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Five Dimensional Cosmological Implications Of A Decay Law For Λ-Term: Expressions For Some Observational Quantities

Engineering

KEYWORDS :

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ABSTRACT

Implication of cosmological model with decay law . The cosmological tests pertaining to proper distance, luminosity distance, angular diameter distance and look back time in the framework of five dimensional space time. This work has generalized to higher dimensions the well known result in Five dimensional space time.

1. Introduction

The dimensionality of the world has long been a subject of discussion, since while our senses perceive one time and three space dimensions; there is nothing in the equations of relativity that restrict them to four dimensions and hence, the proliferation in recent years of various versions of Kaluza-Klein theory, supergravity, superstrings, membrane and Brane theory. However, while these theories have abstract appeal, such as a prospect of unification of all the interactions of physics, it has proven difficult to isolate effect of higher dimensions that are clear-cut and might be tested.

Kaluza and Klein (1921, 1926) indepently were the first who initiated the study of unify gravity with electromagnetic interaction by introducing the extra dimension. Kaluza-Klein theory is essentially an extension of Einstein general relativity in five dimensions which is much interest in particle physics and cosmology. The development of superstring theory [Weinberg (1986 (a), 1986 (b)] provides great interest among theoretical physicist and mathematician.

Several ansatz have been proposed in which the Λ term decays with time [Gasperini (1987), Gasperini (1988), Berman (1990 (a), 1990 (b), 1991), Berman and Som (1990), Berman and Gomide (1990), Berman et al. (1989), Freese et al. (1987), Ozer and Taha (1987), Peebles and Ratra (1988), Chen and Wu (1990), Abdussattar and Vishwakarma (1996), Gariel and Le Denmat (1999), Pradhan and Kumar (2001), Pradhan and Yadav (2002)]. Of the special interest is the ansatz $\Lambda \propto a^{-2}$ by Chen and Wu (1990), which has been considered / modified by several authors [Abdel-Rahman (1990, 1992), Carvalho et al. (1992), Waga (1993), Silveira and Waga (1994), Vishwakarma (2000)]. However, not all vacuum decaying cosmological models predict acceleration. Al-Rawaf and Taha and Al-Rawaf (1996, 1998) and Overduin and Cooperstock (1998) proposed a cosmological model with a cosmological constant of the form $\Lambda = \beta \frac{\ddot{a}}{a}$, where a is the scale factor of the universe and β is a constant. Following the same decay law recently Arbab (2003 (a), 2003(b)) has investigated cosmic acceleration with positive cosmological constant and also analyzed the implication of a mode built-in cosmological constant for four-dimensional space time. The cosmological consequences of this decay law are very attractive. This law provides reasonable solutions to the cosmological puzzles presently known. One of the motivations for introducing Λ term is to reconcile the age parameter and the density parameter of the universe with recent observational data.

Waga (1993) investigated the flat cosmological model with cosmological term vary $\Lambda = \alpha R^{-2} + \beta H^2 + \gamma$.

It has been also discuss the cosmological tests pertaining to proper distance, luminosity distance, angular diameter distance and look back time. The Pradhan et al. (2006) present the cosmological consequences for the term $\Lambda = \alpha \left(\frac{\dot{a}}{a}\right)^2$, $\Lambda = \beta \frac{\ddot{a}}{a}$, $\Lambda = \gamma\, \rho$ and $\Lambda = \mu\, \dot{H}$ in the frame work of higher dimensional space time. With this motivation in the present chapter by considering cosmological implication of decay law $\Lambda \sim \frac{\ddot{a}}{a}$, we discuss the cosmological tests pertaining to proper distance, luminosity distance, angular diameter distance and look back time in the framework of five dimensional space time. The results for the cosmological tests are found to be compatible with the present observations.

2. The Metric and Field Equation

Consider the line element

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) + (1-kr^2)\, d\varphi^2\right] \qquad (2.1)$$

where $a(t)$ is the scale factor and k = 0, ± 1 is the curvature factor.

