

# Formulation of Mathematical Modeling to Characterize the Aluminium Metals Using Ultrasonic Non-Destructive Techniques.



## Physics

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## ABSTRACT

*Predicting the type of aluminium metals and composition of elements present in the aluminium samples through Nondestructive testing (NDT) is a matter of very importance for aluminium Industry. The unique method to determine grade of the aluminium sample is required to characterize the aluminium metals. Therefore a technique is required to predict the percentage of aluminium, Iron, Copper, Manganese of aluminium metals so as to categorize into different grades and applications. In Aluminium samples percentage of Aluminium plays very important role which may help to decide the grade of the aluminium metals hence its applications. The present work is focused on how the percentage of aluminium in aluminium samples can be calculated by adopting the mathematical modeling technique.*

*An attempt is being made to formulate an approximate mathematical model which will certainly predict the percentage of aluminium in aluminium samples. The reliability of the model has also been estimated.*

## 1. INTRODUCTION

For the purpose of detecting defects inside metals, Ultrasonics techniques is the most practical alternative to heavy doses of x-rays or gamma rays. Ultrasonics can also be used to infer much information about the bulk microstructure of metals [1,2] and material properties [3]. Signal processing involves techniques that improve our understanding of information contained in the received ultrasonic data [4]. The formulation of mathematical model based on experimental model is a new approach to study the material. The theory of experimentation as suggested by Hilbert [5] is a good approach of representing the response of any phenomenon in terms of proper interaction of various inputs of the phenomenon. This approach finally establishes an experimental data based model for any phenomenon. As suggested in this article the experimentation has been carried out and the models are formulated. Aluminium percentage in the aluminium samples is determined to characterize the samples. This objective is only achievable by formulation of such models.

## 2. EXPERIMENTAL APPROACH

In order to characterize the aluminium metals, different grades needs to be identified which is possible only if aluminium percentage and composition of other elements are known. It is observed that in aluminum percentage in the aluminium metals are varying with respect to other observed NDT parameters. The value of the parameters either increase or decrease with respect to other parameters. In order to develop the relationship between aluminium percentages with one of the observed NDT parameters is not possible. Hence the individual parameter is not sufficient to characterize the aluminium samples. However the combine effect of all these NDT observed parameters is utilized to characterize aluminium metals. Hence one is left with only alternative of formulating experimental data based models to be more specific field data based models. Normally the approach adopted for formulating generalized experimental model suggested by Hilbert for any complex physical phenomenon involves following steps. Hence, this aspect in general instigates to investigate a mathematical model, which can predict the percentage of aluminium. Indeed the model will be useful for researcher as well as industry to work on prominent variables by which they can estimate aluminium percentage.

## 3. AN APPROACH TO FORMULATE MATHEMATICAL MODEL:

The mathematical model can be established by an approach of experimental data based model formulation suggested by Schenck. H. Jr., in a modified form is adopted for this purpose. The modification is to the extent of covering only and it is done by

- Identification of variables
- Establishment of dimensionless pie terms
- Formulating the model for prediction of aluminium percentage using other observed NDT parameters.

### Identification of Variables:

The first step in this process is identification of variables .The parameters of the phenomenon is called variables. These variables are of three types

- (1) Independent variables,
- (2) Dependent variable, and
- (3) Extraneous variable.

The independent variables are those which can be changed without changing other variables of the phenomenon. Whereas, the dependent variables are those, that can only change if any change in the independent variables. The extraneous variables change in a random and uncontrolled manner in the phenomenon . This correlation is nothing but a mathematical model as a designed tool for such situation and this model is used to predict the aluminium percentage in aluminium metals which may help to identify its applications. As far as this phenomenon is concerned the dependent or response variable is Percentage of aluminium in the aluminium samples while the phenomenon is influenced by following variables.

S.N	Name of Variables	Symbol Used	Dependent/Independent
1	Percentage of aluminium in sample	Al% or Y	Dependent
2	Hardness	H	Independent
3	Density	P	Independent
4	Velocity	V	Independent
5	Attenuation	A	Independent
6	Modulus of elasticity	MOE	Independent
7	Peak amplitude of Time signal	TS(Y)	Independent
8	Peak ampl. of FFT	FFT(Y)	Independent
9	Peak Amp. of PSD	PSD(Y)	Independent
10	Peak amplitude time of time signal	TS(x)	Independent
11	Peak amplitude frequency of FFT	FFT(x)	Independent
12	Peak amplitude frequency of PSD	PSD(x)	Independent

**Table1: List of Dependent and Independent Variables and their Symbols**

#### 4.Reduction of Independent variables Analysis

Deducing the dimensional equation for a phenomenon reduces the number of independent variables in the experiments. The exact mathematical form of this dimensional equation is the targeted model. This has been achieved by applying Buckingham's  $\pi$  theorem (Hibert, 1961). When this theorem has been apply to a system involving  $n$  independent variables, ( $n$  minus number of primary dimensions viz. L, M, T, Q) i.e.( $n-4$ ) numbers of  $\pi$  terms are formed. When  $n$  is large, even by applying this theorem number of  $\pi$  terms will not be reduced significantly than number of all independent variables. Thus much reduction in number of variables is not achieved. It is evident that, if we take the product of the terms it will also be dimensionless number and hence a  $\pi$  term. This property has been used to achieve further reduction of the number of variables. Dimensional analysis has been used to reduce the variables. These Independent variables have been reduced into a group of  $\pi$  terms is the second step of this process. The Equation 1 shows the dimension less pie terms for the phenomenon.

