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Relativistic Effect of Geodesic Mechanics the Newtonian Mechanics and Their Consequnces

Agriculture

KEYWORDS : gravity Space-time, curve-Space, distance, tensor notation

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ABSTRACT

In this paper a study of consequence of Newtonian 2nd law of motion in gravity and relativistic effect on equation of Geodesics was carried out .This paper answer some important questions on the link between Newtonian theory and geodesic gravitation With mathematical details we further discuss local inertial system and the motion of a particles along world line under gravity. In addition we state the line element equation that enable Astrophysics used to determine all possible effects of Newtonian gravitation and to possibly compute the red shift of spectral line that ultimately helped us in discussing quantization of events taking place in the celestial space of Astrophysics particle dynamics in Planck's scale geometry .This paper reveals the theory of geodesic and Newtonian theory of gravity is a manifestation of curvature of space time when considering a particle is moving slowly in weak unchanging gravitational field .It is also shown that ,if the motion of free particle has to follow space time geodesics, then the expression for the gravity acceleration is determine uniquely. Also the letter further tell us that every particle in space influence the behaviour of its neighbouring particle hence creating correlation. It depends on the variation of the metric with space and time and it involves the velocity of the particle.

INTRODUCTION; 1.1

Historically Isaac Newton theory initiated in classical mechanics and the extension to celestial mechanics to describe the motion of massive object later they appeared in electrodynamics when describing the motion of charged particles in electric and magnetic fields (Electromagnetism theory) .With the advent of general relativity, Newton theory became modified. In all these cases the deferential equations were in terms of a functions describing the particle's trajectory in terms of space and time coordinates, as influenced by force or energy transformations[1] From the nitty-gritty of Isaac Newtonian dynamics, it is presupposed that a single particle moving freely under-gravity (ie freefall) admit the Newtonian theory of gravity.

$$\mathcal{M}\ddot{x} = -m\nabla\phi \qquad 1.1.1$$

Where $\mathcal{M}$ is the inertial mass, that here equals the passive gravitational mass m Thus motion under gravity is independent of mass and composition. If a Gravitational field is specified as.

$$\bar{g} = -\nabla\Phi \qquad 1.1.2$$

is constant in space and time then all particle have a constant acceleration;

$$\bar{a} = \bar{g} \qquad 1.1.2a$$

Superimposed on the gravity-free motion with vector translational displacement given as;

$$\bar{x} = \bar{x}^1 + \tfrac{1}{2}\bar{a}\,t^2 \qquad 1.1.3$$

Where $\bar{x}^1$ could be regarded as the position in an inertial frame with no gravitational field. Conversely uniform acceleration, $\bar{a}$ applied to the coordinates gives the illusion of uniform gravitational field $\bar{a}$

Thus uniform gravitational fields" are fictitious" as they can be eliminated by a change of coordinates. Hence, we note that in any gravitational field, if an observer falls freely in anon-rotating laboratory frame, then he or she will see objects in the laboratory moving essentially on a straight line, meaning that the local gravitational field has been eliminated.

Therefore, a freely falling non-rotating laboratory provides a local inertial frame allowing inertial coordinate $(\bar{x}, t)$ to set up near the laboratory. There are limitations on local inertial frame. For instance, on an astronomical scale, nearby particles at position with coordinates $\bar{x}$ and $\bar{x} + \bar{\xi}$, have relative tidal acceleration, given as.

$$\frac{d^2\xi^i}{dt^2} = -\Phi_{,ij}\xi^j \qquad 1.1.4$$

(N/B tidal acceleration is acceleration due to force of astronomical origin operating in the gravitational field).In a true" non-uniform gravitational field tidal forces cannot be eliminated by coordinate transforms, and there are many different local initial frame with relative accelerations; and this was a fact which Newton quite acknowledged, but remain indifferent to it, thereby giving room to other scientist and mathematicians to exploit and developed. One such leading scientist and mathematician that revolutionized modern physics ,Albert Einstein who developed General relativity theory that relate space, and time to gravitation. In Einstein theory of gravity, however, space and time are intertwined into one single fabric that can be distorted manipulated

In a notch shell, Einstein summarized the whole of Newton work vis-a-visa relative acceleration in a gravitational field in a law which he called the 1st postulate of relativity,(also called the law of Equivalent principle".