The usual energy momentum tensor is modified by addition of a term

$$T_{ij}^{vac} = -\Lambda(t)\, g_{ij}\ , \qquad (2.2)$$

where $\Lambda(t)$ is the cosmological term and g_{ij} is the metric tensor. For the perfect fluid distribution Einstein field equations with the cosmological constant Λ and gravitational constant G may be written as:

$$G_{ij} = R_{ij} - \frac{1}{2} R g_{ij} = -8\,\pi\, G\, T_{ij} + \Lambda(t)\, g_{ij}\ . \qquad (2.3)$$

The energy momentum tensor T_{ij} in the presence of a perfect fluid has the form

$$T_{ij} = (p+\rho) u_i u_j + p g_{ij}\,, \qquad (2.4)$$

where p and ρ are respectively, the energy and pressure of the cosmic fluid and u_i is the fluid five-velocity such that $u^i u_i = -1$.

Therefore, from equation (2.4) the energy momentum tensor T_j^i can be expressed as

$$T_o^o = -\rho, \quad T_1^1 = T_2^2 = T_3^3 = T_4^4 = p\,. \qquad (2.5)$$

The Einstein's tensor $G_{ij} = R_{ij} - \frac{1}{2} R g_{ij}$ gives

$$G_0^0 = \frac{6}{a^2\left(k + \dot{a}^2\right)}\ , \qquad (2.6)$$

$$G_1^1 = G_2^2 = G_3^3 = G_4^4 = \frac{1}{a^2}\left[3\ddot{a}a + 3k + 3\dot{a}^2\right] \qquad (2.7)$$

where over dot indicates a derivative w. r. to ‘t’.
From equation (2.6) and equation (2.7) with the help of equations (2.5) the field equations (2.3) can be expressed for (k = 0) as

$$\frac{6\dot{a}^2}{a^2} = 8\pi\, G\rho + \Lambda\,, \qquad (2.8)$$

$$\frac{3\ddot{a}}{a} + \frac{3\dot{a}^2}{a^2} = -8\pi\, G\, p + \Lambda\,. \qquad (2.9)$$

To solve the field equations (2.8) and (2.9) it is observed that, the numbers of equations are less than number of unknown therefore for the complete determinacy of the system; we consider the following two equations

(i) $p = \omega\rho$ (2.10)

(ii) $\Lambda = \alpha\left(\frac{\dot{a}}{a}\right)^2, \Lambda = \beta\left(\frac{\ddot{a}}{a}\right),$

$\Lambda = 8\pi\, G\gamma\rho$ and $\Lambda = \mu\, \dot{H}$ (2..11)

where ω, equation of state parameter can be taking the constant values 0, 1/3 and +1 respectively for dust, radiation and stiff fluid and α, β, γ and μ the free parameters.

The energy conservation equation $T^i_{j\,;\,i} = 0$ leads to

$$\left[8\,\pi\, G\, T_j^i + \Lambda\, \delta_j^i\right]_{;\,i} = 0\,,$$

$$8\,\pi\left[G_{\,;\,0}\, T_j^i + G\, T^i_{j\ ;i}\right] + \Lambda_{\,;\,i}\, \delta_j^i = 0\,,$$

$$4\frac{\dot{a}}{a}(\rho + p) = -\left(\frac{\dot{G}}{G}\rho + \dot{\rho} + \frac{\dot{\Lambda}}{8\pi G}\right) \qquad (2.12)$$

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3. Solution of the Field Equation

By using equation (2.10), equation (2.8) and equation (2.9) can be expressed as

$$3\frac{\ddot{a}}{a}+3(1+2\omega)\frac{\dot{a}^2}{a^2}=(1+\omega)\Lambda. \quad (3.1)$$

We propose a phenomenological decay law for Λ of the form

(i) Model for $\Lambda \sim \left(\frac{\dot{a}}{a}\right)^2$ or $\Lambda=\alpha\left(\frac{\dot{a}}{a}\right)^2=\alpha H^2$

where α is a constant, then equation (3.1) is reduce to

$$a\ddot{a}+\frac{1}{3}\left[\omega(6-\alpha)+(3-\alpha)\right]\dot{a}=0. \quad (3.2)$$

