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Numerical Study on the Zero-Velocity Surface of G2 Star--Kepler-452b--Spacecraft System

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ABSTRACT

For the G2 star--Kepler-452b--spacecraft system, this paper illustrates the relationship between geometry structures of zero velocity surface and Jacobi constant by carrying out several numerical simulations. The obtained results indicate that "total energy" of the spacecraft in G2-K system is only 0.054 percent greater than Sun-Earth system at their own libration points L_1 and L_2 , and 0.00038 percent larger at libration point L_3 . This implies that the Earth and Kepler-452b are very much alike in a certain

0. Introduction

In deep space exploration, restricted three-body problem means that the mass of a celestial body or spacecraft is small enough to be neglected when compared with the other two primaries. If the two main bodies are moving around their common center of mass at a certain angular velocity, it is called circular restricted three body problem [1, 3-6]. With the boom of the second round of deep space exploration, terrestrial planets and giant planets are gradually becoming the focus of exploration [2, 7, 8]. Jenkins et al. [2] reported an important discovery on Kepler-452b, which has likely been in the habitable zone. Compared with the Earth, Kepler-452b has several similar characteristics.

Due to the existence of zero velocity surfaces in the restricted three body problem, the infinite celestial body or spacecraft cannot fly anywhere in the space. In this paper, the geometric structure of the zero velocity surface of the G2 star--Kepler-452b--spacecraft (G2-K) system is considered, and the influence of Jacobi constant C on the forbidden flight area of the spacecraft is discussed based on numerical simulations. As C can be considered as "total energy" of the spacecraft relative to the rotating coordinate system, we find that this "total energy" of the spacecraft in G2-K system is only 0.054 percent larger than Sun-Earth system at their own libration points L_1 and L_2, and 0.00038 percent greater at corresponding libration point L_3. These indicate that the Kepler-452b and the Earth are far more alike in a certain extent.

1. Model of the Circular Restricted Three-body Problem

We first introduce two important coordinate systems: inertial frame $OXYZ$ and rotating frame $oxyz$. Consider the system consisting of the G2 star, Kepler-452b and a spacecraft in the inertial coordinate system $OXYZ$, the origin O of the system lies in the center of mass of G2 star and Kepler-452b (as shown in Figure 1), where the X-Y plane is the orbital plane of the system, G2 star and Kepler-452b do circular motion around their common center of mass. Spacecraft is only attracted by the gravity of the G2 star and Kepler-452b. The X-axis is directed by G2 star and Kepler-452b, and the Y-axis is perpendicular to the X-axis and complies with the right-hand rule (the Z-axis is default perpendicular to the orbital plane X-Y), the masses of spacecraft, G2 star and Kepler-452b are m_s, M and m, respectively, where $m_s \ll M$, m.

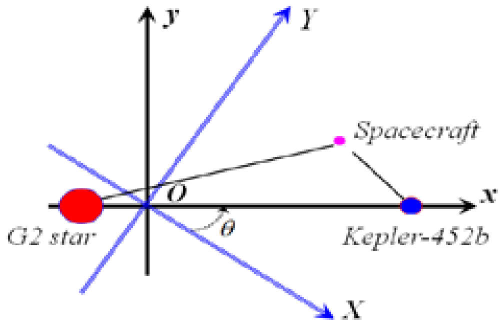

Figure 1. Inertial frame $OXYZ$ and rotating frame $oxyz$

For convenience, assume that $[m]$, $[L]$ and $[T]$ are the mass unit, length unit and time unit, respectively, L_{G2K} is the average distance between the G2 star and Kepler-452b, ω is the angular velocity of their relative motion. In general, the system can be nondimensionalized according to the rules of $[m]=M+m$, $[L]=L_{G2K}$ and $[T]=\left[L_{G2K}/(G(M+m))\right]^{1/2}=1/\omega$. In Figure 1, θ is the turning angle of the connected line between G2 star and Kepler-452b around the center of mass, mass ratio $\mu = m/(M+m)$ (please see Figure 2 for detail), then the dimensionless distance from the G2 star and Kepler-452b to center of mass can be denoted as $1-\mu$ and μ, respectively.

