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A Designing Procedure for Beattie Continuous Sampling Plan

Statistics

KEYWORDS :

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ABSTRACT

Continuous sampling plan was devised for process involving a continuous or nearly continuous flow of products. Harald F. Dodge first proposed continuous sampling plan as acceptance or rectification inspection procedure to control and improve the Average Outgoing Quality (AOQ) of a product. The Beattie Continuous sampling plan although not devised to deal with a product that is rectifiable, can be used in that situation and possess certain advantages over other plans. This Procedure was developed for situation when it is devised to accept or reject a continuously produced product based on the number of defectives and where the number of items available for testing in relatively small. This paper attempts to design a simple procedure formulated by reducing cost of inspection and as well increasing the quality levels for both producer and consumer.

Introduction

One of the major areas of statistical quality control is acceptance sampling. "Acceptance sampling is a methodology that deals with procedures, by which decision to accept or reject is based on the inspection of samples".

"Dodge (1943) innovated the concept of continuous sampling inspection plans that are applicable to situations where there is continuous flow of consecutive articles or consecutive batches of articles and these articles are submitted for inspection in the order of production".

Beattie Continuous Sampling Plan

"Beattie (1962) proposed the continuous acceptance sampling procedure based upon the cumulative sum chart for the number of defectives. The Beattie continuous sampling plan although not devised to deal with a product is rectifiable, can be used in that situation and possesses certain advantages over other plans.

The Beattie procedure was developed for situations where it is desired to accept or reject a continuously produced product based on the number of defectives observed and where the number of items available for testing is relatively small.

The procedure sets up two zones, an acceptance zone and rejection zone; and accepts or rejects according to the value of a cumulative sum (cusum). From a continuous production process, a sample of size n is selected at regular intervals and the number of defectives y_i is recorded. The cusum $S_m = \Sigma(y_i - k)$ is then computed according to the rules below and plotted on a graph. The rules for decision and plotting are as follows.

1. Start the cusum at zero
2. Accept the product as long as $S_m < h$; when $S_m < 0$, return the cusum to zero
3. Reject the product when $S_m >= h$, restart S_m at $(h + h^*)$ and continue reject the product until $S_m <= h$. when $S_m > (h + h^*)$, return S_m to $(h + h^*)$
4. When h is crossed or reached from above, accept the product and restart cusum at 0

Although Beattie pointed out that his continuous acceptance sampling plan was devised to deal with a production process in which it is not feasible to rectify, this does not prevent one from using it in a case where rectification is practiced. Furthermore the procedure has certain advantages over other plans.

The Beattie continuous sampling plan is based on parameters n, k, h and h*. In addition, the sampling rate in the acceptance zone, r_a and the sampling rate in the rejection zone r_r can be specified. For example, in the acceptance zone, $r_a = 0.2$ in n out of 5n items are sampled. Using values for these parameters and numerical methods, the average run length (ARL) that is the average number of samples in the acceptance zone L(p) ad the ARL in the rejection zone L*(p) can be computed".

"Praire and Zimmer (1973) computed the Average outgoing quality limit (AOQL) for Beattie plans when the sample units rectified because of a screening inspection. This is done by first specifying the average fraction of the total product inspected in the long-run and then relating the AOQ to the incoming quality p (quality of the product entering the inspection procedure).

1. If $r_r < 1$ (less than 100 percent inspection in the rejection zone) and $r_r > r_a$, the average fraction of total product inspected in the long run is

$$F = \frac{L(p)+L*(p)}{(1/r_a)L(p)+(1/r_r)L*(p)} \quad \text{and}$$

$$AOQ = p(1-F) = (1-r_r)p + (r_r - r_a)p\,P_a(p)$$

Where $f = r_a/r_r$ and $P_a(p)$, the probability of acceptance of incoming quality p, is given as

$$P_a(p) = \frac{(1/r_a)L(p)}{(1/r_a)L(p)+(1/r_r)L*(p)} = \frac{L(p)}{L(p)+f\,L*(p)}$$

2. If $r_r = 1$(100 percent inspection in the rejection zone) or if with $r_r < 1$, the rejected material is diverted, then

$AOQ = (1-r_a)p\,P_a(p)$ where in both cases $P_a(p)$ is computed with $r_r = 1$"

2. If $r_r = 1$(100 percent inspection in the rejection zone) or if with $r_r < 1$, the rejected material is diverted, then

$AOQ = (1-r_a)p\,P_a(p)$ where in both cases $P_a(p)$ is computed with $r_r = 1$"

Selection of Beattie Continuous Sampling Plan indexed through MAPD and MAAOQ Quality Levels In this paper, a procedure for construction and selection of Beattie Continuous Sampling Plan is given. Tables for determining AOQL and selection of plans indexed with Maximum Allowable Percent Defective (MAPD) and Maximum Allowable Average Outgoing Quality (MAAOQ) are also provided.