Integrating equation (3.2) we get

$$a(t)=\left[\frac{1}{3}(6-\alpha)(1+\omega)C_1\, t\right]^{\frac{3}{(6-\alpha)(1+\omega)}}, \quad (3.3)$$

$$a(t)=\left[\frac{C_1.t}{A_0}\right]^{A_0}, \quad (3.4)$$

where C_1 is an integrating constant and constant A_0 has the value

$$A_0=\frac{3}{(6-\alpha)(1+\omega)}.$$

Now, from equation (2.8) we have

$$\rho(t)=\frac{9}{8\pi G(6-\alpha)(1+\omega)^2}\frac{1}{t^2} \quad (3.5)$$

$$\Lambda(t)=\alpha\frac{\dot{a}^2}{a^2}=\frac{9\alpha}{(6-\alpha)^2(1+\omega)^2}\frac{1}{t^2}, \quad (3.6)$$

$$H=\frac{3}{(6-\alpha)(1+\omega)}\frac{1}{t}$$

$$t=\frac{3}{(6-\alpha)(1+\omega)}\frac{1}{H}. \quad (3.7)$$

The deceleration parameter q is defined as,

$$q=\frac{-\ddot{a}a}{\dot{a}^2},$$

$$q=\frac{(1-A_0)}{A_0}=\frac{\omega(6-\alpha)-\alpha+3}{3}. \quad (3.8)$$

Using t from equation (3.7) in equation (3.5) we get

$$\Omega_{m\alpha}=\frac{8\pi G\rho}{6H^2}=1-\frac{\alpha}{6}. \quad (3.9)$$

Again, using equation (3.7) in equation (3.6) we get

$$\Omega_{\Lambda\alpha}=\frac{\Lambda}{6H^2}=\frac{\alpha}{6}. \quad (3.10)$$

Adding equation (3.8) and equation (3.9) we get

$$\Omega_{m\alpha}+\Omega_{\Lambda\alpha}=1. \quad (3.11)$$

(ii) Model for $\Lambda \sim \left(\frac{\ddot{a}}{a}\right)$ or $\Lambda=\beta\left(\frac{\ddot{a}}{a}\right)$,

where β is a constant, then equation (3.1) is reduce to

$$a\ddot{a}+\frac{3(1+2\omega)}{(3-\omega\beta-\beta)}\dot{a}^2=0.$$

Integrating above equation we get

$$a(t)=\left[\frac{(1+\omega)(6-\beta)}{(3-\omega\beta-\beta)}C_2.t\right]^{\frac{(3-\omega\beta-\beta)}{(1+\omega)(6-\beta)}}, \quad (3.12)$$

$$a(t)=\left[\frac{C_2\, t}{B_0}\right]^{B_0},$$

where C_2 is an integrating constant and constant B_0 has the value

$$B_0=\left(\frac{3-\beta\omega-\beta}{(1+\omega)(6-\beta)}\right).$$

Now from equation (2.8) we obtained

$$\rho(t)=\frac{3(\beta+\omega\beta-3)}{8\pi G(\beta-6)(1+\omega)^2}\frac{1}{t^2}, \quad (3.13)$$

$$\Lambda(t)=\beta\frac{\ddot{a}}{a}=\left[\frac{3\beta(1+2\omega)(\beta+\omega\beta-3)}{(1+\omega)^2(\beta-6)^2}\right]\frac{1}{t^2}, \quad (3.14)$$

$$H=\frac{(3-\beta\omega-\beta)}{(1+\omega)(6-\beta)}\frac{1}{t},$$

$$t=\frac{(3-\beta\omega-\beta)}{(1+\omega)(6-\beta)}H^{-1}. \quad (3.15)$$

The deceleration parameter q is defined as

$$q=\frac{-a\ddot{a}}{\dot{a}^2}=\left(\frac{1-B_0}{B_0}\right),$$

$$q=\frac{6\omega+3}{3-\omega\beta-\beta}. \qquad (3.16)$$

Using value of t from equation (3.15) in equation (3.13) we get

$$\Omega_{m\beta}=\frac{8\pi\,G\rho}{6H^2}=\frac{(\beta-6)}{2(\beta+\omega\beta-3)}. \qquad (3.17)$$

Again, using value of t from equation (3.15) in equation (3.14) we get

$$\Omega_{\Lambda\beta}=\frac{\Lambda}{6H^2}=\frac{\beta(1+2\omega)}{2(\beta+\omega\beta-3)} \qquad (3.18)$$