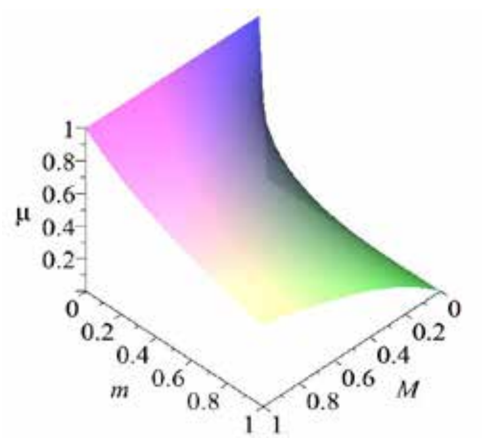

Figure 2. Mass ratio μ of G2 star and Kepler-452b

The origins of the above two frames coincide at o and O, x-axis and y-axis rotate counterclockwise in unit of angular velocity around its center of mass. Furthermore, we also suppose that the two coordinate systems coincide with each other at $t=0$, then $\theta=t$. If the (x, y, z) indicates the position of the spacecraft in the rotating coordinate system, we have

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R_t \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad (1)$$

where $R_t = \begin{pmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Differentiating both sides of equation (1) with respect to time t, we get

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} = \dot{R}_t \begin{pmatrix} x \\ y \\ z \end{pmatrix} + R_t \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = -R_t J \begin{pmatrix} x \\ y \\ z \end{pmatrix} + R_t \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = R_t \begin{pmatrix} \dot{x} - y \\ \dot{y} + x \\ \dot{z} \end{pmatrix}, \qquad (2)$$

where $J = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

Therefore, Lagrangian function in the rotating frame can be represented as

$$L = \frac{1}{2}\left((\dot{x}-y)^2 + (\dot{y}+x)^2 + \dot{z}^2\right) - U(x, y, z), \qquad (3)$$

where

$$U\left(x, y, z\right) = -\frac{1-\mu}{r_1} - \frac{\mu}{r_2}, \quad r_1 = \left[(x+\mu)^2 + y^2 + z^2\right]^{\frac{1}{2}}, \quad r_2 = \left[\left(x-1+\mu\right)^2 + y^2 + z^2\right]^{\frac{1}{2}}.$$

Substituting equation (3) into Euler-Lagrange equation (please see reference [4], pp.32-34), the governing equation of the spacecraft in the rotating coordinate $oxyz$ can be written as

$$\ddot{x} - 2\dot{y} = x - U_x, \qquad (4a)$$
$$\ddot{y} + 2\dot{x} = y - U_y, \qquad (4b)$$
$$\ddot{z} = -U_z. \qquad (4c)$$

We define generalized potential energy $\Omega(x, y, z)$ as follows

- 3 -

$$=\frac{1}{2}\left(x^2+y^2\right)+\frac{1-\mu}{r_1}+\frac{\mu}{r_2}. \qquad (5)$$

$$\Omega(x, y, z)=\frac{1}{2}\left(x^2+y^2\right)-U(x, y, z)$$

Then, the dimensionless equations of motion of the spacecraft in the rotating coordinate system $oxyz$ can be reduced to

$$\ddot{x}-2\dot{y}=\Omega_x, \qquad (6a)$$
$$\ddot{y}+2\dot{x}=\Omega_y, \qquad (6b)$$
$$\ddot{z}=\Omega_z. \qquad (6c)$$

2. Analysis on the Zero-Velocity Surface of G2-K system

Multiplying both sides of equations (6a)~(6c) by $\dot{x}$, $\dot{y}$ and $\dot{z}$, respectively, we obtain

$$\ddot{x}\dot{x}-2\dot{y}\dot{x}=\Omega_x\dot{x}, \qquad (7a)$$
$$\ddot{y}\dot{y}+2\dot{x}\dot{y}=\Omega_y\dot{y}, \qquad (7b)$$
$$\ddot{z}\dot{z}=\Omega_z\dot{z}. \qquad (7c)$$

