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Research Paper

Dynamic Balancing of the Rotor System Using Influence Coeffcient Method and Finite Element Method

Engineering

KEYWORDS : Modal Analysis, FEA, Frequency Response, Influence

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ABSTRACT

In all the method, influence coefficient measured from the experiment, which is later used to calculate the balance correction mass along with the phase angle. Measuring the Unbalance response each time is difficult. In this project the balancing of rotor is achieving by the using the influence coefficient method along the Finite element methodology in which response of the structure is calculated by frequency response analysis. Influence coefficient suggested by Thearle in 1934 is two planes balancing method need not required the rotor and bearing properties to perform the calculation. Only the assumption is that the vibration measured is linear product of the unbalance and influence coefficients.To use the influence coefficient method at least two planes are required with arbitrary unbalance in the rotor. By using this method balance correction along with the relative phase angle is calculated which is used to simulate the modified model of rotor system in the finite element. Modal analysis and frequency response analysis is conducted in FE software to generate the mode shape and response in the structure. The simulation is carried on original as well as on the modified design to compare the response. The analysis performed is similar to the lateral analysis of the rotor and similar boundary condition is applied to the rotor shaft.

INTRODUCTION

Lateral analysis of shaft is performed in the finite element model. The dynamic analysis of rotor system is performed to determine the Eigen frequencies, mode shape and response of the rotor system. Response is evaluated at the original rotor system and by using the influence coefficient method, position of correction weight in the rotor is determined along with the phase angle.

Based on the result of the influence coefficient method, modification is implemented in the rotor system and finite element analysis is performed again to evaluate the response. Purpose of this work is to reduce the vibration in the rotor system after the modification.

Complete rotor assembly is shown in Figure 1. Rotor assembly composed of Two Disc, shaft and unbalance masses. Detail of individual component of the rotor and physical dimension of the rotor is tabulated in Table 1. In the original condition of the rotor assembly, unbalance is created by two unbalance masses mounted on different disc. Unbalance mass of 3gm is mounted on disc 1 at phase angle of 0° (θ_1)and 5gm is mounted on disc 2 at phase angle of 240°(θ_2). These phase angle is measured clockwise when viewed from the positive end of Y. Detail of the angular position of unbalance mass in original condition of the rotor is shown in Figure 2.

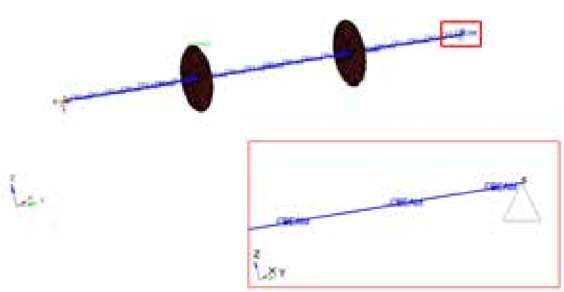

Figure 1: Geometry – Rotor System

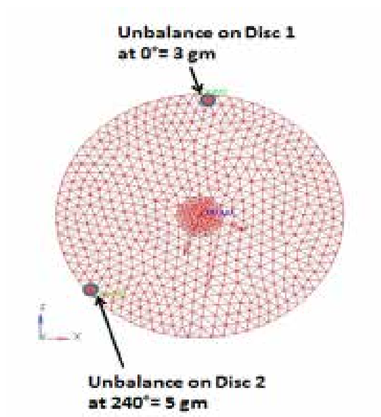

Figure 2: Rotor: Original condition

TABLE – 1
Physical dimension of rotor system

Rotor Dimension	
Shaft Diameter	10 mm
Length of shaft	400 mm
Mass of each Disc	800 gm
Distance between left bearing and left disc	137.5 mm
Distance between Disc1 and Disc2	157 mm
Diameter of Disc1 and Disc2	60 mm
Phase angle of Disc1 (θ1)	0°
Initial Phase angle of Disc2 (θ2)	240°
Speed Range	0-4000 rad/s
Mass Moment of inertia Ixx (ton-mm2)	78.6
Mass Moment of inertia Iyy (ton-mm2)	4.6
Mass Moment of inertia I_{zz}(ton-mm^2)	78.6

SCOPE OF WORK

Scope of work is to develop the methodology of balancing the rotor system using combination of analytical calculation and finite element method. Analytical calculation is performed using the influence coefficient method and response is predicted using finite element method. This paper can be used to understand the methodology used to create the model for influence coefficient method and finite element model.

