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Research Paper

A Note on the Structural Estimation of Auctions

Economics

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ABSTRACT

In this paper, we outline the basic technical knowledge required to carry out structural estimation of auctions, which has been the focus of a lot recent empirical research in auctions. This is because identifying the data-generating process of valuations of potential buyers is a pre-requisite for implementing the optimal (revenue-maximizing) selling mechanism. The optimal auction format depends on bidder valuations which are random draws from distributions that are unknown to the mechanism designer / seller. However, since the equilibrium bid strategy of a potential buyer is usually an increasing function of his valuation, assuming that potential buyers bid according to Bayesian-Nash equilibrium bid strategies, it may be possible to estimate the latent distribution of bidder valuations using bid data from a cross-section of auctions. The estimated distribution(s) can then be used to determine the optimal selling mechanism.

1. Introduction

The use of auctions in the conduct of human affairs has ancient roots. The earliest evidence of an auction as recorded by Greek historian Herodotus dates back to fifth century B.C. in Babylonia. Around second century A.D., during the decline of the Roman Empire, the auction of plundered booty was common.

While historically, auctions were used to sell art and antique 'one-of-a-kind' objects, in today's world, auctions are the preferred mode of selling a wide range of commodities. Plantation products such as tea, tobacco, spices, fish, grains, fresh flowers being sold through auctions is now common practice. The Central Banks of several countries sell long-term securities and bonds through weekly auctions to finance the borrowing needs of the government. The most investigated and debated has perhaps been the use of auctions to transfer publicly held assets to private entities. The rights to use natural resources from public property such as coal and mineral fields, off-shore oil-fields, etc. were the first instances of such transfers. Disinvestment of public-sector enterprises such as heavy-equipment manufacturing, road transport, civil aviation, etc. was the next to follow. The most recent examples include auctions of rights to use the electromagnetic spectrum for communication. In the private sphere, the volume and value of internet-based auction activity has burgeoned.

The increasingly large volume of transactions arranged using auctions has made it crucial that auction theory be well developed and the relationship between auction theory and traditional competitive theory be thoroughly explored.

2. Why Structural Estimation?

Given the monopoly power of the seller in an auction, there has been considerable theoretical research on devising selling mechanisms that enable the seller to extract maximum revenue for himself (mechanism design literature). The optimal mechanism however depends upon random variables whose distributions are typically unknown to the analyst. So for instance, to design the optimal auction format for a given market, the auctioneer needs to know what the data-generating process of bidders' valuations is like. The expected revenue of a seller from an auction format is a function of the valuations of the bidders. Even for the symmetric independent private values (IPV) paradigm, where all the auction formats are revenue-equivalent, the auctioneer can seek to maximize the revenue by choosing an appropriate reserve price. But the determination of the optimal (revenue-maximizing) reserve price also requires knowledge of the distribution of bidders' valuations.

This recognition of the importance of identifying the probability law governing the valuations of potential buyers has led to an active research in the structural estimation of auction data. The structural estimation approach is essentially an econometric identification strategy and is based on the twin hypotheses of optimizing behaviour and market equilibrium to identify the distribution of valuations. That is, this approach relies upon the hypothesis that observed bids are the equilibrium bids of the auction model under consideration. The equilibrium concept typically used is Bayesian-Nash and potential bidders are assumed to maximize the expected utility from winning the auction. The theory is assumed to hold and the equilibrium behavioural relationships are imposed upon the data to determine the underlying probability law. Once the distribution of valuations of bidders in a market has been estimated given the informational environment, the alternative auction formats can be compared for their revenue implications. Also, the optimal reserve price can be determined for a given format.

3. Literature

Though the first academic work on auctions dates back to two seminal papers by Friedman (1956) for the case of single strategic bidder and Vickrey (1961) for the equilibrium game-theoretic approach, but it was only after the clarification of the Bayesian-Nash equilibrium concept by Harsanyi (1967) that the theory of auctions was massively developed. While Vickrey had restricted his analysis to the IPV paradigm, his game-theoretic approach to auctions was developed for symmetric common values (CV) by Rothkopf (1969) and Wilson (1969,1977) and for asymmetric CV by Wilson (1967,1969). A general paradigm for auction models was developed by Milgrom and Weber (1982) under the name of the affiliated values (AV) paradigm, which includes the IPV and CV as polar cases.