Adding equation (3.17) and (3.18) we get

$$\Omega_{m\beta}+\Omega_{\Lambda\beta}=1. \qquad (3.19)$$

(iii) Model for $\Lambda\sim\rho$ or $\Lambda=8\pi\,G\gamma\,\rho$,

where γ is a free parameter, then equation (3.1) is reduce to

$$\ddot{a}a-\left(\frac{\gamma-2\omega-1}{(1+\gamma)}\right)\dot{a}^2=0 \quad .$$

Integrating above equation we get

$$a(t)=\left[\frac{2(1+\omega)}{(1+\gamma)}C_3t\right]^{\frac{1+\gamma}{2(1+\omega)}}, \qquad (3.20)$$

$$a(t)=\left[\frac{C_3t}{C_0}\right]^{C0},$$

where C_3 is an integrating constant and constant C_0 has the value $C_0=\frac{1+\gamma}{2\,(1+\omega)}$.

Now, from equation (2.8), we get

$$\rho(t)=\left[\frac{3\,(1+\gamma)}{16\pi\,G\,(1+\omega)^2}\right]\frac{1}{t^2} \qquad (3.21)$$

$$\Lambda(t)=\frac{3\gamma\,(1+\gamma)}{2\,(1+\omega)^2}\,\frac{1}{t^2}, \qquad (3.22)$$

$$H=\frac{(1+\gamma)}{2(1+\omega)\,t},$$

$$t=\frac{(1+\gamma)}{2(1+\omega)\,H}. \qquad (3.23)$$

The declaration parameter q is defined as

$$q=\frac{-a\ddot{a}}{\dot{a}^2}=\left(\frac{1-C_0}{C_0}\right),$$

$$q=\frac{1+2\omega-\gamma}{1+\gamma} \qquad (3.24)$$

Using the value t from (3.23) in equation (3.21) we get

$$\Omega_{mr}=\frac{8\pi\,G\rho}{6H^2}=\frac{1}{1+\gamma}. \qquad (3.25)$$

Using the value t from (3.23) in equation (3.22) we get

$$\Omega_{\Lambda\gamma}=\frac{\Lambda}{6H^2}=\frac{\gamma}{1+\gamma}. \qquad (3.26)$$

Adding equation (3.25) and (3.26) we get

$$\Omega_{m\gamma}+\Omega_{\Lambda\gamma}=1. \qquad (3.27)$$

(iv) Model for $\Lambda\sim\dot{H}$ or $\Lambda=\mu\,\dot{H}$,

where μ is a free parameter.

Now, from equation

$$H=\frac{(3-\mu-\mu\omega)}{6\,(1+\omega)\,t}, \quad \text{we get}$$

Integrating above equation we get

$$a(t)=(Dt)^{\frac{(3-\mu-\mu\omega)}{6\,(1+\omega)}},$$

$$a(t)=(Dt)^{K_0} \qquad (3.28)$$

where D is an integrating constant and constant K_0 has the value .

$$K_0=\frac{(3-\mu-\mu\omega)}{6\,(1+\omega)}$$

From equations,

$$\rho(t)=\frac{(3-\mu-\mu\omega)}{16\pi\,G(1+\omega)^2}\,t^{-2}, \qquad (3.29)$$

$$\Lambda(t)=\frac{-\mu\,(3-\mu-\mu\omega)}{6(1+\omega)}\,t^{-2} \qquad (3.30)$$

$$H=\frac{(3-\mu-\mu\omega)}{6\,(1+\omega)\,t},$$

$$t=\frac{(3-\mu-\mu\omega)}{6\,(1+\omega)\,H}. \qquad (3.31)$$

The deceleration parameter q is defined as

$$q=\frac{-a\ddot{a}}{\dot{a}^2}=\frac{(1-K_o)}{K_o},$$

$$q = \frac{3+\mu(1+\omega)+6\omega}{3-\mu-\mu\omega}. \quad (3.32)$$

Using the value t from equation (3.31) in equation (3.29) we get

$$\Omega_{m\mu} = \frac{8\pi G\rho}{6H^2} = \frac{-3}{(\mu+\mu\omega-3)} \quad (3.33)$$

Using the value t from equation (3.31) in equation (3.30) we get

$$\Omega_{\Lambda\mu} = \frac{\Lambda}{6H^2} = \frac{\mu(1+\omega)}{(\mu+\mu\omega-3)}. \quad (3.34)$$