Then, we have

$$\ddot{x}\dot{x}+\ddot{y}\dot{y}+\ddot{z}\dot{z}=\Omega_x\dot{x}+\Omega_y\dot{y}+\Omega_z\dot{z}, \qquad (8)$$

Integrating both sides of equation (8), manifold of the spacecraft can be read as

$$2\Omega(x, y, z)-v^2=C, \qquad (9)$$

where $v^2=\dot{x}^2+\dot{y}^2+\dot{z}^2$ represents velocity of the spacecraft, and C is the Jacobi constant.

Due to the spacecraft's flight region is determined by $v^2\geq 0$, its flight region will be depends upon the above manifold. A series of three dimensional surfaces correspond to equation (9) are called zero-velocity surfaces or Hill surfaces, or known as zero-velocity curves while they are mapped on the two-dimensional plane. According to the data of Jenkins, $m\approx 5\times 5.9726e+24 \;=29.863e+24kg$, $M\approx 1.037\times 1.98855e+30 = 2.06212635e+30kg$, and then the system's mass ratio

μ=1.4481444$e-5$ (see Figure 2).

As the three-dimensional forbidden flight region of the spacecraft shown in Figure 3, energy consumption of the system increases gradually with the decreasing value of Jacobi constant C, which indicates that the flight region of the spacecraft will become progressively expansive. On the contrary, if the spacecraft has no enough energy, it will be trapped inside a potential well-like region encircling the G2 star or Kepler-452b.

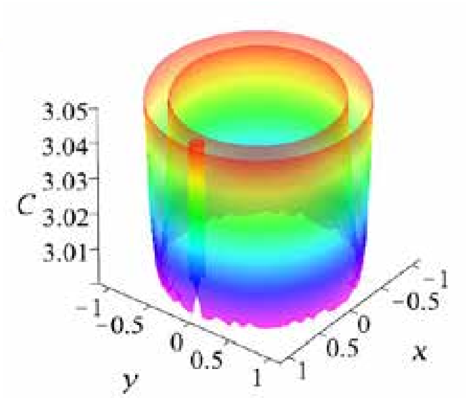

Figure 3. Zero-velocity surface of G2-K system for $C \in [3, 3.05]$

As shown in Figures 4 and 5, zero-velocity curves are composed of three separate "circles" in two-dimensional plane for $C = 3.003$. The outer peripheral circle surrounding another two, of which the bigger one is G2 star region and the other is Kepler-452b region. In this case, the spacecraft can only flight around G2 star or Kepler-452b, but cannot transfer between them.

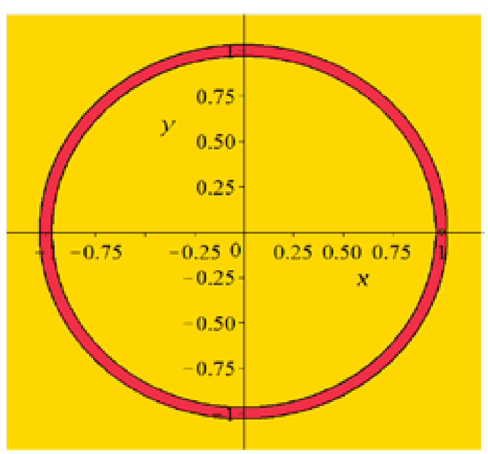

Figure 4. Zero-velocity curve of the G2-K system for $C = 3.003$

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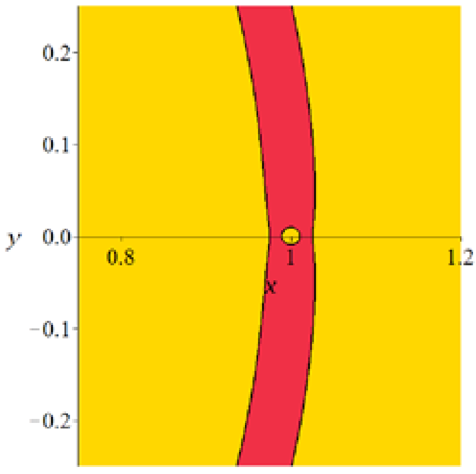