FE methodology covers Geometry import, geometry clean up, procedure to create meshed model, frequency response analysis, material and mass application, boundary conditions application, post processing results, interpretation of results and comparing the result of original design with the modified design.

METHODOLOGY

- **Influence coefficient method**

To use the influence coefficient method at least two planes are required with arbitrary unbalance in the rotor. By using this method balance correction along with the relative phase angle is calculated which is used to simulate the modified model of rotor system in the finite element.

To employ the influence coefficient method of balancing, rotor is spun at fix speed and amplitude of response along with the phase angle is recorded. Initial response at two planes for the original condition at fixed speed is as follow

$$Z_1(omega) = Z_1\angle\theta 1 \ , Z_2(omega) = Z_2\angle\theta 2$$

Where Z is the amplitude measured and θ is the relative phase angle. Relation of vibration reading and unknown balances are represented as;

$$Z_{11} = a_{11}(omega)U_1 + a_{12}(omega)U_2$$
$$Z_{21} = a_{21}(omega)U_1 + a_{22}(omega)U_2$$

Where Z_{11} and Z_{21} are new vibration reading recorded at disc1 and disc2 for the first trial weight on the disc1.From the above equations influence coefficient is as follow;

$$a_{11} = \frac{Z_2 - Z_1}{u_{r1}} \qquad a_{21} = \frac{Z_{21} - Z_2}{u_{r1}}$$

First trial weight from the disc1 is removed and second trial weight is added on the disc2 and vibrations reading are recorded again. The resulting vibration will be as follow

$$Z_{12} = a_{11}(omega)U_1 + a_{12}(omega)(U_1 + U_{r2})$$
$$Z_{21} = a_{21}(omega)U_1 + a_{22}(omega)(U_2 + U_{r2})$$

New influence coefficients are as follow;

$$a_{12} = \frac{Z_{12} - Z_1}{u_{r2}} \ , \ a_{22} = \frac{Z_{22} - Z_1}{u_{r2}}$$

In the matrix form this equation will be as follow;

$$\begin{Bmatrix} Z_1 \\ Z_2 \end{Bmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{Bmatrix} U_{r1} \\ U_{r2} \end{Bmatrix}$$

And balance correction weights are

$$\begin{Bmatrix} U_{r1} \\ U_{r2} \end{Bmatrix} = -\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1} \begin{Bmatrix} Z_1 \\ Z_2 \end{Bmatrix}$$

So, the balance correction weight for the disc1 is given by

$$U_{r1} = \frac{Z_1 a_{22} - Z_2 a_{12}}{delta}$$

delta is determinant of the influence coefficients. Balance correction weight for the disc2 is given by

$$U_{r2} = \frac{Z_2 a_{11} - Z_1 a_{21}}{delta}$$

This U_{r1} and U_{r2} represent the position of the correction weight on the disc1 and disc2 along with the phase angle.

- **Finite Element Method**

The rotor shaft is modeled as 1 dimensional beam element, as the shaft is long member compared to the other dimension. To execute the rotor dynamics problem in finite element six degree of freedom is required in each node. Thus the rotor shaft is meshed with the 1D beam element. Both the disc is meshed with the higher order shell element (Tria6). Unbalance masses on the disc are modeled by using the lumped mass which is node merged with the shell node at the outer periphery. Steel material is considered for the analysis. To perform the modal analysis, dynamic characteristic of the lumped masses is assigned by the mass moment of inertia to the lumped masses. Mass and FE modeling strategy is described in detail in Table 2