Structural estimation of auctions emerged as a field of research almost two decades back with Hendricks and Porter (1988) and Paarsch (1989). The initial research was restricted to parametric estimation approaches requiring that joint distributions belong to particular families of distributions. Paarsch (1989) used generalized method of moments (GMM) to estimate parametric symmetric IPV, extended it to symmetric CV in Paarsch (1992). Donald and Paarsch (1995) used a parametric maximum likelihood estimation (MLE) approach to estimate symmetric IPV models. Laffont et al. (1995) proposed a simulated nonlinear least-squares estimation method for symmetric IPV models.

Though statistical literature on nonparametric identifiability as well as nonparametric estimation techniques began emerging almost three decades back, the applications of nonparametric econometrics to auction data are of a relatively recent vintage.

The basic idea of nonparametric estimation methods is to estimate the bid distribution in a first step. In a second step, the latent distribution of valuations is recovered from the estimated bid distribution by inverting the bid function assuming that it is

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part of the Bayesian-Nash equilibrium of an auction.

Guerre et al. (1995) used such a two-step nonparametric procedure for estimating the distribution of private valuations in symmetric IPV models for first-price auctions. This was later applied to estimation of a symmetric affiliated private values (APV) model by Li and Vuong (1996) and to an asymmetric IPV model by Laffont et al. (1996).

Brendstrup and Paarsch (2006) employ a semi-nonparametric (SNP) MLE procedure, developed by Gallant and Nychka (1987) to estimate an asymmetric IPV model with observed object heterogeneity across auctions.

The Method of Structural Estimation

In order to carry out structural estimation, we start by formulating an auction as a non-cooperative game of incomplete information and specifying the informational environment. This is followed by computing for a bidder, the Bayesian-Nash equilibrium strategy specific to the auction format and the informational environment. We then check for identifiability of the valuation structure of bidders given the limited data from auctions. The game-theoretic structure which underlies the observed bids being the optimal bids is then invoked to invert the bids and recover the values. This essentially is the *structural estimation* of bidders' values. We now describe the steps in this method in some detail.

IV.1. Auctions as Bayesian Games

In an *auction*, the object is sold to the highest bidder. Auctions can take different forms. Bids may be called out sequentially (going from lowest to highest or vice-versa) or may be submitted in sealed envelopes. A game-theoretic analysis helps in understanding the consequences of various auction designs; it suggests, for example, the design likely to be most effective at allocating resources, and the one likely to raise the most revenue. But often in an auction, the bidders may not be perfectly informed of the other bidders' valuation for the object on sale. We thus model an auction as a Bayesian game and then define its Nash equilibrium.

The following four **auction formats** are the most common:

First-price Sealed-Bid Auction

Bidders submit bids simultaneously, with the object going to the highest bidder at a price equal to his bid.

Oral Descending-price (Dutch) Auction

At oral descending-price auctions, the price is initially set very high and then allowed to drop continuously. The auction ends at the point when a bidder offers to buy the object at the current price.

Second-price Sealed-Bid Auction

Bidders submit bids simultaneously, the object is sold to the highest bidder at a price equal to his nearest opponent's bid.

Oral Ascending-price (English) Auction

The most frequently used format is the oral ascending-price auction. The standard model of an ascending auction is the so-called *clock-auction* or *button-auction* model of Milgrom and Weber (1982), where the price rises continuously and exogenously. Bidders indicate their willingness to continue bidding by for example, raising their hands or pressing a button as the price rises. As the auction proceeds, bidders exit observably and irreversibly until only one bidder remains. This final bidder obtains the good at the price at which his last opponent exited; i.e., the auction ends at a price equal to the second-highest exit price (bid).

IV.2. Specification of the Informational Environment

The joint distribution of bidders' valuations and signals is characterized using the following three categorizations:

Based on valuation structures - Private versus Common Values

Bidders have private values if no bidder has private information relevant to another's expected utility. A bidder's valuation for the object depends only on his own signal. Knowledge of other bidders' signals would not alter his valuation. In a common values model, each bidder would update his beliefs about his valuation if he learnt an opponent's signal in addition to his own signal. Indeed this structure of valuations leads to the *winner's curse.* Winning a common values auction reveals (in equilibrium) to the winner that his signal was more optimistic than those of his opponents. Rational bidders anticipate this information when forming expectations of the utility they would receive by winning.

Based on distribution of signals - Independence versus Affiliation

If the bidders' signals are correlated then their values are said to be affiliated, while if the signals are not correlated, it is an independent values paradigm.

Based on Bidder Symmetry - Symmetric versus Asymmetric Bidders

The bidders at an auction are symmetric if their values can be modeled as being draws from the same distribution, if not, then they are asymmetric.

