

Mathematical Modeling: From Times Being



Mathematics

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ABSTRACT

The transmission of knowledge through models is not a new phenomenon, rather the humans have made models from times being for communicating their ideas in a better way and for making their life better. This paper traces modeling from prehistoric times to the age of modernization. Models such as maps and diagrams have been used since prehistoric times. One can dispute artifacts like the Lebombo bone or the Ishango bone as true models as they belong to prehistoric times, but how can we ignore Yajurvedasamhitā (1200–900 BCE), where the numbers as high as have been included. Modeling is the art which converts real life into concepts simple to understand. Not only humans but animals also follow the art. An acute mathematical mind can easily find mathematics in the pattern by which pigeons occupy their places. If 10 pigeons have 9 positions, at least two will occupy the same position and this is the famous pigeonhole principle. This principle is exemplified in real-life by such truisms as there must be at least two males or two females in a group of three people. The principle is indeed a counting model which can be applied to many formal problems, including ones involving infinite sets that cannot be put into one-to-one correspondence. The model $\frac{dI}{dt} = \lambda I - \mu I^2$ which allows the calculation of instantaneous rates of change, has revolutionized the mathematical thinking. According to Stewart, "More than any other mathematical technique, it has created the modern world." While admitting that no model can be perfect, this paper traces famous models and also discuss future prospects like fuzzy modeling.

Introduction

"Philosophy is written in this grand book - I mean the universe - which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other mathematical figures, without which it is humanly impossible to understand a single word of it; without these one is wandering about in a dark labyrinth."

Galileo Galilei II Saggiatore [1623]

The truth is not far away, it is not distant. It is near us, close and closer than we are to ourselves, but still we go on missing it, and if one knows, it becomes more difficult to tell others. The one who knows, first starts searching the tools, the figures, the characters or the models to communicate. The transmission of knowledge through models is not a new phenomenon, rather the humans have made models from times being to communicate and make their life better.

Prehistoric Time

The oldest known possibly a mathematical model is the Lebombo bone, consisting of 29 distinct notches cut into a baboon's fibula. It is discovered in 1970, in the Lebombo mountains of Swaziland and dated back to approximately 35,000 BC. Now the question arises why someone will make the exact 29 tally marks on a bone. It is conjectured that this is the earliest model to quantify time. The Universal Book of Mathematics provides the following information about the Lebombo Bone:

"One of the oldest mathematical artifacts known, a small piece of the fibula of a baboon, found near Border Cave in the Lebombo Mountains between South Africa and Swaziland. Discovered in the 1970s during excavations of Border Cave and dated about 35,000 B.C., the Lebombo bone is marked with 29 clearly defined notches. This suggests it may have been used as a lunar phase counter, in which case African women may have been the first mathematicians, because keeping track of menstrual cycles requires a lunar calendar. Certainly, the Lebombo bone resembles calendar sticks still used by Bushmen in Namibia."

Similar is the Ishango bone, found near the headwaters of the Nile river (northeastern Congo), which may be as much as 20,000 years old. It consists of a series of tally marks carved in three columns running the length of the bone. It is interpreted that the Ishango bone shows either the earliest known demonstration of sequences of prime numbers or a six month lunar calendar.

These are the few ones known to humanity, many such proofs have been lost. But one thing is certain that human consciousness and their models become more and more sophisticated with the passage of time. Who can deny the bricks and the ruler of the Indus Valley Civilization. Excavations at Harappa, Mohenjodaro and other sites have uncovered evidence of the use of manufactured bricks whose dimensions were in the proportion 4:2:1, which is favourable for the stability. The inhabitants of Indus civilization also tried to standardize measurement of length to a high degree of accuracy. They designed a ruler—the Mohenjodaro ruler—whose unit of length (approximately 1.32 inches or 3.4 centimetres) was divided into ten equal parts. Bricks manufactured in ancient Mohenjodaro often had dimensions that were integral multiples of this unit of length.

Vedic Time

Since prehistoric times models such as maps and diagrams have been used. We can dispute these artifacts as they belong to prehistoric times, but how can we ignore Yajurvedasamhitā (1200–900 BCE), where the numbers as high as 10^{12} have been included. The mantra (sacrificial formula) at the end of the *annahoma* ("food-oblation rite") performed during the *asvamedha*, and uttered just before, during, and just after sunrise, invokes powers of ten from a hundred to a trillion: "Hail to śata ("hundred," 10^2), hail to sahasra ("thousand," 10^3), hail to parārdha ("one trillion," 10^{12} , "beyond parts"), hail to the heaven (*svarga*), hail to the world (*martya*), hail to all." According to (Hayashi 2005, p. 363), the Śulba Sūtras contain "the earliest extant verbal expression of the Pythagorean Theorem in the world, although it had already been known to the Old Babylonians."