TABLE – 2
Mass and FE details

Components	Total Mass [kg]	FE detail
Shaft	260	1D Beam element
Disc1	800	Shell Mesh (2nd order Tria element)
Disc2	800	Shell Mesh (2nd order Tria element)
Unbalance mass1 (on disc1)	3	Lumped Mass
Unbalance mass2(on disc2)	5	Lumped Mass

The FE model is meshed in Hypermesh software and FEA results are pre-processed and post processed within the Altair software. The input deck file is created in hyper mesh. Then this input deck is submitted to Nastran and output file (.op2) is generated. Influence coefficient method is used to calculate the balance correction weight. All the vibration reading referred from the FE output of the frequency response analysis. Various Iterations were performed in which the trial masses are kept in both the plane one by one and performed the frequency response analysis. Based on the vibration reading from the FE data, vibration reading along with the phase angle is used in the influence coefficient method and correct balancing weight along with the phase degree is calculated. This provide the next guideline for the modified rotor system

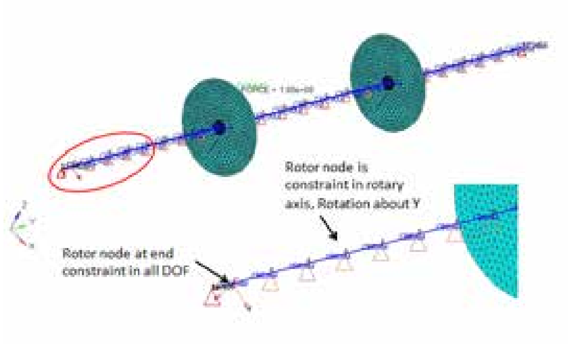

Figure 3: Boundary condition: Lateral Analysis

RESULT AND DISCUSSION

Influence coefficient method is used to calculate the balance correction weight. To employ the influence coefficient method of balancing, rotor is spun with a speed of 2000 rad/s which kept remain constant. This condition is treated as the original condition of the rotor. Hence first the reading is measure at the original condition.

Initial response at the two planes for the original condition along with the phase are measured from FE output which is given in Eq.1 and Eq.2

$$Z_1 = 2.5mm\angle 172^\circ \qquad .... (1)$$

$Z_2 = 1.7mm\angle 188^\circ$ .... (2)

To proceed with the influence coefficient calculation, First trial weight of 3 gram is added on disc1 at phase angle of 95° as shown in Eq.3

$= 3gm\angle 95^\circ$ .... (3)

, which can be written as

$u_{r1} = 3(\cos 95 + j\sin 95)$

$u_{r1} = 2.98 - j0.2614$

With adding trial weight on disc1, Response is evaluated from the FE model and measured at disc 1 and disc 2. Resultant responses are

$Z_{11} = 5.3mm\angle 127^\circ$, $Z_{21} = 3.3mm\angle 7.2^\circ$ .... (4)

The Influence coefficient a_{11} and a_{21} at disc1 and disc2 respectively is calculated using following expression

$$a_{11} = \frac{(4.22 - j3.22) - (0.348 - j2.475)}{2.98 - j0.2614}$$

$$a_{11} = 1.312 - j0.135(mm/gm)$$

$$a_{11} = \frac{(1.7\cos 188^\circ + 1.7\sin 188^\circ) - (2.5\cos 172^\circ + 2.5\sin 172^\circ)}{3\cos 95^\circ + 3\sin 95^\circ}$$

Similarly a_{21} is calculated using;

Now first trial weight is removed and second trial weight of 3

$$a_{21} = \frac{3.3\angle 7.2^\circ - 1.7\angle 188^\circ}{3\angle 95^\circ} = 1.13 + j0.8(mm/gm)$$

gram is added on disc2 at phase angle of 273° as shown in Eq.

