Research Paper

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Analysis of The Role of Catastrophes and Failure Processes in A Multi - Server Queue

Mathematics

KEYWORDS : Multi server, catastrophe, failure process, impatient units, mean and variance.

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ABSTRACT

A multi server Markovian queue with catastrophe and failure model is considered. Due to the occurrence of catastrophe in this system, all the units in the system are rejected as well as servers stop working. A repair process and the arrival of impatient units follow exponential distribution with different parameters. The mathematical expressions for the probability generating functions under the system is down and that is functioning are obtained mean size under the above conditions are derived.

1. INTRODUCTION

The queueing Theory has played an important role in the theory of probability and related concepts. Its applications have been utilised varies fields like communication system, industrial sector and so on. Human beings, telephone calls flow of finished products, failed machines and so on may be considered as queueing units. In modern days, the queueing models have been analysed by assuming the telephone calls as the units for demanding service.

The behaviour of telecommunications over a time period, the time dependent analysis of queues has been required, since the steady state relates are inappropriate in situations where the time horizon of operations is finite.

Transient state measures are very important to track down the functioning of the system at any instant of time. In telecommunication systems, especially in wide area network, data are combined into a signal which is then divided by the network management packets and is sent through the network path. The routing is more complex and dynamic. At each node the processor detects the packet address and sends it through the best available path to the next node. As the transmission channel is an electrical medium (e.g. wires, coaxial cable and optical fiber) packets are often subjected to distortion. We have “repeaters” in the transmission system which rebuild or regenerate the packet (signal) into its original form. This is a situation which demands the time dependent analysis of feedback queues. The detection, decision and path routing are done for each packet in real time and so the packets do not pile up for service. Even if there is a reasonable size buffer or queue, they would soon overflow resulting the packet (message) loss or refusal. At times, packets are held up in nodes due to unavailability of paths in the network. This causes delay of several seconds or minutes for the complete message to reach the intended receiver. Finally this ends up in server service degradation.

Consider a multi server Markovian queueing system which operates on telecommunications. This system suffers random failures, when occurring catastrophes, all connections are cut and all existing requests are rejected and lost. The system then goes through a repair process whose duration is random. Meanwhile, while the system is down, the stream of newly arriving request (customers) continues, but the customers become impatient; each customer ‘activates’ his own ‘timer’ with random duration T, such that, it the system is still down when the timer expires, the customer abandons the system never to return.

2. REVIEW OF LITERATURE

Models with customers impatience in queues have bean studied by varies researchers. **Krishna Kumar** and **Arivudainambi (2000)** have analysed transient solution M /M/ 1 queue with catastrophes. The expression for the probability of the server being ideal, mean queue size and steady state probabilities are derived. **Krishna Kumar** and **Pavaimadheswari (2002)** have analysed two server queueing model with transient behaviour. The presence of catastrophes in the ongoing service and the feature effects are discussed.

The source of impatience either a long Waite already experience in the queue or a long Waite anticipated by a customer on arrival. **Altman and Yechiali (2005)** and **(2006)** have analysed with customers impatiences when the server on vacation and unavailable for service.

Sudhesh (2010) have argued that the queueing models with disasters teem to be appropriated in some computer network or telecommunications applications and derived transient solution of a single server queue with system disaster and customer impatience.

Chandrasekaran and **Saravanarajan (2012)** have used the continued fraction technique to obtain the transient solution of the M/ M/ 1 queue with feedback subject to catastrophe, server failures and repairs.

Ayyappan et al (2013) have discussed a bulk service queue under catastrophe. The probability generating function and Rouche’s Theorem are in the designed model. The analytical solutions for mean and variance of the number of customers in the system are derived. In addition that these measures are discussed based on different parameters.

Vidhya (2013) has studied a single server Bulk size Markovian queue along with break down due to disastrous. The probability generating function of the model and expected queue length have been derived.

Nompazy and **Yechiali (2014)** have studied an M/M/1 queue in a multi-phase random environment, where the system suffers a disastrous failure, causing on present job to be lost. The system at once moves to repair phase. After the repair is over it moves to i^{th} phase. The probability generating methods have been applied to study the behaviour of the system.