Figure 5. Zero-velocity curve around Kepler-452b for $C = 3.003$

As the value of C gradually decreases from 3.003, the two inner circles around G2 star and Kepler-452b respectively are close to each other in Figure 6. When C is reduced to 3.0025224724, the outline of two regions appears to be figure ∞. At the connected point, the spacecraft's flight area has been enlarged, but the cross point $(0.9831000442, 0)$ at the waist of figure ∞ becomes a "traffic fortress" between G2 star and Kepler-452b for the spacecraft (see Figure 7), the spacecraft can not visit the other star by passing through the fortress.

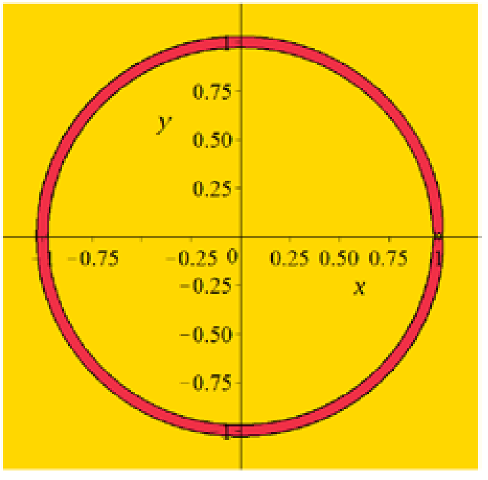

Figure 6. Zero-velocity curve of the G2-K system for $C = 3.0025224724$

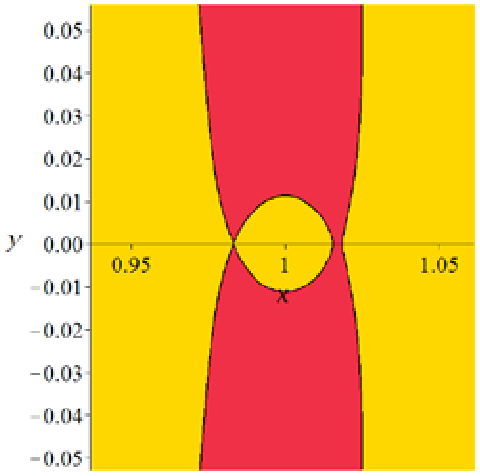

Figure 7. Zero-velocity curve around Kepler-452b for $C = 3.0025224724$

For $C < 3.0025224724$, the "traffic fortress" connected the fields around G2 star and Kepler-452b respectively will be opened (as shown in Figure 8 for the case $C = 3.00252$), and then the spacecraft cannot only fly in the region around G2 star or Kepler-452b, but also can fly along with a transfer orbit passing through the fortress from one star to another.

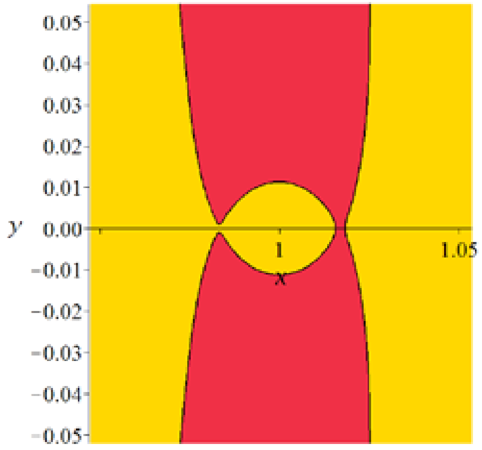

Figure 8. Zero-velocity curve around Kepler-452b for $C = 3.00252$

When C is reduced to 3.0025032545, another "traffic fortress" locating at $(1.017111987, 0)$ is evolved (as shown in Figures 9 and 10). Although the forbidden region of the spacecraft is enlarged yet, it cannot fly to outer space.