$3gm\angle 273^\circ$ .... (5)

, Which can be written as;

$u_{r2} = -2.995 + j0.157$

With adding this trial weight, Response is evaluated from the FE model and measured at disc 1 and disc 2. Resultant responses are

$Z_{12} = 1.7mm\angle 140^\circ$, $Z_{22} = 0.26mm\angle 212^\circ$

The influence coefficient a_{12} and a_{22} at disc1 and disc2 respectively is calculated using following expression

, whereas a_{22} by using the conjugate method of vector.

$$a_{12} = \frac{1.7\angle 140^\circ - 2.5\angle 172^\circ}{3\angle 95^\circ} = -0.226 - j0.403(mm/gm)$$

$$a_{22} = \frac{0.26\angle 212^\circ - 2.5\angle 172^\circ}{3\angle 95^\circ} = -0.007 - j0.503(mm/gm)$$

Balance correction weight at disc1 (Ur1) is calculated by expression as shown;

$$U_{r1} = \frac{(0.348 - 2.475j)(-0.007 - 0.503j) - (-0.236 - 1.68j)(-0.226 - 0.403j}{-0.13 - 0.029j}$$

$$U_{r1} = -1.57 + 14.89j$$

$U_{r1} = 14.9gm\angle 89^\circ$ .... (6)

Similarly balance correction weight at disc2 (Ur2) is calculated by expression as shown;

$$U_{r2} = \frac{(-0.236 - 1.68j)(1.312 - 0.135j) - (0.348 - 2.475j)(1.13 + 0.8j)}{-0.13 - 0.029j}$$

$$U_{r2} = 2.62 - 0.9j$$

$U_{r2} = 2.77gm\angle 19^\circ$ .... (7)

U_{r1} and U_{r2} are the calculate balance weight on each disc along with the relative phase angles. From the analytical calculation, Balance correction weight at plane is 14.9gm (U_{r1}) at phase angle of 89° and 2.77gm (U_{r2}) at phase angle of 19°. Detail of the correction weight is shown in Figure 4.

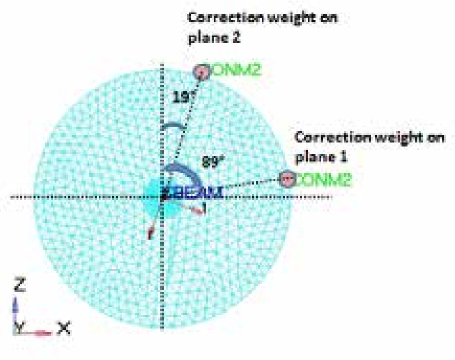

Figure 4: Correction weight at phase angle by influence coefficient method

- **Modal Analysis**

Modal analysis is performed on the original model of rotor system with set unbalance to evaluate the Eigen frequencies and mode shapes of the rotor system. From the modal analysis the mode shape of the rotor system is evaluated. Using this method different Eigen vector has been identified. From the result following observation can be drawn. This analysis in significant to decide whether system is achieving response at same frequencies.

First mode is the bending mode with zero nodes forming at the axis of rotation. First fundamental frequency or critical speed of the rotor is 88 Hz i.e. 5280 rpm. Details of the mode shape are shown in Figure 5.

Second mode is the bending mode with one node forming at the axis of rotation. Second fundamental frequency of the rotor is 190 Hz i.e. the second critical speed of the rotor is at 190 Hz i.e. 11400 rpm. Detail of the mode shape is shown in Figure 6.

Third mode is the bending mode with two node forming at the axis of rotation. Third fundamental frequency of the rotor is 422 Hz i.e. the third critical speed of the rotor is at 422 Hz i.e. 25320 rpm. Detail of the mode shape is shown in Figure 7.

In Table 3 all the extracted modes and critical frequencies are tabulated.