The mean queue sizes, the mean waiting times and fraction of lost customers have been derived.

In this paper, a multi server Markovian queueing system with catastrophes and customer impatience when the system is down are considered. This model is an extension of **Yechiali (2007)** and **Sudhesh (2010)**. The probability generating functions of the system under the system is down and that is functioning and serving the units. The mean queue size, factorial moments and Variance of the system under the above said conditions are derived.

3. DESCRIPTION OF THE MODEL

Consider a multi server Markovin queue with different techniques. The inter arrival times of units fallow exponential distribution with mean arrival rate . The service times are distributed exponentially with parameter . During the service process of units the system may breakdown due to the occurrence of catastrophe which follows Poisson distribution with parameter . It is noted that all the units present in this system are rejected and lost, in addition that, all servers stop working whenever the system breakdown. A repair process starts at once when occurring the failure which is also exponentially distributed with parameter . When the system is undergoing a repair proess, newely arriving units become impatient and each unit activates a timer which is exponentially distributed ith parameter .

The above process forms a two dimensional continuous time Markov process with state space S = {(j, n): j = 0, 1; n = 0,1,2,....}. j = 0 indicates that the system is down undergoing repair process and j = 1 denotes that the system is functioning and serving the units. n = 0,1,2,..... denotes the number of units in the system. $P_{j,n}$ (t) denotes the transient state probability that the system is in the state j = 0 or 1 and n, (n = 0,1,2,.....) units in the system at time t.

4. TRANSIENT AND STEADY - STATE EQUATIONS

Based on the above stated parameters and transition probabilities, Kolmogorov differential difference equations are framed.

$$P\_0,0^{\wedge\prime}(t) = -(\lambda+\gamma)\, P\_0,0\,(t) + \epsilon\, P\_0,1\,(t) + \eta\textstyle\sum\_(n=0)^\wedge\infty \; [P\_(1,n)\,(t)] \qquad (1)$$

$$P'_{0,n}(t) = -(\lambda+\gamma+n\epsilon)P_{0,n}(t) + \lambda\, P_{0,n-1}(t) + (n+1)\epsilon P_{0,n+1}(t); \quad n \geq 1 \qquad (2)$$

$$P\_1,0^{\wedge\prime}(t) = -(\lambda+\gamma)\, P\_1,0\,(t) + \gamma\, P\_0,0\,(t) + \mu P\_1,1\,(t) \qquad (3)$$

$$P'_{1,n}(t) = -(\lambda+n\mu+\eta)P_{1,n}(t) + \lambda\, P_{1,n-1}(t) + (n+1)\mu P_{1,n+1}(t) + \gamma\, P_{0,n}(t) \quad 1 \leq n \leq c-1 \qquad (4)$$

$$P'_{1,n}(t) = -(\lambda+c\mu+\eta)P_{1,n}(t) + \lambda\, P_{1,n-1}(t) + c\mu P_{1,n+1}(t) + \gamma\, P_{0,n}(t), \quad c \leq n \leq \infty \qquad (5)$$

For applying large t in the equations from (1) to (5) which give the following steady - state equations.

$$-(\lambda+\gamma)\, P\_0,0 + \epsilon P\_0,1 + \eta\textstyle\sum\_(n=0)^\wedge\infty \; [P\_(1,n) = 0] \qquad (6)$$

$$-(\lambda+\gamma+n\epsilon)P_{0,n} + \lambda P_{0,n-1} + (n+1)\epsilon\, P_{0,n+1} = 0\,;\; n \geq 1 \qquad (7)$$

$$-(\lambda+\gamma)\, P\_1,0 + \gamma P\_0,0 + \mu P\_1,1 = 0 \qquad (8)$$

$$-(\lambda+n\mu+\eta)\, P\_(1,n) + \lambda P\_(1,n-1) + (n+1)\mu\, P\_(1,n+1) + \gamma\, P\_(0,n) = 0\,; \quad 1 \leq n \leq c-1 \qquad (9)$$

$$-(\lambda+c\mu+\eta)P_{1,n} + \lambda P_{1,n-1} + c\mu\, P_{1,n+1} + \gamma\, P_{0,n} = 0\,; \quad c \leq n \leq \infty \qquad (10)$$