Adding equation (3.33) and equation (3.34) we get

$$\Omega_{m\mu} + \Omega_{\Lambda\mu} = 1 \quad (3.35)$$

According to high red shift supernovae and CMB, the preliminary result from the advancing field of cosmology suggest that the universe may be accelerating universe with a dominant contribution to its energy density coming in the form of cosmological Λ term. The result, when combined with CMB anisotropy observation on intermediate angular scales, strongly support a flat universe

$$\Omega_m + \Omega_\Lambda = 1 \quad (3.36)$$

For the three different cases ω = 0, 1, 1/3 values of the matter energy density ρ, vacuum energy density $\Lambda(t)$, deceleration parameter q, the matter density parameter Ω_m, vacuum density parameter Ω_Λ, present age of universe t_0 for five dimensional space are given in the following table:

(i) Model for $\Lambda \sim \alpha\left(\frac{\dot{a}}{a}\right)^2$

Values	$\omega=0$	$\omega=1$	$\omega=\frac{1}{3}$
$\rho(t)$	$\frac{9}{8\pi G(6-\alpha)t^2}$	$\frac{9}{32\pi G(6-\alpha)t^2}$	$\frac{81}{128\pi G(6-\alpha)t^2}$
$\Lambda(t)$	$\frac{9\alpha}{(6-\alpha)^2 t^2}$	$\frac{9\alpha}{4(6-\alpha)^2 t^2}$	$\frac{81\alpha}{16(6-\alpha)^2 t^2}$
q	$\frac{(3-\alpha)}{3}$	$\frac{(9-2\alpha)}{3}$	$\frac{(15-4\alpha)}{9}$
Ω_m	$\frac{(6-\alpha)}{6}$	$\frac{(6-\alpha)}{6}$	$\frac{(6-\alpha)}{6}$
Ω_Λ	$\frac{\alpha}{6}$	$\frac{\alpha}{6}$	$\frac{\alpha}{6}$
t_0	$\frac{3}{(6-\alpha)H_0}$	$\frac{3}{2(6-\alpha)H_0}$	$\frac{9}{4(6-\alpha)H_0}$

(ii) Model for $\Lambda \sim \left(\frac{\ddot{a}}{a}\right)$

Values	$\omega=0$	$\omega=1$	$\omega=\frac{1}{3}$
$\rho(t)$	$\frac{3(\beta-3)}{8\pi G(\beta-6)t^2}$	$\frac{3(2\beta-3)}{32\pi G(\beta-6)t^2}$	$\frac{9(4\beta-9)}{128\pi G(\beta-6)t^2}$
$\Lambda(t)$	$\frac{3\beta(\beta-3)}{(\beta-6)t^2}$	$\frac{9\beta(2\beta-3)}{4(\beta-6)t^2}$	$\frac{15\beta(4\beta-9)}{16(\beta-6)t^2}$
q	$\frac{-3}{(\beta-3)}$	$\frac{-9}{(2\beta-3)}$	$\frac{-15}{(4\beta-9)}$
Ω_m	$\frac{(\beta-6)}{2(\beta-3)}$	$\frac{(\beta-6)}{2(2\beta-3)}$	$\frac{3(\beta-6)}{2(4\beta-9)}$
Ω_Λ	$\frac{\beta}{2(\beta-3)}$	$\frac{3\beta}{2(2\beta-3)}$	$\frac{5\beta}{2(4\beta-9)}$
t_0	$\frac{(\beta-3)}{(\beta-6)H_0}$	$\frac{(2\beta-3)}{2(\beta-6)H_0}$	$\frac{(4\beta-9)}{4(\beta-6)H_0}$