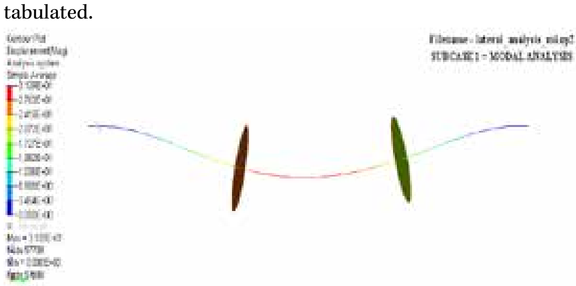

Figure 5: Rotor with first mode shape at 88 Hz or 5280 rpm

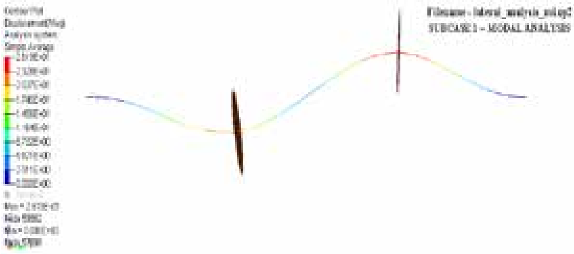

Figure 6: Rotor with second mode shape at 190 Hz or 11400 rpm

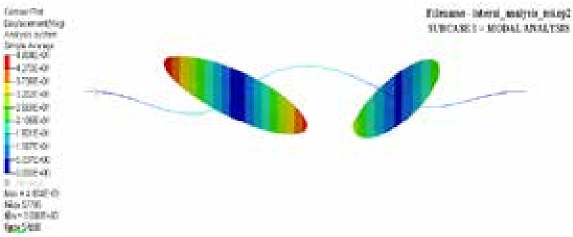

Figure 7: Rotor with third mode shape at 422 Hz or 25320 rpm

TABLE – 3
Eigen Value and mode shape behaviour

Mode	Frequency [Hz]	Mode Shape
1	88	1st mode shape of rotor; Bending of rotor with zero node
2	190	2nd mode shape of rotor; Bending of rotor with one node
3	422	3rd mode shape of rotor; Bending of rotor with two node

- **Frequency Response Analysis**

Original Design
Frequency response analysis is performed on the original model of the rotor system to measure the response. This amplitude is measured in form of displacement (mm) along with the phase angle. Centrifugal forces are considered with varying speed of rotor to excite the rotor system. Response is measured at various location of the rotor system such as rotor shaft, Disc1 and Disc2. Detail response on the disc1 is shown in Figure 8. First peak of the response is observed in between the 85-90 Hz, which is same as frequency observed in the modal analysis. From the graph it is observed that the maximum response on the disc1 is 3mm @88 Hz. Similarly Second peak of the response is observed at 190 HZ. No response is observed at 422 Hz; hence this frequency is not the critical speed of the rotor.

Maximum amplitude of 3mm @88Hz is also observed at Disc2. Detail response on the disc2 is shown in Figure 9. Vibration is measured at various location of rotor, such as left side of disc1, middle of disc1 and disc2 and right side of disc2. Maximum vibration observed on the rotor is 1.6 mm@88 Hz. Detail vibration on the rotor shaft is shown in Figure 10.