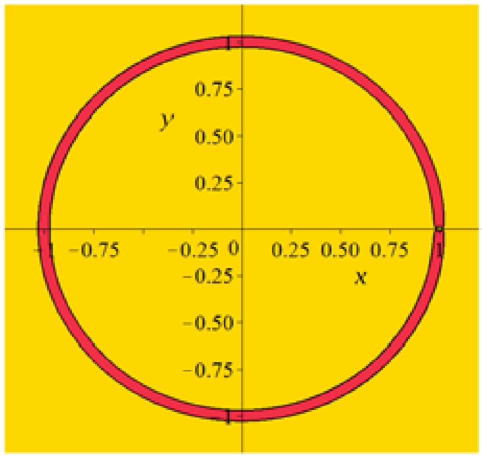

Figure 9. Zero-velocity curve of the G2-K system for $C = 3.0025032545$

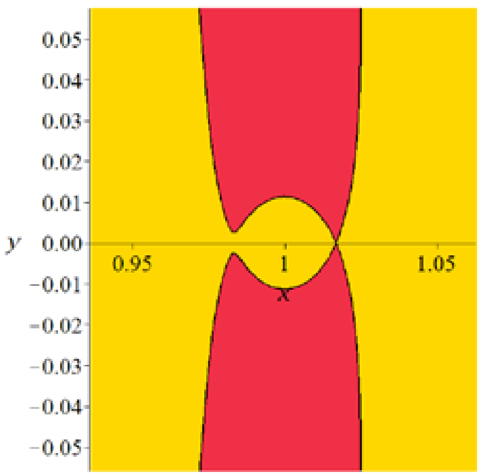

Figure 10. Zero-velocity curve around

As C continues to decrease gradually, the aforementioned another "traffic fortress" will be opened. For $C = 3.0025$, the spacecraft cannot only fly back and forth between G2 star and Kepler-452b, but also to explore the outer space by the opening "fortress" (as shown in Figures 11 and 12).

Kepler-452b for C = 3.0025032545

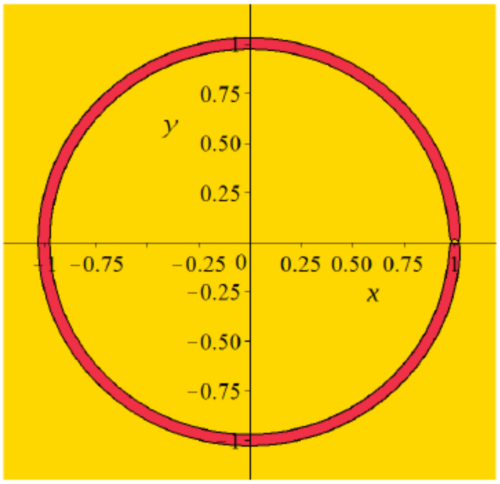

Figure 11. Zero-velocity curve of the G2-K system for $C = 3.0025$

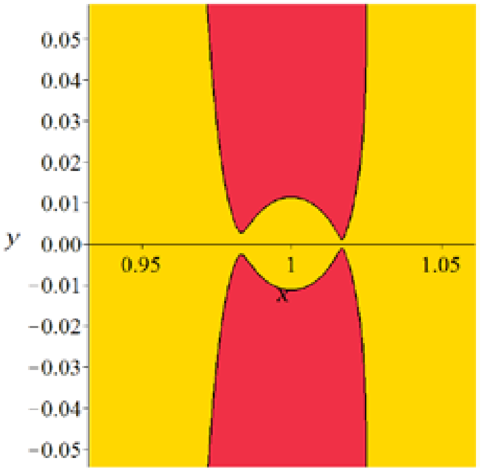

Figure 12. Zero-velocity curve around Kepler-452b for $C = 3.0025$

When C continues to be reduced to 3.0000144824, forbidden region of the spacecraft has been reduced to two connected curved arc-shaped zone at the critical point $(-0.9999871276, 0)$ (as shown in Figure 13). At this time, although the spacecraft can through the aforementioned opened "fortress" gap to outer space, but not by the point where two curved arc-shaped are connected together. As C is getting smaller, the forbidden area of the spacecraft becomes smaller and smaller until they degrade into two points (as shown in Figures 13 and 14).