(iii) Model for $\Lambda \sim \rho$

Values	$\omega=0$	$\omega=1$	$\omega=\frac{1}{3}$
$\rho(t)$	$\frac{3\,(1+\gamma)}{16\,\pi\,G\,t^2}$	$\frac{3\,(1+\gamma)}{64\,\pi\,G\,t^2}$	$\frac{27\,(1+\gamma)}{256\,\pi\,G\,t^2}$
$\Lambda(t)$	$\frac{3\gamma\,(1+\gamma)}{2\,t^2}$	$\frac{3\gamma\,(1+\gamma)}{8\,t^2}$	$\frac{27\,\gamma\,(1+\gamma)}{32\,t^2}$
q	$\frac{(1-\gamma)}{(1+\gamma)}$	$\frac{(3-\gamma)}{(1+\gamma)}$	$\frac{(5-3\gamma)}{3\,(1+\gamma)}$
Ω_m	$\frac{1}{(1+\gamma)}$	$\frac{1}{(1+\gamma)}$	$\frac{1}{(1+\gamma)}$
Ω_Λ	$\frac{\gamma}{(1+\gamma)}$	$\frac{\gamma}{(1+\gamma)}$	$\frac{\gamma}{(1+\gamma)}$
t_0	$\frac{(1+\gamma)}{2\,H_0}$	$\frac{(1+\gamma)}{4\,H_0}$	$\frac{3\,(1+\gamma)}{8\,H_0}$

(iv) Model for $\Lambda \sim \dot{H}$

Values	$\omega=0$	$\omega=1$	$\omega=\frac{1}{3}$
$\rho(t)$	$\frac{-\,(\mu-3)}{16\,\pi\,G\,t^2}$	$\frac{-\,(2\mu-3)}{64\,\pi\,G\,t^2}$	$\frac{-9\,(4\mu-9)}{256\,\pi\,G\,t^2}$
$\Lambda(t)$	$\frac{\mu\,(\mu-3)}{6\,t^2}$	$\frac{\mu\,(2\mu-3)}{12\,t^2}$	$\frac{\mu\,(4\mu-3)}{24\,t^2}$
q	$\frac{-(\mu+3)}{(\mu-3)}$	$\frac{-3}{(2\mu-3)}$	$\frac{-(4\mu+15)}{(4\mu-9)}$
Ω_m	$\frac{-3}{(\mu-3)}$	$\frac{-3}{(2\mu-3)}$	$\frac{-3}{(4\mu-9)}$
Ω_Λ	$\frac{\mu}{(\mu-3)}$	$\frac{2\mu}{(2\mu-3)}$	$\frac{4\mu}{(4\mu-9)}$
t_0	$\frac{-(\mu-3)}{6\,H_0}$	$\frac{-(2\mu-3)}{12\,H_0}$	$\frac{-(4\mu-3)}{24\,H_0}$

4. Neoclassical tests (Proper distance d (z))

A photon emitted by a source with co-ordinate $r = r_1$, and $t = t_1$, and received at a time t_0 by an observer located at $r = 0$. The emitted radiation will follow null geodesics on which (θ, ϕ, φ) are constant. The proper distance between the source and observer is given by

$$d(z) = a_0 \int_a^{a_0} \frac{da}{a\dot{a}} \tag{4.1}$$

$$r_1 = \int_t^{t_0} \frac{dt}{a} = \frac{a_0^{-1} H_0^{-1} . A_0}{(1-A_0)} \left[1 - (1+z)^{\frac{A_0-1}{A_0}} \right]$$

where

$$A_0 = \frac{3}{(6-\alpha)(1+\omega)}.$$

Hence

$$d(z) = r_1 a_0 = H_0^{-1} \left(\frac{A_0}{1-A_0} \right) \left[1 - (1+z)^{\frac{A_0-1}{A_0}} \right], \tag{4.2}$$

where $(1+z) = \frac{a_0}{a}$ = redshift and a_0 is the present factor of the universe

For small z equation (4.2) reduces to,

$$H_0\ d(z) = z - \frac{1}{2\ A_0} z^2 + ... \tag{4.3}$$

By using equation (3.8) we get

$$H_0\ d(z) = z - \frac{1}{2} (1+q) z^2 + ... \tag{4.4}$$

From equation (4.2), it is observed that the distance d is maximum at $z = \infty$. Hence

$$d(z=\infty) = H_0^{-1} \left(\frac{A_0}{1-A_0} \right) \tag{4.5}$$

5, Luminosity distance

Luminosity distance is the another important concept of theoretical cosmology of a light source The luminosity distance is a way of expanding the amount of light received from a distant object It is the distance that the object appears to have assuming the inverse

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square low for the reduction of light intensity with distance holds. If is d_L the luminosity distance to the object, then

$$d_L = \left(\frac{L}{4\pi l}\right)^{\frac{1}{2}} , \qquad (5.1)$$

where L is the total energy emitted by the source per unit time, l is the apparent luminosity of the object. Therefore one can write

$$d_L = r_1 a_0 = d(1+z).$$

Using equation (4.2), equation (5.1) reduces to

$$H_0\, d_L = (1+z)\left(\frac{A_0}{1-A_0}\right)\left[1-(1+z)^{\frac{A_0}{1-A_0}}\right] . \qquad (5.2)$$

