

Equations of motion for a high speed particle.



Mathematics

KEYWORDS: The expression of instantaneous mass, velocity, acceleration and displacement for a high speed particle

Ayan Banua

C/O – Sanatan Banua, Vill- Thakurchak P.O.-Dhanyaghar Pin-721643, India

ABSTRACT

A constant force is applied on a particle for infinitive time from rest respect to the frame 'A'. Now a person at 'A' measuring instantaneous velocity, acceleration of the particle and measuring time by a clock in 'A'. Suppose at a instant velocity of the particle is 'v' and mass of the particle is 'm' and the particle take 't' seconds respect to 'A'

Then the expression of 'v' will be $v = c / \{ \sqrt{1 + (c/a0t)^2} \} = ca0t / \{ \sqrt{(a0t)^2 + (c)^2} \}$

The expression of instantaneous acceleration (a) will be $a = (a0) / \{ 1 + (a0t/c)^2 \}^{3/2}$

The expression of displacement will be $X = (c/a0) \sqrt{(a0t)^2 + (c)^2} - (c)^2 / a0$

The expression of instantaneous mass will be $m = m0 \{ 1 + (a0t/c)^2 \}^{1/2}$

Introduction:

We know the equation

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Now suppose a constant force is applied on the particle

for infinitive time from rest respect to 'A'. As it is difficult to measure velocity respect to 'A' so it is difficult to measure Instantaneous mass. So we have to form a relation between instan mass and time to measure instan mass easily. Side by side we have to form somerelation between instantaneous velocity/and time to measure Instantaneous velocity/easily. To obtain these equations I used the relativistic formulae

$$F = \frac{d(mv)}{dt} = m_0 \frac{a}{\left\{ 1 - \left(\frac{v}{c}\right)^2 \right\}^{3/2}}$$

[a=instan~~t~~aneous acceleration (dv/dt)]

At first I compare them and then using

calculus I obtained these equations

Literature Survy:

I am trying to generalize our basic equation for last 2.5 years. So I used relativistic concept and tried to form them. It is the result of my works for 3 years

Methodology:

Suppose a particle is moving respect to the frame 'A'. A constant force is applied on the particle for infinitive time from rest respect to the frame 'A'. Now a person at 'A' measuring Instantaneous velocity, acceleration of the particle and measuring time by a clock in 'A'. Suppose at a instant velocity of the particle is 'v' and mass of the particle is 'm' and the particle take 't' seconds respect to 'A' So the force acting on the particle is

$$F = \frac{d(mv)}{dt}$$

$$= \frac{v dm}{dt} + m \frac{dv}{dt}$$

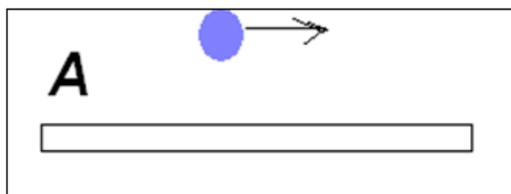
$$= m_0 \frac{v}{\left(1 - \frac{v^2}{c^2} \right)^{3/2}} + m a$$

[a=instananeous acceleration(dv/dt)]

$$= m_0 \left[\frac{v^2}{\left(1 - \frac{v^2}{c^2} \right)^{3/2}} + \frac{a}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \right]$$

$$= m_0 \frac{a}{\left(1 - \left(\frac{v}{c}\right)^2 \right)^{3/2}}$$

Using this formulae we get that at rest v=0



and $F = m_0 a_0$ [a₀=primary] As I consider that the force is constant so,

$$m_0 \frac{a}{\left(1 - \left(\frac{v}{c}\right)^2 \right)^{3/2}} = m_0 a_0$$

or $a = a_0 \left\{ 1 - \left(\frac{v}{c}\right)^2 \right\}^{3/2}$

or $dv/dt = a_0 \left\{ 1 - \left(\frac{v}{c}\right)^2 \right\}^{3/2}$

or $-\int \frac{dv}{\left[\left(\frac{v}{c}\right)^2 - 1 \right] \sqrt{1 - \left(\frac{v}{c}\right)^2}} = \int a_0 dt$

$$= \int \frac{dv}{\left[\left(\frac{v}{c}\right)^2 - 1 \right] \sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Or $\int \frac{cz}{\left[(1-z^2) \sqrt{z^2-1} \right]} dz = \int a_0 dt$

[putting (v/c)=(1/z)]

$$-c \int (m^{-2}) dm = \int a_0 dt$$

$$[\text{putting } z^2 - 1 = (m)^2]$$

then

$$\frac{c}{\sqrt{\left(\frac{c}{v}\right)^2 - 1}} = a_0 t + k \quad [k = I.C.]$$

At t=0, v will be 0 and k=0 (as the particle

starts from rest

By calculation we shall get that

$$v = \frac{c}{\sqrt{1 + \left(\frac{c}{a_0 t}\right)^2}} = \frac{c a_0 t}{\sqrt{\{(a_0 t)^2 + (c)^2\}}}$$

Result:

instantaneous velocity

$$v = \frac{c}{\sqrt{1 + \left(\frac{c}{a_0 t}\right)^2}} = \frac{c a_0 t}{\sqrt{\{(a_0 t)^2 + (c)^2\}}}$$

Discussion:

If we put t=0 we will get v=0 and if we put t=∞ we will get v=c. When v= (our known equation). The equation shows that if a particle moves under a constant force velocity does not increase uniformly and there is a maximum velocity that is C for any magnitude of force. So we may take that the acceleration of the particle decreases with time to 0.