In the literature, the average outgoing quality limit (AOQL) is designated as the worst average quality that the consumer will receive in the long run, when the defective items are replaced by non-defective items. The proportion defective corresponding to the inflection point of the Operating Characteristic Curve (OC), p* is defined as the maximum allowable percent defective.

The desirability for developing a set of sampling plans indexed with p* has been explained by Mandelson (1962) and Soundararajan (1975).

Dodge (1943) provided the concept of continuous sampling inspection and introduced the continuous sampling plan, originally referred to as the random order plan and later designated as CSP-1 plan by Dodge and Torrey (1951)

Zimmer and Tai (1980) have studied Beattie continuous sampling plan controlling the AOQ.

"One of the desirable properties of an OC curve is that the decrease of $P_a(p)$ should be slower for lesser values of p* and steeper for larger values of p* which provides a better discriminating power of OC curve.

If p* is considered as a standard quality measure, then the above property of desirable OC curve is exactly followed, since p* corresponds to the inflection point of the OC curve.

$$\frac{d^2 P_a(p)}{dp^2} < 0 \qquad p < p^*$$

$$\frac{d^2 P_a(p)}{dp^2} = 0 \qquad p = p^*$$

$$\frac{d^2 P_a(p)}{dp^2} > 0 \qquad p > p^*$$

Where $P_a(p)$ is the probability of acceptance at quality level p fraction defective. Taking into consideration the criticism at AOQL by several authors and considering the importance of the MAPD as a quality measure, this study provides tables for selection of Beattie continuous sampling plan using MAPD as a quality standard and MAAOQ as a measure of outgoing quality as designed by Suresh and Ramkumar (1996)

Definition of the MAAOQ

The MAAOQ of a sampling plan is designated as the Average Outgoing Quality (AOQ) at the MAPD.

AOQ = $p P_a(p)$

Then we have

MAAOQ = AOQ at p = p* which can be rewritten as

MAAOQ = $p^* P_a(p^*)$"

P_{MAQ} = $[p^*(1-p^*)^i (1-f)]/[f+(1-p^*)^i (1-f)]$

Selection of sampling plans

For specified MAAOQ and MAPD Table 1 is used to construct the plans. For any given values of the MAPD (p*) and MAAOQ (P_{MAQ}), one can find the ratio R1 = MAAOQ/MAPD. Find the value in Table 1 under the column R_1 which is equal to or just less than the specified ratio. Then the corresponding values of i and f are noted. From this, the parameters i and f for the Beattie continuous sampling plan can be determined.

In a similar way the parameters of Beattie continuous sampling plan can be determined for a specified value of MAPD and AOQL using the same procedure by interchanging MAAOQ with AOQL, the ratio R_2 = AOQL/MAPD can be found using the Table 1

Illustration:

Given MAAOQ (P_{MAQ}) = 0.05 and MAPD (p*) = 0.08, compute the ratio R_1 = MAAOQ/MAPD = 0.6250 and select the value of R_1 equal to or just less than 0.6250 using Table 1. The corresponding value of R_1 is 0.623082, which is associated with i = 6 and f = 0.2. Thus i = 6 and f = 0.2 are the parameters selected for Beattie continuous sampling plan for a given MAPD of 0.08 and MAAOQ of 0.05 defective.