5. QUEUEING PERFORMANCES

$$G_i(s) = \sum_{n=0}^{\infty} P_{i,n} s^n \,.$$

and

$$G\_i^{\wedge\prime}(s) = \textstyle\sum\_(n=1)^\wedge\infty \; [P\_(i,n) \; [ns] \;^\wedge(n-1); \quad i = 0,1] \qquad (11)$$

Multiply the equations (6) and (7) by proper powers of s and use the equations given in (11) and get,

$$(\epsilon s-\epsilon)G_0'(s) + (\lambda+\gamma-\lambda s)s\, G_0(s) = -\eta \sum_{n=0}^{\infty} P_{1,n} \qquad (12)$$

Apply the same operations in the equations (8), (9) and (10) and use equation (11) and get,

$$G_1'(s) + \left\{\frac{\lambda}{\mu}\sum_{m=0}^{\infty} s^{m+1} - \left(\frac{\lambda+\eta}{\mu}\right)\sum_{m=0}^{\infty} s^m\right\}G_1(s) - s^c \sum_{n=1}^{\infty} n\, P_{1,c+n}s^{n-1} + \frac{\gamma}{\mu}\sum_{n=c}^{\infty} P_{0,n}s^n\left(\sum_{m=0}^{\infty} s^m\right) + \{\gamma P_{0,0} - (\eta-\gamma)P_{1,0}\}\sum_{m=0}^{\infty} s^m = 0 \qquad (13)$$

In order to find $G_0(s)$, we have to solve the linear differential equation given in (12) and get,

$$G_0(s) = AB\frac{s^{n_1+2n_2+1}}{n_1+2n_2+1}\, e^{\frac{1}{\epsilon}\left[(\lambda+\gamma+\epsilon)s-\frac{\lambda}{2}s^2\right]} + k\, e^{\frac{1}{\epsilon}\left[(\lambda+\gamma+\epsilon)s-\frac{\lambda}{2}s^2\right]} \qquad (14)$$

Where $A = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\left(-\frac{1}{\epsilon}(\lambda+\gamma+\epsilon)\right)^{n_1}\left(\frac{\lambda}{2\epsilon}\right)^{n_2}\frac{1}{n_1!\,n_2!}$

and

$$B = -\eta\sum_{n=0}^{\infty} P_{1,n}$$

By using initial condition, the constant k becomes zero and the

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equation (14) reduces to

$$G_0(s) = AB\frac{s^{n_1+2n_2+1}}{n_1+2n_2+1}e^{\frac{1}{\epsilon}\left[(\lambda+\gamma+\epsilon)s-\frac{\lambda}{2}s^2\right]} \quad (15)$$

On differentiating (15) with respect to s and letting s = 1, the expected queue size, the second factorial moment and variance are obtained respectively.

$$G_0'(1) = A\left[B + \frac{\gamma+\epsilon}{\epsilon(n_1+2n_2+1)}\right]e^{\frac{1}{\epsilon}\left(\frac{\lambda}{2}+\gamma+\epsilon\right)} \quad (16)$$

$$G_0''(1) = \frac{A}{\epsilon^2(n_1+2n_2+1)}[(n_1+2n_2)B\epsilon^2 - \lambda\epsilon + (\gamma+\epsilon)\{(1+B)(n_1+2n_2+1)\epsilon+\gamma+\epsilon\}]e^{\frac{1}{\epsilon}\left(\frac{\lambda}{2}+\gamma+\epsilon\right)} \quad (17)$$

$$V_0 = A\,e^{\frac{1}{\epsilon}\left(\frac{\lambda}{2}+\gamma+\epsilon\right)}\Big[(n_1+2n_2+1)^2\epsilon^2(B+1) + (n_1+2n_2+1)(B+1) + (\gamma+\epsilon)(\gamma+2\epsilon) - \lambda\epsilon - \frac{A}{(n_1+2n_2+1)}\{B(n_1+2n_2+1)\epsilon+\gamma+\epsilon\}^2 e^{\frac{1}{\epsilon}\left(\frac{\lambda}{2}+\gamma+\epsilon\right)}\Big] \quad (18)$$