Greek Time

If there is one model known to all mathematically educated people, it is surely the Pythagorean Theorem. It is a property of right angled triangle which states that the square of the hypotenuse equals the sum of the squares of the other two sides i.e. $a^2 + b^2 = c^2$ (1), and conversely, a solution of (1) by positive numbers a, b, c can be realized by a right-angled triangle with sides a, b and hypotenuse c. This theorem is the first hint of a hidden, deeper relationship between arithmetic and geometry. Arithmetic is based on counting which is the epitome of a discrete process, whereas geometry involves continuous objects like lines, curves, and surfaces. By stretching a closed rope with 12 equally spaced knots we can obtain a (3, 4, 5) right angled-triangle. This relationship has always aroused curiosity to obtain such triples. Plimpton 322, a clay tablet, at least as far back as 1800 BCE in Babylonia, systematically lists a large number of integer pairs (a,

c) for which there is an integer b satisfying the theorem.

Pigeonhole Theorem

Mathematical modeling is exciting and rewarding, and it is a significant slice of the intellectual pie. A good education consists of learning different methods of discourse, and certainly modeling is one of the most well-developed and important modes of discourse that we have. Modeling is the art which converts real life into concepts simple to understand. Not only humans but animals also follow the art. An acute mathematical mind can easily find mathematics in the pattern by which pigeons occupy their places. If 10 pigeons have 9 positions, at least two will occupy the same position and this is the famous pigeonhole principle. The pigeonhole principle, also known as Dirichlet's box(or drawer) principle, states that if n pigeons are put into m pigeonholes with $n > m$, then at least one pigeonhole must contain more than one pigeon. More formally, it states that there does not exist an injective function on finite sets whose codomain is smaller than its domain. The theorem is exemplified in real-life by such truisms as there must be at least two males or two females in a group of three people.

Renaissance Time

The most important landmark indicating the transition from the medieval to the modern age, was the great intellectual revolution brought about by what is known as the Renaissance or the revival of classical learning. The intellectual revival was stimulated by the crusades and the explorations. It was manifested in an interest in the past and a desire for understanding the present. Perhaps its greatest attribute was the development of inquisitiveness which is necessary for intellectual progress. Interest in what earlier civilizations had contributed was great and the classics were revived. Not only was the modeling in mathematics but in astronomy, physiology and medicine was also investigated with sound scientific procedure. That was the time in which the natural, the human and the sensual were given precedence over the ascetical, the supernatural, and the theological. For the first great contributions to the mathematical sciences we must, therefore, look to Italy and Germany. In Italy brilliant accessions were made to algebra, in Germany to astronomy and trigonometry. She produced Regiomontanus, Copernicus, Kepler, and Tycho Brahe.

Enlightenment age

The 17th and 18th centuries in Europe is referred to as the Age of Enlightenment because during this period people came out of the shadow of superstitions and prejudices and began to judge things on the basis of reason. Calculus, we currently know was described around 17th century by Newton and Leibnitz. The model,

$$\frac{df}{dt} = \lim_{h \rightarrow 0} \frac{f(t+h) - f(t)}{h}$$

which allows the calculation

of instantaneous rates of change, has revolutionized the mathematical thinking. According to Stewart, "More than any other mathematical technique, it has created the modern world."

Twentieth Century

In any survey of the history of modeling it is hard to ignore the twentieth century. In fact the conception of classical modeling is influenced by the modern ideas or the most of the classical modeling becomes clearer when presented in modern terms. But the greatest problem is that twentieth century modeling is so vast that no one can grasp it all, and even some single models are based on theories too large to be explained in a paper of this size. Even then we have certain models which throw new light on the question "What is mathematics?", and these are sets and logic and more importantly fuzzy sets and fuzzy logic.

21st Century

The sets of best batsmen in the world, beautiful girls in a competition, and intelligent students in the class are often regarded as not well defined sets because they don't fulfill the criterion of belongingness. Such problems can't be discarded as such. They demand equal attention. The first such step which was indeed revolutionary was taken by Lotfi A. Zadeh (1965) who introduced fuzzy sets which is an extension of the classical notion of sets. In classical notion of set theory the membership of elements in relation to a set is assessed in crisp terms according to a bivalent condition.

Conclusion

The grand book, this universe is limitless and unmeasurable and man makes models to limit it. It is interesting to watch who wins, the unknown or the instinct to know.

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