Construction of Tables

The operating characteristic expression for a Beattie continuous sampling plan is

$$P_a(p) = \frac{L(p)}{L(p) + fL*(p)}$$

$$= \frac{(1-p)^i}{f + (1-p)^i(1-f)}$$

Where f = r_a/r_r, r_r = 1 ; L(p) = 1/p = fv and L*(p) = $(1-q^i)/(pq^i)$ = u, p+q = 1

r_a - sampling rate in the acceptance zone

r_r - sampling rate in the rejection zone

L(p) - average number of samples in the acceptance zone

L*(p) - average number of samples in the rejection zone

The second order derivative $P_a(p)$ with respect to p is equated to zero, where

$$p* = \frac{[i(1-2f)+1]}{[(1-f)(i^2+1)]}$$

The AOQ is defined as

$$AOQ = p[1-F] = (1-r_r)p + (r_r - r_a)p\,P_a(p)$$

$$F = \frac{L(p)+L*(p)}{(1/r_a)L(p)+(1/r_r)L*(p)}$$

$$AOQ = (r_r - r_a)p\,P_a(p)$$

$$AOQ = \frac{(1-f)p(1-p)^i}{f+(1-p)^i(1-f)}$$

The MAAOQ is defined as

MAAOQ = AOQ at p = p* which is

$$MAAOQ = \frac{(1-f)p*(1-p*)^i}{f+(1-p*)^i(1-f)}$$

R_1 = MAAOQ/MAPD

R_2 = AOQL/MAPD

By definition, the relative slope is given as

$$h = \frac{-p}{P_a(p)}\frac{dP_a(p)}{dp} \qquad p = p*$$

and hence

$$h* = \frac{p*fi}{(1-p*)[f+(1-p*)^i(1-f)]}$$

which is a function of f and i. Then for any specified values of i and f the unique value of R_1 is listed in Table 1, which is also used to select the sampling plan for given AOQL and MAPD values.

Table 1: Beattie continuous sampling plan values of MAAOQ/MAPD and AOQL/MAPD for specified values of i and f

I	f	R_1 = MAAOQ/MAPD	R_2 = AOQL/MAPD

4	0.2	0.606117	0.691764
5	0.3	0.506192	0.566505
	0.2	0.616076	0.730053
6	0.3	0.519244	0.623512
	0.2	0.623082	0.747991
7	0.3	0.527681	0.656574
	0.2	0.627906	0.757105
8	0.3	0.533539	0.678018
	0.2	0.631578	0.763654
9	0.3	0.537815	0.693877
	0.2	0.633295	0.767154
10	0.3	0.541095	0.705479
	0.2	0.636477	0.771069
11	0.3	0.543913	0.716486
	0.2	0.637500	0.773611
12	0.3	0.546232	0.724315
	0.2	0.639269	0.774733
13	0.3	0.548022	0.730696
	0.2	0.640728	0.778145
14	0.3	0.548254	0.735115
	0.2	0.641071	0.776785
15	0.3	0.550111	0.739420
	0.2	0.641074	0.779270
16	0.3	0.551558	0.743405
	0.2	0.642710	0.779475
17	0.3	0.552699	0.748071
	0.2	0.641921	0.780287
18	0.3	0.554945	0.752747
	0.2	0.645011	0.781902
	0.3	0.553935	0.752186
19	0.2	0.644607	0.781186
	0.3	0.557275	0.758513
20	0.2	0.645994	0.782945
	0.3	0.555555	0.761437
21	0.2	0.644021	0.728608
	0.3	0.556700	0.759450
22	0.2	0.646723	0.783475
	0.3	0.555956	0.761732
23	0.2	0.647761	0.785074
	0.3	0.556818	0.765151
24	0.2	0.644859	0.785046
	0.3	0.559523	0.769841
25	0.2	0.646103	0.785714
	0.3	0.557851	0.768595
26	0.2	0.645270	0.783783
	0.3	0.560344	0.771551
27	0.2	0.647887	0.788732
	0.3	0.560538	0.771300
28	0.2	0.645985	0.784671
	0.3	0.561039	0.772093
29	0.2	0.647772	0.787878
	0.3	0.560386	0.772946
30	0.2	0.647059	0.788235
	0.3	0.560000	0.775000

Conclusion

Acceptance sampling is the technique, which deals with the procedures in which decisions to accept or reject lots or process are based on the examination of samples. The work here mainly relates to the new procedure for construction and selection of tables for Beattie continuous sampling plans indexed through MAPD and MAAOQ quality levels. The study also takes into consideration both incentive and filter effects simultaneously. The tables provided are simple and facilitate easy selection of plan parameters to shop floor conditions. There is a good scope for further study on this topic.

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