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On Slightly b*-Continuous functions

Mathematics

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ABSTRACT

In this paper we apply the concept of b-open sets in topological spaces to study a class of functions called slightly b*-continuous functions. Some characterizations and several properties concerning slight b*-continuity are obtained.*

INTRODUCTION

The notion of continuity has shown its significance not only in general topology but also in other branches of mathematics. So different forms of weak and strong continuities have been discussed over years. Levine [9] initiated this study by introducing the notion of semi-open sets. Many mathematicians have put their efforts in this direction. Later on Jain [7] made his contribution by giving the concept of slightly continuous functions. Many weak forms of slightly continuous functions have been studied. For instance slightly semi-continuous [15] slightly β-continuous [13], slightly γ- continuous [6], slightly ω- continuous [14] and slightly b-continuous functions [4] have been investigated. In this paper we investigate some weak forms of open sets namely b*-open sets[10]. By means of these sets we investigate some concepts of classes of functions, namely slightly b*-continuous functions.

2. NOTATION, DEFINITIONS AND PRELIMINARIES

Throughout the paper spaces mean topological spaces and f: (X, τ) → (Y, σ) or (f: X → Y) mean a function of a space (X, τ) into a space (Y, σ). Let X be a topological space and B ⊂ X. cl (B) and int (B) are used to denote the closure of B and interior of B respectively. B is called clopen set [18] of X if it is both open and closed in X.

Definition 2.1: A subset B of X is called α-open [12] if B⊂int(cl(intB))).

Definition 2.2: A subset B of X is called δ*-open [16] if for each x ∈ B there exists an open and closed set V of X such that x∈V⊂B.

Definition 2.3: A subset A of a topological space (X, τ) is called a b*-closed set[10] if int(cl(A)) ⊆ U , whenever A ⊆ U and U is b-open.

Definition 2.4: A function f is said to be slightly continuous [7] if for every clopen subset U of Y, the set f^{-1} (U) is open in X.

Definition 2.5: f is slightly b*-continuous at a point x ∈ X if for each clopen subset U of Y containing f(x), there exists b*-open subset V in X containing x such that f (V) ⊂U. f is said to be slightly b*-continuous if it is slightly b*-continuous at each point of X.

Definition 2.6: function f: X → Y is called
(i) b*-irresolute if for every b* -open subset U of Y, f^{-1}(U) is b* -open in X,
(ii) b*-open if for every b* -open subset V of X, f(V) is b* -open in Y,
(iii) Weakly b* -continuous if for each point x∈X and each open subset U of Y containing f(x), there exists b*-open subset V in X containing x such that f(V) ⊂ cl(U),
(iv) Contra b* -continuous if f^{-1}(H) is b* -open for each closed set H of Y.

Definition2.7: A space X is said to be
(1) Ultra Hausdorff, [17] if every two distinct points of X can be separated by

disjoint clopen sets,
(2) Extremely disconnected [3] if the closure of every open set of X is open in X,
(3) Locally indiscrete [11] if every open set of X is closed in X,
(4) 0-dimensional [18] if its topology has a base consisting of clopen sets,

Theorem 3.1 The following properties hold for a function f: X → Y:

(1)f is slightly b* -continuous;

(2)For every clopen set U⊂Y, f^{-1} (U) is b*-open;

(3)For every clopen set U ⊂ Y, f^{-1} (U) is b* -closed;

(4)For every clopen set U⊂ Y, f^{-1} (U) is b* -clopen;

(5) For every δ*-open set U⊂ Y, f^{-1} (U) is b* -open;

(6)For every δ* -closed set U ⊂ Y, f^{-1} (U) is b* -closed.

Proof. (1) ⟹ (2): Let U be a clopen subset of Y and let x ϵ f^{-1} (U). Since f is slightly b* -continuous, by (1) there exists b* -open set Vx in X containing x such that f (Vx) ⊂ U; hence Vx ⊂f^{-1} (U). We obtain that f^{-1} (U) =∪ [{Vx |x ϵ f^{-1} (U)].Thus f^{-1} (U) is b* –open.