Thus:

All local inertia frames are equivalent for the performance of all experiments". All non-gravitational laws of physics therefore take their special-relativistic forms in local inertial frames (as provided by the usual arguments of special relativity. We thus study course like fluid dynamics, Quantum mechanics, and electrodynamics electromagnetism in a gravitational field by using the special -relativistic laws and local inertial frames The speed of light is therefore a constant c" and distance and time are measured by Minkowski metric as

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt \qquad 1.1.5$$

The obvious consequence is that light can deflected by gravitational fields (just like ordinary matter) light move in a straight-line in local inertial frame which accelerate with respect to global coordinates. Again there is also a gravitational frequency shift .For instance, Consider a light of height accelerating downwards at a rate g, with respect to time. The earth and the light has speed 0 at $t = o$ and a light ray of frequency λ is emitted from the base of the light at $t = 0$, at $t = hc^{-1}$ .The light ray is at the top of the lift and has an observed frequency λ'in that frame (by equivalence principle).The light then has speed $\frac{gh}{c}$ and so the light has a Doppler shift frequency of $\lambda(1 - \frac{gh}{c^2})$ measured from the earth frame with resulting equation of the directrix of the ray system given as

$$\frac{d\lambda}{\lambda} = -\frac{d\phi}{c^2} \qquad 1.1.6$$

And the equation hold symmetrically for light emitted from other directions .Now Equation 1.1.6 can be integrated directly to obtain

$$\frac{\lambda}{\lambda_0} = exp(\frac{\phi_0 - \phi}{c^2}) \qquad 1.1.7$$

For a beam of (photon) of light emitted at a point p_0 with a frequency λ_0 and observed a frequencyλ' at p.

Thus we expect that clocks in a potential well will appear to go slow (gravitational time dilation) this is astronomically observed for spectral lines in some white dwarf stars, but not necessarily a big effect.
To this end the obvious inquiry encountered next, naturally led to the question". What is the connection between special relativity and gravitation?".In the sense of Albert Einstein, we recast the question thus;". Can we fix up special relativity so that it holds over an extended region containing gravitational fields?.A cogent argument that regards to this as a reply is gravitational time dilation implies that a good clock at rest measures a time.

$t_m = t_c exp\left(\frac{\emptyset}{c^2}\right), where(x_c y_c z_c t)$ Are special relativity coordinate, the theory can only be made to admit the Lorentz invariance principle if all measurements obey the distance rule as.

$$ds_m^2 = exp(\frac{\emptyset}{c^2})ds_c^2 \qquad 1.1.8$$

Now statement equation 1.1.8 is completely equivalent to a special type of curved space time theory (not GR) with the metric;

$$ds^2 = exp\left(\frac{\emptyset}{c^2}\right)(dx^2 + dy^2 + dz^2 - c^2 dt) \qquad 1.1.9$$

And this given as a more natural view point Hence, we see that attempt at combining special relativity and Newtonian gravity lead to naturally to curve space time, and the concept of Geodesics as a geometrical principle of relativity.(M.Arminjon 1996) Here we define Geodesics" as the generalization of straightest possible curve space time between two point on the earth's surface. For instance, if points are not too far apart we can find the geodesics, γ, by eternizing the length $\int ds$.we note that geodesics are intrinsic to the surface (they depend on the metric). As an example, great circles on the sphere are geodesics. We can find the intrinsic curvature of any surface at a point p by drawing all geodesics from p out to a distance r.We evaluate the circumference $C(r)$and thus area A(r) and then use the equation of curvature $\mathcal{K}_r$at that point is given as

$$\mathcal{K} = \lim_{r\to 0}\frac{32\pi - G(r)}{r^3}$$

$$\text{or}\mathcal{K} = \lim_{r\to 0}\frac{12\pi r^2 - A(r)}{r^4} \;] \qquad 1.1.10$$