For small z equation (5.2) gives

$$H_0\, d_L = z + \frac{1}{2}(1-q)\, z^2 + ... \qquad (5.3)$$

or by using equation (3.8)

$$H_0\, d_L = z + \left[1-(1+\omega)\,\Omega_m\right] z^2 + ... \qquad (5.4)$$

The luminosity distance depends on the cosmological model we have under discussion and hence can be used to tell us which cosmological models describe our universe. Unfortunately, however, the observable quantity is the radiation flux density received from an object, and this can only be translated into a luminosity distance if absolute luminosity of the object is known. This problem can however be circumvented if there are a population of objects at different distances which are believed to have the same luminosity ; even if that luminosity is not known, it will appear merely as an overall scaling factor.

6. Angular Diameter Distance

The angular diameter distance d_A is defined as the ratio of an objects physical transverse size to its angular size. It is used to convert angular separations in telescope images into proper separations at the sources. The angular diameter d_A of a light source of proper distance d is given by,

$$d_A = d(z)(1+z)^{-1} = d_L\,(1+z)^{-2} . \qquad (6.1)$$

Applying equation (5.2) we obtain

$$H_0\, d_L = \frac{A_0}{1-A_0}\left[\frac{1-(1+z)^{\frac{A_0-1}{A_0}}}{(1+z)}\right]. \qquad (6.2)$$

Usually d_A has a minimum for maximum for some $z = z_m$. The angular diameter and luminosity distance have similar forms, but have a different dependence on redshift.

7. Look back time

The time in the past at which the light we now receive from a distant object was emitted is called the look back time. How *long ago* the light was emitted (the look back time) depends on the dynamics of the universe. The radiation travel time (or look back time) ($t - t_0$) for photon emitted by a source at instant t and received at t_0

$$t - t_o = \int_a^{a_0} \frac{da}{\dot{a}} . \qquad (7.1)$$

Equation (3.4) can be written as

$$a = B_0\, t^{A_0}, \;\; B_0 = \text{constant}. \qquad (7.2)$$

This follows that,

$$\frac{a_0}{a} = 1 + z = \left(\frac{t_0}{t}\right)^{A_0} \qquad (7.3)$$

The above equation (7.3) gives

$$t = t_0\,(1+z)^{\frac{-1}{A_0}} . \qquad (7.4)$$

This equation can also be expressed as

$$H_0\,(t_0 - t) = A_0\left[1-(1+z)^{\frac{-1}{A_0}}\right]. \qquad (7.5)$$

For small z one obtain

$$H_0\,(t_0 - t) = z - \left(1+\frac{q}{2}\right) z^2 + ... \qquad (7.6)$$

From equation (7.4) and equation (7.6), we observe that $z \to \infty$, $H_0\, t_0 = A_0$ (constant).

8. Conclusion

In this paper we have presented the cosmological consequences of equation (2.11) for three different cases $\omega = 0, 1, \frac{1}{3}$ in the frame work of Kaluza-Klein theory of gravitation. The nature of the cosmological constant Λ and energy density ρ have been examined and in the context of five dimensional space-time. In all the four cases we observed that ρ and Λ vary inversely with square of time, which matches its natural units.

It is also shown that $\Lambda = \mu \dot{H}$ model is equivalent to other three types of phenomenological models, *viz.* $\Lambda = \alpha \left(\frac{\dot{a}}{a}\right)^2$, $\Lambda = \beta \frac{\ddot{a}}{a}$ and $\Lambda = \gamma \rho$. Also, for the different cases $\omega = 0, 1, \frac{1}{3}$ values of the matter energy density ρ, vacuum energy density $\Lambda(t)$, deceleration parameter q, the matter density parameter Ω_m, vacuum density parameter Ω_Λ, present age if universe t_0 are calculated and given in the tabular form in the frame work of five dimensional space time.

The proper distance, the luminosity distance-redshift, the angular diameter distance-redshift and look back time-redshift for the models are also presented in the frame work of Kaluza-Klein theory of gravitation.

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