(2) ⟹ (3): Let U be a clopen subset of Y. Then Y \U is clopen. By (2) f^{-1} ((Y \ U) = X \ f^{-1} (U) is b* -open. Thus f^{-1} (U) is b* -closed.

(3) ⟹ (4): It can be proved easily.

(4) ⟹ (5): Let Ube δ*-open open set in Y and let x ϵ f^{-1} (U). Then f(x) ϵ U. Since U is δ*-open there exists a F ϵ CO(Y) such that f(x) ϵ F ⊂ U. This implies that x ϵ f^{-1} (F) ⊂ f^{-1} (U). By (4), f^{-1} (F) is b* -clopen. Hence f^{-1} (U) is b*- neighbourhood of each of its points. Consequently, f^{-1} (U) ϵ b*O(X).

(5) ⟹ (6): It is clear from the fact that the complement of δ*-closed is δ*-open.

(5) ⟹ (1): It is clear from the fact that every clopen set is δ*-open.

Lemma3.2: let A and X_0 be subsets of X.If A ϵ b*O(X) and X_0 ϵα O(X), then A∩ X_0 ϵ b* O(X_0).

Theorem 3.3: Let f: X →Y be slightly b*-continuous and B ϵα O(X) , then the restriction f|B: B→Y is slightly b* -continuous.

Proof. Let U be a clopen subset of Y. We have $(f|B)^{-1}(U) = f^{-1}(U) \cap B$. Now f^{-1} (U) ϵ b*O(X) and B ϵα O(X).So by lemma 3.2 f^{-1} (U) ∩B ϵ b* O (B). Therefore $(f|B)^{-1}$(U) is b*-open in the relative topology of B. Thus f|B is slightly b*-continuous**.**

Lemma 3.4: Let A⊂ X_0 ⊂X .If A ϵ b*O(X_0) and X_0 ϵα O(X),then A ϵ b*O(X).

Theorem 3.5: Let f: X →Y be a function and xϵX. If ∃U ϵα O(X) such that x ϵU and the restriction of f to U is slightly b*-continuous at x,then f is slightly b*-continuous at x.

Proof: Let F be a clopen subset of Y containing f(x). Then ∃ V ϵ b*O (U) containing x such that f(V) ⊂ F.Since V ϵ b*O(U) , U ϵα O(X) and V ⊂ U ⊂X.So by lemma3.4,V ϵ b*O(X).From which It follows that f is slightly b*-continuous at x.

Theorem 3.6: Let f: X →Y be a function and σ={U_i: i ϵI} be a cover of X such that U_i ϵα O(X) for each i ϵI. If f|Ui is slightly b* -continuous for each i ϵI .Then f is slightly b*-continuous function.

Proof: Let V be clopen set in Y.Since f|U_i is slightly b* -continuous for each i ϵI, $(f|U_i)^{-1}$(V) ϵ b*O(U_i) for each i ϵI.Now f^{-1}

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$(V) = \cup_{i\in I}(\ f^{-1}(V) \cap U_i\ )$

$= \cup_{i\in I}(f|U_i)^{-1}(V)$.Now $U_i \in \alpha O(X)$ for each $i \in I$.It follows from lemma 3.4, $f^{-1}(V) \in b^*O(X)$.

Theorem 3.7: Let f: X → Y and function g: Y → Z be functions. Then the following properties hold:

(1) If f is b*-irresolute and g is slightly b*-continuous, then gof: X → Z is slightly b*-continuous.

(2) If f is slightly b*–continuous and g is slightly continuous, then gof is slightly b*-continuous.

Proof: . (1) Let U be any clopen set in Z. By the slight b*-continuity of g, $g^{-1}(U)$ is b*-open. Since f is b*-irresolute, $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is b*-open. Therefore, gof is slightly b*-continuous.