Now $\mathcal{K}$, can be negative (for instance at a saddle) Generally let$\xi(r)$ (ie geodesics deviation) be the distance between the ends of two nearby geodesics of length r from point p .
Now on a real surface S^2, with$\xi(r) = asin\frac{r}{a}\delta\emptyset$, then we not that ξ
Satisfies the geodesics deviation equation as

$$\frac{d^2\xi}{dr^2} = -\mathcal{K}\xi \qquad 1.1.11$$

A similar equation hold in any curved space Thus if $\mathcal{K} > 0$, two neighbouring geodesics are cross (eventually), if $\mathcal{K} = 0$ the geodesics are straight line and if $\mathcal{K} < 0$ the geodesics separate exponentially. To conclude this section is noted that our arguments have led to a curve space time with four coordinate x^a a= 1,2, 3, 4 and a metric:

$ds^2 = g_{ab}dx^a dx^b$ Where $g_{ab} = g_{ba}$ depends on position. At any event $p\ in\ space$ time one can find a local inertial frame (ie one can make a coordinate change such that:

$$g_{ab} = g_{ab} = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -c^2 \end{matrix} \qquad 1.12$$

Where $\daleth_{ab}$ is the Minkowski metric (here defined oppositely to electrodynamics) .The metric g_{ab} has the canonical form $+ + + -$ at each point.

The four coordinates x^a (a= 1,2,3,4) used to specify the metric

$ds^2 = g_{ab}dx^a dx^b$ is said to constitute the local frame of Minkowski's world also called the Galilean world or also called the world of Riemannian space time. At the end Einstein fusion relativity and gravitation together led to a new theory was later recognized and called general relativity .In this new theory gravitational field is explained in terms of the curvature of a Riemannian space-time, while other field theories are based on the concept of Poincare invariance .The reason for this difference is the following observational experiment:" The principle of equivalence postulates that a freely falling small system(say satellite etc)constitute a local inertial system

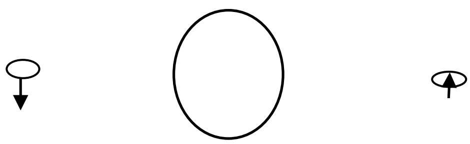

Fig. 1

Two Satellite as local inertial system.

Here effects of gravity are absent in such a system, if it is sufficiently small .The relation between these local inertial system at different points in space-time is obviously rather complicated as in fig. 1 above.

Shows the relation which described the curvature of space-time and the curvature in turn is determined by the mass distribution, which governs the relation between freely falling system at different points. There is thus no global inertial system and the concept of Poincare invariant field theories have to be abandoned.

1. AL INERTIAL SYSTEM & THE MOTION OF A PARTICLE A LONG WORLD LINE UNDER GRAVITY.

Since the Newtonian dynamics is essentially about the translational motion of a particle along the coordinate axes of local inertial frame in the universe of discourse. To determine the gravitational acceleration of a particle in local inertial frame along the world lines, we start by define the line element in a local inertial frame as;

$$ds^2 = \daleth_{ik}dy^i dy^j \qquad 1.2.1$$

Where $\daleth_{ik} = diag(1, -1, -1, -1)$ (where here $c = 1$) is the metric of flat space-time (ie the magnitude of Galelian metric).This line element is valid ,however, only in a small neighbourhood of axes $y = o$

$$ds^2 = \eta_{ik} dy^i dy^j \qquad 1.2.1$$

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Now, we have to find an expression for ds^2 which is valid globally
(ie in all of space-time).To this end, we introduce some global coordinate system x^i.The relation between y^iandx^i is in general .Rather complicated, since the x^i-system can be any curvilinear coordinate system, and the y^i-system is accelerated w r t it.

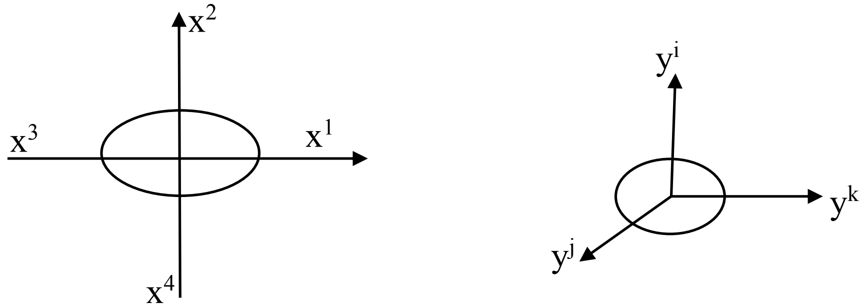

Fig. 2-earth with one local inertial system (y^i)and global coordinate system (x^i).
The functions determine the coordinate relations between (y^i)and(x^i) systems is generally given as

$$y^i = f^i(x^k) \qquad 1.2.2$$

And there functions are generally very complicated but can at least in principle be evaluated experimentally by reading of the x and y coordinates. Simultaneously, while the local inertial system is falling Substituting equation 1.2.2 into 1.2.1,we obtain.

