented in figure 4 as follows.

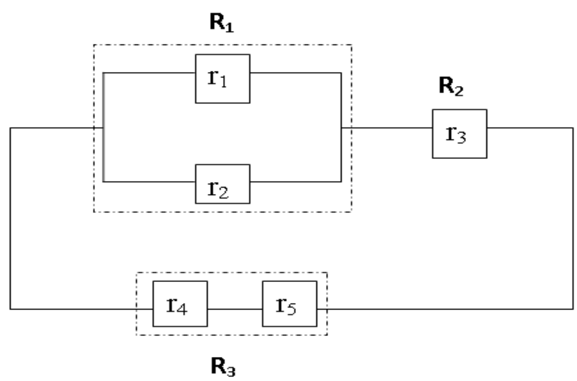

Fig.4Design Configuration of transmission system Network

It consists of three main units (subsystems) connected in mixed system. The first unit consisting of 2 components in parallel, secondunit consists of single component connected with series of first system and third component consisting of 2 components in series connected with parallel to first two units.In reliability optimization model, assume that cost components, cost constraints and system weight are fuzzy in nature, where the cost components and cost constraints are taken as weighted trapezoidal fuzzy number.

For Unit1, the cost components are c_1 = (18, 18.5, 19.0, 19.5: 0.7),

c_2 = (29.25, 30.0, 31.0, 31.75:0.8) and cost constraints is C_1 = (43.0, 43.75, 44.5, 45:0.85)

For Unit2, the cost components is c_3 = (25.0, 25.50, 26.5 27.0: 0.75) and cost constraints is C_2 = (24, 24.5,25.0, 25.5:0.8)

For Unit3, the cost components are c_4 = (16, 16.5, 17.5, 18: 0.9),

c_5 = (33, 33.75, 34.4, 35:0.8) and cost constraints is C_3 = (51.0, 51.5, 52.5, 53:0.85)

The cost components and constraints of left and right interval α-cut membership function aretabulated as follows.

Unit (i)	Cost components(c_i)	Cost constraints(C_i)
1	$c_1 = (c_1^L, c_1^R)$ = (18.0+0.714286α , 19.5-0.714286α) $c_2 = (c_2^L, c_2^R)$ = (29.5+0.937500α , 31.5-0.937500α)	$C_1 = (C_1^L, C_1^R)$ = (43.0 +0.882353α, 45.0-0.588235α)
2	$c_3 = (c_3^L, c_3^R)$ = (25+0.666667α , 27.0-0.6666667α)	$C_2 = (C_2^L, C_2^R)$ = (24 +0.625000α , 25.5- 0.625000α)
3	$c_4 = (c_4^L, c_4^R)$ = (16+0.555555α , 18.0 -0.555556α) $c_5 = (c_5^L, c_5^R)$ = (17.5+0.625000α , 19.5-0.937500α)	$C_3 = (C_3^L, C_3^R)$ = (30.0+0.588235α, 32.5- 0.588235α)

The following table 1 and table 2show the left and right interval optimal solution of transmission network reliability system through fuzzy parametric non-linear programming.

Table 1: Left intervaloptimal solution of fuzzy membership value of α

α	r_{11}	r_{12}	R_1	R_2	r_{31}	r_{32}	R_3	R
0.0	0.875000	0.923729	0.990466	0.960000	0.953125	0.871429	0.830581	0.991673
0.2	0.871743	0.921619	0.989947	0.959881	0.950203	0.868586	0.825333	0.991307
0.4	0.868647	0.919535	0.989431	0.959763	0.947321	0.865783	0.820174	0.990940
0.6	0.865381	0.917478	0.988891	0.959646	0.944478	0.863019	0.815102	0.990568
0.8	0.862274	0.915446	0.988355	0.959530	0.941673	0.860294	0.810116	0.990194
1.0	0.859214	0.913438	0.987813	0.959416	0.938907	0.857606	0.805212	0.989817

Table 2: Right interval optimal solution of fuzzy membership value of α

α	r_{11}	r_{12}	R_1	R_2	r_{31}	r_{32}	R_3	R
0.0	0.839744	0.901575	0.984227	0.944444	0.902778	0.833333	0.752315	0.982550

0.2	0.844067	0.904360	0.985087	0.944479	0.905096	0.838377	0.758812	0.983212
0.4	0.848431	0.907178	0.985931	0.944451	0.907445	0.843522	0.765450	0.983854
0.6	0.852872	0.910030	0.986763	0.944549	0.909822	0.848767	0.772227	0.984522
0.8	0.857381	0.912917	0.987580	0.944584	0.912230	0.852118	0.777328	0.985048
1.0	0.861958	0.915839	0.988382	0.944620	0.914669	0.859576	0.786228	0.985815

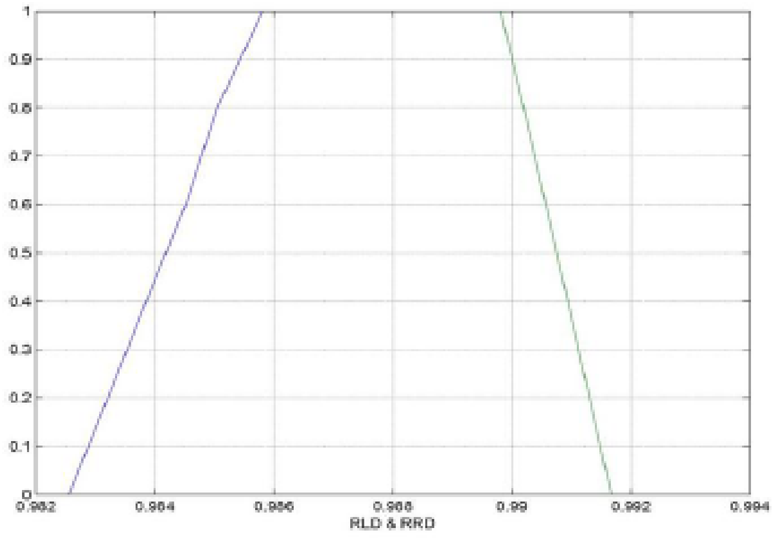

Figure 5: Interval Valued Reliability with Weighted Trapezoidal Fuzzy Function

7. Conclusion

To analyze the fuzzy system reliability, the reliability of each component of the system is represented as fuzzy number in nature. The reliability evaluation of a system design with high reliability and low requirement, reveals the system designer can adopt series, parallel and mixed system technique to improve system reliability with available constraints such as cost, weight etc. Transmission network system is a good example of mixed system reliability. This paper attempts to provide the reliability optimization problem of the transmission system network under the fuzzy environment, where we maximize the reliability subject to the available cost and the cost component and cost constraint of each subsystem is taken as weighted Trapezoidal fuzzy number. Kuhn-Tucker conditions are taken into concern to solve the non-linear programming problem with fuzzy components to find out the optimal solutions for left and right interval valued membership function of α. This can be maximized for the system reliability subject to the available cost. Table 1, 2 shows that the left and right interval optimal solution of transmission network system reliability subject to the cost constraints, which identifies that maximum reliability with different values of the membership function. The proposed methodology used for mixed system reliability involves optimization.

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