#### 6. MODEL FORMULATION BY IDENTIFYING THE CONSTANT AND VARIOUS INDICES OF PI TERMS:

The multiple regression analysis helps to identify the indices of the different  $\pi$  terms in the model aimed at, by considering three independent  $\pi$  terms and one dependent  $\pi$  term. Let model aimed at be of the form,

$$(AI\%) = k \times \left[ \frac{H \times MOE}{\rho^2 \times v^4} \right]^a \left[ \frac{PSD \times Y \times FFTX^2 \times a}{\rho \times v^4} \right]^b \left[ \frac{TSK \times TSY \times PSDX}{FFTY} \right]^c \quad (1)$$

$$(Y) = K * ((\pi_1)^a * (\pi_2)^b * (\pi_3)^c) \quad (2)$$

The regression equations become as under.

$$\begin{aligned} \sum Y &= n K 1 + a \sum A + b \sum B + c \sum C \\ \sum YA &= K 1 \sum A + a \sum A^2 + b \sum AB + c \sum AC \\ \sum YB &= K 1 \sum B + a \sum AB + b \sum B^2 + c \sum BC \\ \sum YC &= K 1 \sum C + a \sum AC + b \sum BC + c \sum C^2 \end{aligned} \quad (3)$$

In the above equations  $n$  is the number of sets of readings,  $A, B$ , and  $C$  represent the independent  $\pi$  terms  $\pi_1, \pi_2$ , and  $\pi_3$ . While,  $Y$  represents, dependent  $\pi$  term. Next, calculate the values of independent  $\pi$  terms for corresponding dependent  $\pi$  term, which helps to form the equations in matrix form.

$$[y] = [X] x [a] \quad (4)$$

Solving, the above matrix by using MATLAB software, the values of constant and different indices of the proposed model have been found out

$$Y = 89.742894 * (\pi_1)^{-0.0042} * (\pi_2)^{-0.0018} * (\pi_3)^{0.0005}$$

#### 7. INTERPRETATION OF MODEL:

Interpretation of model is being reported in terms of several aspects viz (1) order of influence of various inputs (causes) on outputs (effects) (2) Interpretation of curve fitting constant  $K$  (3) Sensitivity of causes (4) reliability

##### (A) Order of influence of various observed NDT parameters

The above equation is established using the various observed NDT parameters. It indicates that which is related to the signal analysis parameters of aluminium metals has highest influence as 0.0005 on the measurement of aluminium percentage in aluminium samples. The least influence is seen for  $\pi_1$  as -0.0042, which is related to physical properties of aluminium metals. The negative sign indicate the inverse relation. The  $\pi_2$ , which is related to properties and signal analysis parameters has moderate influence as -0.0018 on the measurement of aluminium percentage in the samples.

##### (B) Interpretation of curve fitting constant (K)

The value of curve fitting constant in this model is 89.742894. This collectively represents the combined effect of all, which are influenced on the dependent terms. Further, as it is positive, this indicates that there are good numbers of NDT parameters which have influence on estimating the aluminium percentage in the samples.

##### (C) Sensitivity Analysis

Sensitivity analysis is done to evaluate the sensitiveness of each  $\pi$  term involved in the model. This analyze by introducing percentage change in each independent  $\pi$  term, one can see the change in effect on the dependent  $\pi$  term. Hence, in the present work change of  $\pm 10\%$  is introduced in individual  $\pi$  terms independently (one at a time). Therefore total rate of introduced change is 20%. After introducing the change in independent  $\pi$  term, one may have obtained the change in the dependent  $\pi$  term. The percentage of change in dependent  $\pi$  term by introducing 20% change in each independent  $\pi$  terms is shown graphically in Figure (1). The sensitivity is evaluated and discussed below.

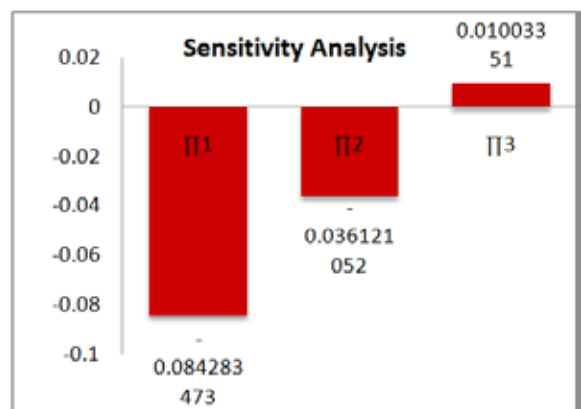


Figure (1): Sensitivity Chart of independent  $\pi$  terms

##### (D) RELIABILITY OF MODELS:

In general reliability is a term associated with the chance of failure. Hence reliability also finds value, which is used to show the performance of the model. The reliability of model is evaluated as follows.

For available mathematical model, the known value of independent variables has substituted in the mathematical model. So that, one will obtain, the required values of dependent variable, which is known as calculated value of dependent variable. Now, one can find error in the calculated value of dependent variable and observed value of dependent variable. For this, it is necessary to subtract calculated value from the observed value of dependent variable. Once the error is calculated, then one can calculate the reliability of model by calculating the mean error.

This can be done by using following formula,

Reliability= 1- Mean error

Where, Mean error=  $\sum XIFI / \sum FI$

Where,  $\sum XIFI$ = Summation of the product for percentage of error and frequency of error occurrence and  $\sum FI$ = Summation of frequency of error occurrence.

Hence for the present model reliability is obtained as 99.40351% This model may help to identify the aluminium percentage present in the aluminium sample.

#### 8. CONCLUSION.

This mathematical model provides and helps to make the NDT monitoring system for the analysis of characterization of alu-

minium metals to predict the percentage of aluminium, in the aluminium metals. This may help to identify the types of aluminium and its applications

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