$$ds^2 = \eta_{ik} dy^i dy^k \quad = \eta_{ik} \frac{\partial y^i}{\partial x^l} \frac{\partial y^k}{\partial x^m} dx^l dx^m$$

$$= g_{lm}(\mathrm{x}) dx^l dx^m \qquad 1.2.3$$

Where $g_{lm}$is the (covariant) metric tensor of space-time? Now, is a mathematical abstraction, and for all practical purpose, a large number of local inertial system (ie satellites) will obviously be necessary to determine g_{lm} is the method described here so as equation 1.2.3 is valid only along the path of the local inertial system; and Thus only by combining the information from these local system can we truly determine the global metric $g_{lm}(x)$. when only a limited class of local system is consider we obtain an incomplete or partial description of space-time ,a typical example of such is the geometry of the Schwarzschild solution, for which ds^2 is obtained from the path sliced long the universe of the a star decoupled into a 3-dimention Cartesian space. Next, our task of generating the relativistic motion of particle under the Newtonian theory of gravitation is thus to determine the metric tensor g_{ik} as a function of the mass-energy distribution and to calculate the equations of motion of given masses in the gravitational field. For masses which are amorphous or structure less i.e. (have spine uncharged etc) the exercise will be very simple as test masses as the motion will be nearly rectilinear in flat space-time. Thus we restrict our selves here to particles undergoing uniform rectilinear motion in a local inertial system (ie a long straight line through space time) (see fig 3). When particles move uniformly along straight line in the local inertial system then the particle instantaneous displacement along the local coordinate

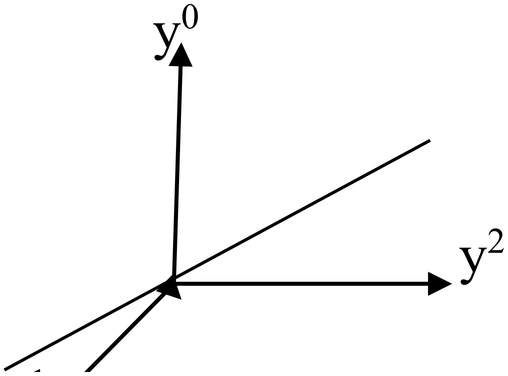

Fig. 3: motion of particles through local inertial system is in a straight lines in space-time. Line in space-time is given by:

$y^i = v^i s$ 1.2.4

Translating this into motion along arbitrary curvilinear coordinates in a global inertial system, involve taking the variation in various line elements in local inertial systems making up the global system into consideration .To do this
We start from the variation principal that specifies the general variation as.

$\delta \int ds = 0$ 1.2.5

And this fulfilled equation 1.2.4 globally. Now substituting equation1.2.3 into equation1.2.5 we obtain.

$\delta \int \sqrt{g_{ik}} \dot{x}^i \dot{x}^k d\lambda \equiv \delta \int \mathcal{L} d\lambda = 0$ 1.2.6.

Where λ is an arbitrary parameter along the world line of the particle and $\dot{x}^i = \frac{dx^i}{d\lambda}$ is the Newtonian tensor derivative, and$\mathcal{L} = (g_{ik}\dot{x}^i\dot{x}^k)^{\frac{1}{2}}$ is here treated as a functional of line element distance along the world line. Extremizing the functional, we obtain Euler-Lagrange's equation of variation principle as.

$\frac{dL}{\partial x^i} = \frac{d}{d\lambda}\frac{\partial L}{\partial \dot{x}^i}$ 1.2.7

Expanding out in the manner of equation 1.2.3 we obtain.