locate on the line connected G2 star and Kepler-452b, also known as collinear libration points, and the libration points L_4 and L_5 are symmetrical with respect to the axis and compose an equilateral triangle with G2 star and Kepler-452b, respectively, also known as triangular libration points [1,5,9].

For the Sun-Earth-spacecraft system, the geometric structure of zero-velocity curve can also be simulated in Figure 15.

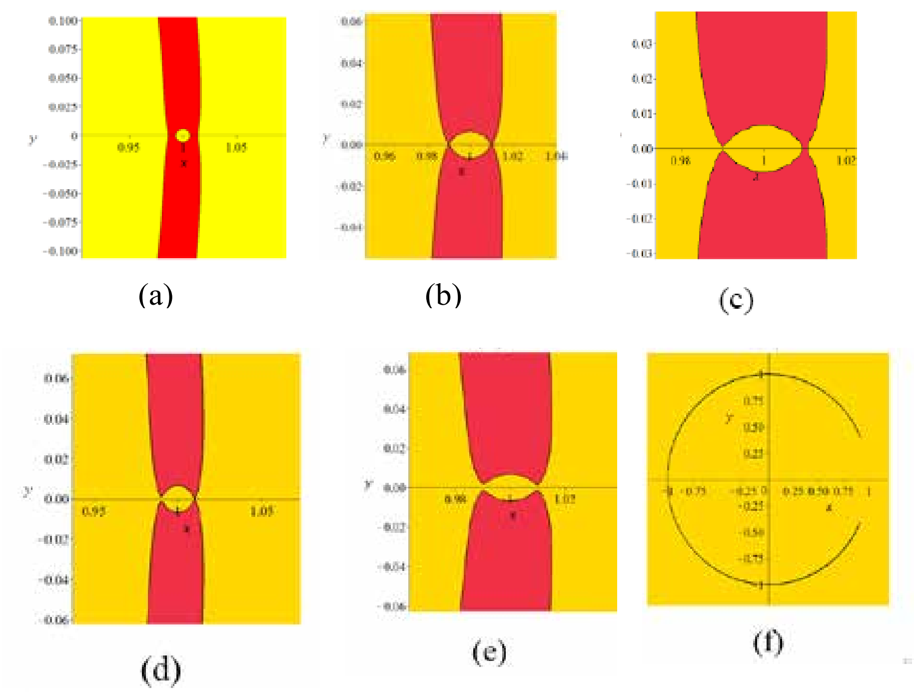

Figure 15. Zero-velocity curve of the Sun-Earth system for: (a) $C = 3.001$, (b) $C = 3.0008909760$, (c) $C = 3.00089$, (d) $C = 3.0008867285$, (e) $C = 3.00088$, (f) $C = 3.0000030047$.

3. Conclusions

This paper discusses the relationship between zero velocity surfaces and Jacobi constant C for the G2-K system, the results indicate that the spacecraft is reduced gradually with the value of C increases little by little. In contrast, the region will becomes bigger and bigger as the value of C decreases. In addition, from the values of C illustrated by Figures 7, 10, 13 and 15, “total energy” of the spacecraft in G2-K system is only 0.054% larger than Sun-Earth system at their own libration points L_1 and L_2, and 0.00038% greater than Sun-Earth system at libration point L_3. This suggests that the Earth and her 'Older Cousin', Kepler-452b, are very much alike in a certain extent.

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