Methodology:

So of the particle is dv/dt.

$$v = \frac{c a_0 t}{\sqrt{\{(a_0 t)^2 + (c)^2\}}}$$

$$\frac{dv}{dt} = c a_0 \left[\frac{1}{\sqrt{\{(a_0 t)^2 + (c)^2\}}} - \frac{(a_0 t)}{\{[(a_0 t)^2 + (c)^2]\}^{3/2}} \right]$$

$$a = \frac{a_0 (c)^3}{\{[(a_0 t)^2 + (c)^2]\}^{3/2}}$$

Result: instantaneous acceleration

$$a = \frac{a_0}{\left\{1 + \left(\frac{a_0 t}{c}\right)^2\right\}^{3/2}}$$

Discussion:

If we put t=0 we will get a= a_0 and if we put t=∞ we will get a=0. The equation shows that if a particle moves under a constant force its

acceleration decreases with time to 0.

Methodology:

Then what will be its displacement?

It should be

$$\int v dt = \int \frac{c}{\sqrt{1 + \left(\frac{c}{a_0 t}\right)^2}} dt$$

$$\text{Or } \int dx = 0.5 \int \frac{2c a_0 t}{\sqrt{\{(a_0 t)^2 + (c)^2\}}} dt$$

displacement

$$x = \left(\frac{c}{a_0}\right) \sqrt{\{(a_0 t)^2 + (c)^2\}} + D \quad [D = I.C.]$$

At t=0, X=0 and we get D = -(c)^2 / a_0

Result:

displacement -

$$X = \left(\frac{c}{a_0}\right) \sqrt{\{(a_0 t)^2 + (c)^2\}} - (c)^2 / a_0$$

Discussion:

The equation appears with Einstein's reformation. Here Einstein's reformation is not necessary. Because if we reform it, the displacement of the photon will be less than earth's circumference. That is not possible. So we have no need to reform it again. If we put t=0 we will get X=0 and if we put t=∞ we will get X=∞ Now

$$X = \frac{(c)^2}{a_0} \sqrt{\left\{\left(\frac{a_0 t}{c}\right)^2 + 1\right\}} - (c)^2 / a_0$$

When a_0 t << c, then

$$x = \frac{(c)^2}{a_0} \left[1 + \frac{1}{2} \left(\frac{a_0 t}{c}\right)^2 \right] - \frac{(c)^2}{a_0}$$

$$\text{Or } x = \frac{1}{2} a_0 (t)^2$$

[our known equation]

Methodology:

What will be the relation between mass & time?

We know that

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$1 - \left(\frac{v}{c}\right)^2 = \left[\frac{m_0}{m}\right]^2$$

$$1 - \frac{(a_0 t)^2}{\{(a_0 t)^2 + (c)^2\}} = \left[\frac{m_0}{m}\right]^2$$

Now [parabolic equation]

Result:

instantaneous mass-

$$m = m_0 \left\{ 1 + \left(\frac{a_0 t}{c}\right)^2 \right\}^{1/2}$$

Discussion:

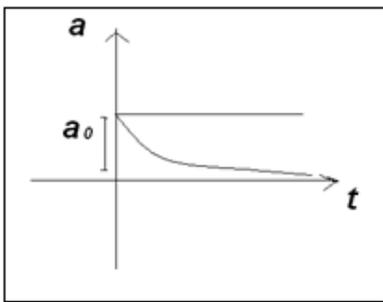
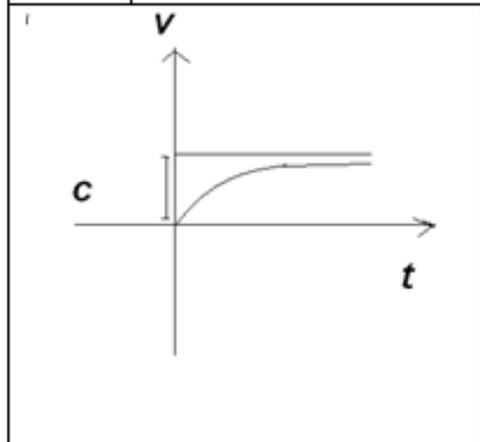
If we put t=0 we will get m= m_0 and if we put t=∞ we will get m=∞. It is clear from the equation that if we apply a constant force on a particle, its mass gradually increases to infinite.

Conclusion:

Future scope:

These equations are highly useful to calculate instantaneous velocity/of an retarding electron during X-Ray production and also emission of a particle during radioactivity & nuclear reaction. These equations will be helpful to form these type of equations in gravitational, electric & magnetic field.

V	$\frac{c}{\left\{ \sqrt{1 + \left(\frac{c}{a_0 t}\right)^2} \right\}}$
a	$\frac{a_0}{\left[\left\{ 1 + \left(\frac{a_0 t}{c}\right)^2 \right\}^{\frac{3}{2}} \right]}$
X	$\left(\frac{c}{a_0}\right)\sqrt{\left\{ \left(a_0 t\right)^2 + \left(c\right)^2 \right\}} - \left(c\right)^2 / a_0$
m	$m_0 \left[\left\{ 1 + \left(\frac{a_0 t}{c}\right)^2 \right\}^{\frac{1}{2}} \right]$



REFERENCE

Relativity-The Special and General Theory | by Albert Einstein | Einstein's Theory of Relativity| by Max Born