As in the case of $G_0(s)$, the linear differential equation (13) gives

$$G_1(s) = \sum_{n=1}^{\infty} n\,P_{1,c+n}\frac{s^{c+n}}{c+n+r+\delta} - \frac{\gamma}{\mu}\sum_{n=c}^{\infty}\sum_{m=0}^{\infty}P_{0,n}\frac{s^{m+n}}{m+n+1+r+\delta} + \{(\eta-\gamma)P_{1,0} - \gamma\,P_{0,0}\}\sum_{m=0}^{\infty}\frac{s^{m+1}}{m+1+r+\delta} \quad (19)$$

On differentiating equation (19) with respect to s and letting s=1, the expected queue size,the second factorial moment and variance are obtained respectively.

$$G_1'(1) = \sum_{n=1}^{\infty} n\,P_{1,c+n}\frac{c+n}{c+n+r+\delta} - \frac{\gamma}{\mu}\sum_{n=c}^{\infty}\sum_{n=1}^{\infty}P_{0,n}\frac{m+n}{m+n+1+r+\delta} + \{(\eta-\gamma)P_{1,0} - \gamma\,P_{0,0}\}\sum_{m=0}^{\infty}\frac{m+1}{m+1+r+\delta} \quad (20)$$

$$G_1''(1) = \sum_{n=2}^{\infty} n\,P_{1,c+n}\frac{(c+n)(c+n-1)}{c+n+r+\delta} - \frac{\gamma}{\mu}\sum_{n=c}^{\infty}\sum_{m=2}^{\infty}P_{0,n}\frac{(m+n)(m+n-1)}{m+n+1+r+\delta} + \{(\eta-\gamma)P_{1,0} - \gamma\,P_{0,0}\}\sum_{m=1}^{\infty}\frac{m(m+1)}{m+1+r+\delta} \quad (21)$$

$$V_1 = \left\{\sum_{n=2}^{\infty} n\,P_{1,c+n}(c+n-1) + \sum_{n=1}^{\infty} n\,P_{1,c+n}\right\}\left(\frac{c+n}{c+n+r+\delta}\right) - \frac{\gamma}{\mu}\sum_{n=c}^{\infty}\left[\sum_{m=2}^{\infty}P_{0,n}(m+n-1) + \sum_{m=1}^{\infty}P_{0,n}\right]\left(\frac{m+n}{m+n+1+r+\delta}\right) + \{(\eta-\gamma)P_{1,0} - \gamma\,P_{0,0}\}\left[\sum_{m=1}^{\infty}\frac{m(m+1)}{m+1+r+\delta} + \sum_{m=0}^{\infty}\frac{m+1}{m+1+r+\delta}\right] - \left[\sum_{n=1}^{\infty} n\,P_{1,c+n}\left(\frac{c+n}{c+n+r+\delta}\right) - \frac{\gamma}{\mu}\sum_{n=c}^{\infty}\sum_{m=1}^{\infty}P_{0,n}\left(\frac{m+1}{m+1+r+\delta}\right)\right]^2 \quad (22)$$

6. CONCLUSION

In this paper, a multi server queue in which the inter arrival times follow exponential distribution and service times follow Poisson distribution has been considered. When service is going on catastrophe occurs and the system breakdown which leads to failures and impatience. For this model, through probability generating functions, the expected queue size, factorial moments and variance under the system is down as well as that is functioning have been derived.

Suppose one may consider a single server queue, then the reduced results are identical with the results of Krishna Kumar and Arivudainambi (2000). In this existing system, if either bulk arrival or bulk service or both may introduce, then the researcher may achieve valuable results.

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