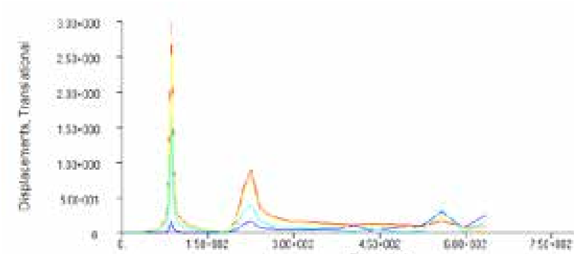

Figure 8: Original Design: Response in the rotor disc1

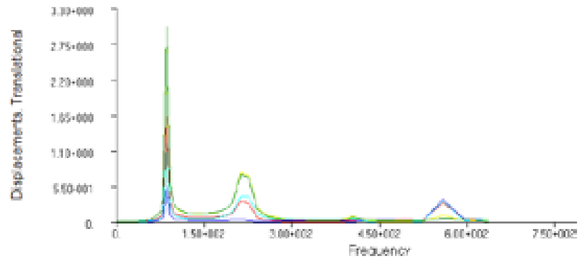

Figure 9: Original Design: Response in the rotor disc2

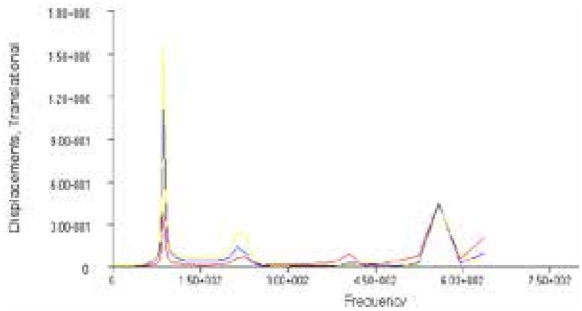

Figure 10: Original Design: Response in the rotor

Modified Design
To reduce the vibration due to unbalance in the rotor system correction weight which are calculated by influence coefficient method is required to be reposition in the rotor system. After the repositioning of the correction weight, frequency response analysis is performed again on the modified rotor and response is measured on the rotor.

From the response graph it is observed that the maximum response on the disc is 1.5mm @88 Hz. As shown in Figure 11. Similarly Maximum amplitude of 1.8mm @88Hz is also observed at Disc2. Detail response on the disc2 is shown in Figure 12. Maximum vibration observed on the modified rotor is 0.33 mm@88 Hz. Detail vibration on the rotor shaft is shown in Figure 13

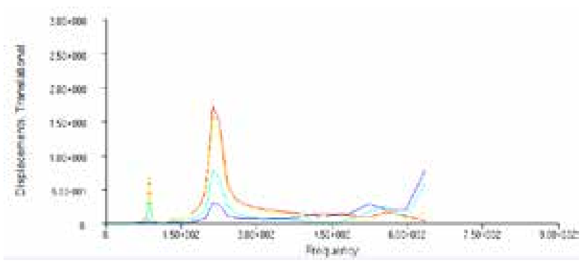

Figure 11: Modified Design: Response in the rotor disc1

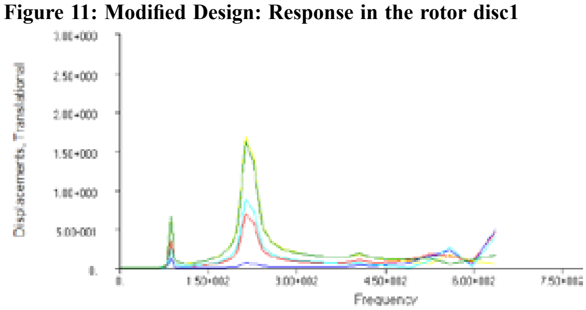

Figure 12: Modified Design: Response in the rotor disc2

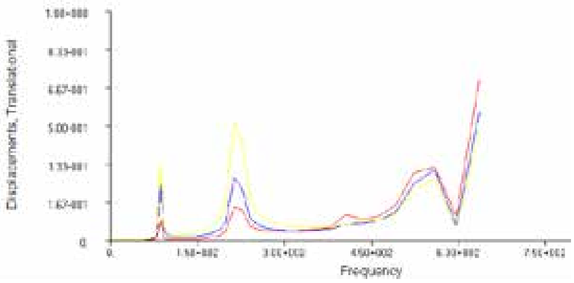

Figure 13: Modified Design: Response in the rotor

Comparison

Frequency response analysis is simulated for both the design to do the comparison. This comparison shows the improvement in the vibration response i.e. reduction in the vibration amplitude due to the rotor balancing which is calculated by the influence coefficient method. In Figure 14 vibration amplitude is compared at the disc1 of the rotor, which shows improvement in vibration reduction by 83% in the 1st harmonic i.e. vibration amplitude is reduced to 0.5mm from 3mm in the original design. Vibration amplitude is increased by 120% in the second harmonics which will be less dominant than the 1st harmonics. Maximum amplitude on disc1 is 1.5mm as compared to 3mm in the original design at second harmonic.