$\frac{1}{L}\dot{x}^l\dot{x}^k\frac{\partial g_{ik}}{\partial x^l} = \frac{1}{L^2}g_{ik}\dot{x}^k\frac{dL}{d\lambda} + \frac{1}{L}\ddot{x}^k g_{ik} = \frac{1}{2L}\frac{\partial g_{kl}}{\partial x^i}\dot{x}^k\dot{x}^l$ 1.2.8

Now choosing the parameterλ to be a time averaged length of arc along the world line then$\lambda = s$, and hence $\frac{dL}{d\lambda} = 0$, and hence we obtain a reduced to equation 1.2.8 as

$\frac{1}{L}\dot{x}^l\dot{x}^k\frac{\partial g_{ik}}{\partial x^i} + \frac{1}{L}\ddot{x}^k g_{ik} = \frac{1}{2L}\frac{\partial g_{kl}}{\partial x^i}\dot{x}^k\dot{x}^l$

$\Longrightarrow \frac{1}{L}\ddot{x}^k g_{ik} + \frac{1}{L}\dot{x}^l\dot{x}^k\frac{\partial g_{lk}}{\partial x^i} - \frac{1}{2L}\frac{\partial g_{kl}}{\partial x^i}\dot{x}^k\dot{x}^l = 0$ $\otimes_1$

But the middle term can be further expanded as

$$\frac{1}{L}\dot{x}^l\dot{x}^k\frac{\partial g_{ik}}{\partial x^i} = \frac{1}{2L}(\dot{x}^l\dot{x}^k\frac{\partial g_{kl}}{\partial x^i} + \dot{x}^l\dot{x}^i\frac{\partial g_{li}}{\partial x^k}) \qquad \otimes_2$$

∴ Substituting $\otimes_2$ into $\otimes_1$ we obtain

$$\frac{1}{L}\ddot{x}^k g_{ik} + \frac{1}{L}\{\frac{1}{2}(\frac{\partial g_{lk}}{\partial x^i} + \frac{\partial g_{li}}{\partial x^k} - \frac{\partial g_{ki}}{\partial x^l})\dot{x}^k\dot{x}^l = 0 \qquad \otimes_3$$

The last term in $\otimes_3$is a quantity Γ_{ikl} defined as

$$\Gamma_{ikl} = \frac{1}{2}(\frac{\partial g_{lk}}{\partial x^i} + \frac{\partial g_{li}}{\partial x^k} - \frac{\partial g_{ki}}{\partial x^l}) \qquad 1.2.9$$

The above equation is called Christofel symbols of the first kind in tensor analysis, and equation$\otimes_2$ .Thus reduces to the form.

$$\frac{1}{L}\ddot{x}^k g_{ik} + \frac{1}{2}\Gamma_{ikl}\dot{x}^k\dot{x}^l = 0 \qquad \otimes_4$$

if we multiply the above equation$\otimes_4$ out with L, we obtain

$$\ddot{x}^k g_{ik} + \Gamma_{ikl}\dot{x}^k\dot{x}^l = o \qquad 1.2.10$$

and equation 1.2.10 now gives the equation of the geodesics, as corresponding equation of motion of the particle under the Newtonian gravity along world lines we will see the next section in Newtonian approximations.

1.3 NEWTONIAN APPROXIMATIONS

To simplify equation 1.2.10 further, we define the contra variant metric tensor $g^{kl}$by ;

$$g_{ik}g^{kl} = \delta_i^l \qquad 1.3.1$$

Thus, multiplying equation 1 2 10 through by $g^{im}$and summing over i we have

$$\ddot{x}^m + \Gamma_{kl}^m\dot{x}^k\dot{x}^l = 0 \qquad 1.3.2$$

Where here Γ_{kl}^m is the christoffel symbol of the second kind defined as:

$$\Gamma_{kl}^m = g^{mi}\Gamma_{ikl} \qquad 1.3.3$$

If we mimic equation 1.3.3 being the equation of the test mass in a gravitational field, to be equivalent to the motion of the planets in Newtonian approximation of the universal gravitation, then we can easily compute the metricg_{ik} corresponding to the sun's gravity field.
Experimental evidence has shown that special relativity is an excellent approximation in the solar system we therefore express $g_{ik}(x)$ in terms of a solar fluctuating function$\psi_{ik}(x)$ as:

$g_{ik} = g_{ik} + 2\psi_{ik}(x)$ 1.3.4

where $|\psi|^2 \ll 1$.