(2) Let U be any clopen set in Z. By the slight-continuity of g, $g^{-1}(U)$ is clopen Since f is slightly b*-continuous, $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is b*-open. Therefore, gof is slightly b*-continuous.

Theorem3.8: Let f: X → Y be b*-irresolute, b*-open surjection and g: Y → Z be a function. Then g is slightly b*-continuous if and only if gof is slightly b*-continuous.

Proof: . Assume that g is slightly b*-continuous. Then by Theorem 3.7, gof is slightly b*-continuous. Conversely, let gof be slightly b*-continuous and U be clopen set in Z. Then $(g\ o\ f)^{-1}(U)$ is b*-open. Since f is [illegible]*-open surjection, then f $((g\ o\ f)^{-1}(U)) = g^{-1}(U)$ is b*-open in Y. It follows that g is slightly b* -continuous.

The following diagram holds for weak b*-continuity, contra b*-continuity and slight continuity, slight b*-continuity.

b*-continuous ⟹ weakly b*-continuous

⇓

Contra b*-continuous ⟹ slightly b*-continuous ⟸ slightly continuous.

These implications need not be reversible as is clear from the following examples:

Example3.9 Let $X=\{a,b,c\}$ and $\tau = \{X, \emptyset, \{a\}\{b\}\{a.b\}, \{a,c\}\}$

Let f: $(X, \tau) \to (Y, \tau)$ be a function defined by ,f (a) =c, f(b)=b, f(c)=a.

Then f is a weakly b*-continuous function which is not b*-continuous.

Example 3.10.The identity function on the real line with usual topology is slightly b*-continuous but not contra b*- continuous since the preimage of any singleton set is not b*-open.

Example 3.11:let X=R and $y=\{a,b,c\}$, and τ = usual topology,

$\sigma = \{Y, \emptyset, \{a\}\{b\}\{a.b\}\}$.

Let f: $(X, \tau) \to (Y, \sigma)$ be a function defined by

$$f(x)=\begin{cases} a, x \in Q \\ b, x \notin Q \end{cases}$$

is not weakly b*-continuous but slightly b*-continuous.

Example 3.12.let X=Y=R, τ = usual topology,

σ = discrete topology, Let f: $(X, \tau) \to (Y, \sigma)$ be an identity function .Then f is slightly b*-continuous. but it is not slightly continuous.

Theorem 3.13. Let f: X → Y be slightly b*-continuous and Y be extremally disconnected, then f is weakly b* -continuous.

Proof. Let x ϵ X and U be an open subset of Y containing f(x). Then Cl (U) is open and hence clopen. Therefore there exists a b*-open set V ⊂X with x ϵ V and f (V) ⊂ Cl (U). It follows that f is weakly b*-continuous.

Theorem 3.14. Let f: X→ Y be slightly b*-continuous and Y be locally indiscrete, then f is b*-continuous and contra b* -continuous.

Proof. Let U be any open set of Y. Since of Y containing f(x). Then Cl ($\bar{U}$) is open and hence clopen. Therefore there exists a b*-open set V $\subset$X with x $\in$ V and f (V) $\subset$ Cl (U). It follows that f is weakly b*-continuous.

Theorem 3.14. Let f: X→ Y be slightly b*-continuous and Y be locally indiscrete, then f is b*-continuous and contra b* - continuous.

Proof. Let U be any open set of Y. Since Y is locally indiscrete, U is clopen and hence f^{-1}(U) is b*-open and b*-closed in X. It follows that f is b*-continuous and contra b*-continuous.

Theorem 3.15. Let f: X → Y be slightly b*-continuous and Y be 0-dimensional, then f is b*-continuous.

Proof. Let x $\in$ X and U $\subset$Y be any open set containing f(x). Since Y is 0-dimensional, there exists a clopen set V containing f(x) such that V $\subset$ U. But f is slightly b*-continuous, so there exists K$\in$ b*O(X, x) such that f(x) $\in$ f (K) $\subset$ V $\subset$ U. Hence f is b*-continuous.

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