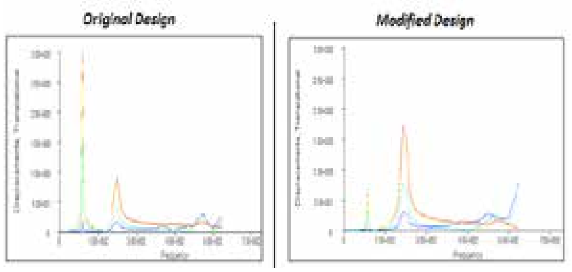

Figure 14: Comparison: vibration amplitude at Disc1

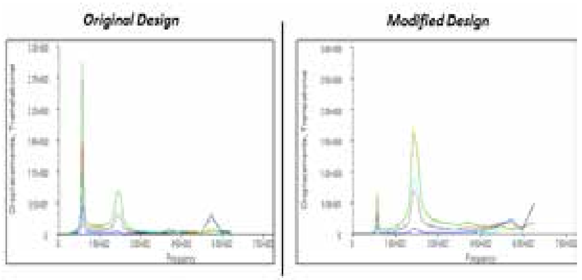

Figure 15: Comparison: vibration amplitude at Disc2

Vibration amplitude is compared at the rotor shaft, which shows improvement in vibration reduction by 79% in the 1st harmonic i.e. vibration amplitude is reduced to 0.33 mm from 1.6 mm in the original design. Vibration amplitude is increased by 72% in the second harmonics which will be less dominant than the 1st harmonics. Detail of the vibration response comparison is shown in Figure 16. Overall vibration is reduce by 68% is the rotor system as shown in Table 4.

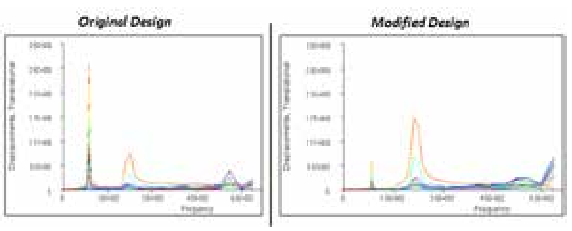

Figure 16: Comparison: vibration amplitude at Rotor

Overall as a rotor system the maximum vibration amplitude on the rotor system is 1.8 mm as compared to 3mm in the original design i.e. rotor vibration is reduced by 40%. Detail comparison of maximum response in the rotor system is calculated in Table 4.

TABLE – 4
Vibration response comparison for the original and modified rotor system

	Amplitude (mm)						
	1st Harmonic		2nd Harmonic		Max Amplitude (mm)	Max Amplitude (mm)	Difference (%)
Measuring Station	Original Design	Modified Design	Original Design	Modified Design	Original Design	Modified Design	
Disc1	3	0.5	0.9	1.5	3.0	1.5	-50.0
Disc2	3	0.58	0.7	1.8	3.0	1.8	-40.0
Rotor	1.6	0.33	0.28	0.52	1.6	0.5	-67.5

CONCLUSIONS

Unbalance in the rotor system is created by arbitrary positioning of the unbalance weight in the form of lumped masses in the finite element. Modal analysis is performed on the rotor system and it is achieving its first critical speed at 88 Hz and second at 190 Hz. Response on the rotor system is also achieving peak at sane critical frequencies. Third harmonic of the rotor system is not critical as rotor is not achieving peak at this frequencies.

From the frequency response analysis, response is measured at the initial condition of the rotor and by using the influence coefficient method, position of the balance correction weight is calculated such that the rotor get balance.

In the original model the maximum vibration level (mm) on the disc1 is 3 mm and maximum vibration level on disc2 is 3 mm.

After the modification, maximum vibration response in the rotor is 1.5 mm on disc and 1.8 mm on disc 2. From the modified rotor design we can conclude that the vibration level at the rotor system is reduced by minimum 40% than the original design.

Vibration level is reduced without changing the mode shape and natural frequency of the actual rotor system. Hence this method is useful in case where the system frequencies can't be change but reducing the response in the system is demanding. This method is useful in design where the manufacturing of part is costly and time consuming. This way balancing of big rotor assembly can be done without undergoing the repeated testing till the balancing is achieved. Testing of the rotor system and measure the actual vibration along with the phase and calculate the influence coefficient. On calculating the influence coefficient from the test data, locate the balance correction weight using influence coefficient method. This data will be useful for validation of the FEA result and accordingly the analysis can be modified. More accurate correction weight can be calculated by taking more trial weight and more data of influence coefficient to predict the balance weight more accurately.

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