All planetary velocities in the solar system are non- relativistic, hence, we can approximate equation 1.3.3 by

$\frac{d^2x^i}{ds^2} \cong \frac{d^2x^i}{dt^2} = -\Gamma^i_{kl}\dot{x}^k\dot{x}^l \cong -\Gamma^i_{00}$ 1.3.5

For the spatial part of the acceleration vector we obtain thus:

$$\frac{d^2x^\alpha}{dt^2} \cong -\Gamma^\alpha_{00} = -g^{\alpha i}\Gamma_{i00} = -g^{\alpha i}\Gamma_{i00} = \Gamma_{\alpha 00}$$

where interchanging α with$i$ yield

$$-\eta^{\alpha i}\Gamma_{i00} = -\eta^{ii}\Gamma_{\alpha 00} = (+1)\Gamma_{\alpha 00}$$

as $-\eta^{ii} = |-dig(1-1-1-1)| = +1$

We have

$$\frac{d^2r^\alpha}{dt^2} = +\Gamma_{\alpha 00}$$

$$= \frac{\partial g_{0\alpha}}{\partial x^0} - \frac{1}{2}\frac{\partial g_{00}}{\partial x^\alpha} \qquad = -\frac{1}{2}\frac{\partial g_{00}}{\partial x^\alpha}$$

$$= -\frac{\partial(\frac{1}{2}g_{00})}{\partial x^\alpha} \qquad = -\frac{\partial \psi_{00}}{\partial x^\alpha}$$

$\frac{d^2x^\alpha}{dt^2} = -\frac{\partial \mathcal{U}_{00}}{\partial x^\alpha}$ 1.3.6

where$\boldsymbol{\mathcal{U}}= \psi_{00} = \frac{1}{2}g_{00}$ where here is interpreted as the gravitational potential or ground state energy driving the particle motion of equation 1.1.1with 1.3.6.Thus in Newtonian approximation, line element becomes expressible as

$ds^2 = (1 + 2\mathcal{U})dt^2 - d\bar{x}^2$ 1.3.7

Where $d\bar{x}^2$ is the translational spatial vector distance component of the line element?
Finally, taking a particular instance for a spherically-symmetric mass distribution (mass m) we have putting the Newtonian gravitational constant $G = 1$ for the earth we have $\boldsymbol{\mathcal{U}}= -\frac{GM}{r} = -\frac{M}{r}$ and hence equation 1.3.7 becomes:

$ds^2 = (1 - -\frac{2M}{r})dt^2 - d\bar{x}^2$ 1.3.8

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Where 2M (or in conventional unit$\frac{2GM}{c^2}$) is thus a length dimensionally and usually called Schwarzchild radius of the mass M.
We note that for the sun $2M = 3km$ land for the earth is $2M = 0.9km$.

CONCLUSION/SUMMARY

In conclusion we state that the line element of equation 1.3.7 enables us to determine all effects of Newtonian gravitation and also to possibly evaluate/compute the red shift of spectral lines that ultimately helped us in discussing quantization of event taking place in the celestial space of astrophysical particles dynamics in Planck's scale geometry. Also this paper exposes to us that every particles in space influence the behaviour of its neighbouring particle hence creating a correlation. Einstein field equation is relativistic generalization of Newton's law of gravitation. Einstein's vision, based on the equality of inertial and gravitational masses, was that there is no gravitational force at all. What is said to be particle in motion under the influence of the gravitational force" in Newton theory is according to the general theory of relativity, free motion along geodesic curve in curved space-time. Newton's gravitational law tell how mass generate gravitational force .Einstein demanded from his field equation that they should tell how matter and energy curve space time. He knew that the energy-momentum conservation of a continuum of matter and energy could be described covariant by vanishing of the divergence of a symmetric energy-momentum tensor of rank 2 thus the field equation must be of the form. A symmetric and divergence-free curvature tensor of rank 2 is proportional to the energy-momentum tensor. The Einstein tensor has just the right properties to represent the geometrical part of Einstein's equation (has physical content) but Newton theory (has no physical) content. These results and solutions provides basic link between Einstein, Newton second law and geodesic assumption

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