IV.3. Equiibrium Bidding Behaviour

Second-price and Ascending Auctions

Equilibrium strategies in second-price and ascending auctions are similar. We discuss here the bidding behaviour in an ascending auction, in which a *bid* is a planned price at which to exit.

In an ascending auction, for bidder i with value v_i ,the payoff from bid b_i is $v_i - \max(b_j)$ if $b_i > \max(b_j)$ and is zero if $b_i < \max(b_j)$. This is because if the bidder wins, he pays a price equal to that at which the last of the bidders, excluding him, dropped out; it is equal to the bid chosen by this last bidder who drops out.

Given this payoff function, it is a weakly dominant strategy for each bidder i to bid according to $b_i = v_i$ (Krishna, 2002). If $b_i < v_i$, i.e., if the bidder chooses to drop out at a price lower than his value, there is positive probability that the winner will be some bidder j who plans to drop out at a bid b_j such that $b_i < b_j < v_i$. Then, bidder i loses, whereas if he had planned to drop out at v_i , he would have won and got a payoff equal to $v_i - b_j$ .On the other hand, bidding $b_i > v_i$ can cause bidder i to win and pay a price higher than his value v_i , with positive probability. As a result, the win price or sale price in an ascending auction is essentially the second-highest bid, which coincides with the second-highest valuation in an independent private values (IPV) setting.

First-price and Descending Auctions

In a first-price auction, for bidder i who submits a sealed bid b_i , the payoff is $v_i - b_i$ if $b_i > \max_{j\neq i} b_j$ and zero if $b_i < \max_{j\neq i} b_j$. If there is more than one bidder with the highest bid, the object goes to each bidder with equal probability.

No bidder would bid an amount equal to his value since this would only guarantee a payoff of zero. Fixing the bidding behaviour of others at any bid (that will neither win for sure nor lose for sure) the bidder faces a tradeoff. An increase in the bid will increase the probability of winning but also reduce the gains from winning. An equilibrium strategy is one where these effects balance out.

Although a descending auction is seemingly different from a first-price auction, the two formats are strategically equivalent. At a descending auction, depending on his valuation for the object, a bidder must decide at what point to stop the auction by signalling his willingness to pay the current price. This situation is strategically identical to that faced by a bidder at a first-price auction, who must decide how high to bid for the object.

IV.4. Identification

The identification problem is concerned with the limits of what can be learned about a structure S from the observable data that it generates combined with a given set of prior information. The data generated by S will reveal, at most, the probability distribution of the observable variables and therefore the nature of S must be inferred from this probability distribution combined with prior knowledge. So if there exists a structure $S^* \neq S$ that is identical to S in terms of what is known *a priori* and that implies the same probability distribution for the observable variables, information will be exhausted without distinguishing S from S^*. In such cases S is termed not identifiable and clearly cannot be estimated.

The parametric identification approach works on the premise that the form of the distribution function from which the sample could have been drawn is known.

When the class of distribution functions under consideration is not confined to be of a particular functional form, we have the problem of identifiability in a nonparametric framework.

When the random variables are independent and identical (i.i.d.) draws from a distribution F(.) (i.e., drawn from the same parent distribution), (nonparametric) identification of F(.) is an immediate implication of known properties of order statistics (Arnold et al. (1992)).

In the symmetric IPV model, the distribution of bidder valuations F(.) (identical across bidders) is identified from the win price (Athey and Haile (2002), Theorem 1).

Essentially, knowledge of the distribution of the win price, in an ascending symmetric IPV auction, is the same as that of the second-highest order-statistic. This distribution lets us uniquely infer the distribution of valuations. Note that this is a special case of identification from any i^{th} order-statistic.

When the random variables are independent draws from different parent distributions, the identification argument is more complex. Such problems typically occur in reliability and survival analysis.

Consider an ascending auction with n bidders. In the asymmetric IPV model, assume that there are J distinct types of bidders. The valuation distribution of bidder i of type j is given by F_j . Assume that each F_j is continuous and that supp[F_j] is the same for all i. Then each F_j is identified if the win price and identity of the winner are observed (Athey and Haile (2002), Theorem 2). Proof follows from reliability analysis (Meilijson (1981), Theorem 1).

IV.5. Estimation

With symmetric bidders, nonparametric estimation of bidder valuations at ascending auctions for the IPV case is based on the nonparametric identification result for the same.

Solving the system of equations corresponding to nonparametric identification of asymmetric bidders' valuation distributions (for IPV) at ascending auctions is tedious since it involves functional in Banach space. A likelihood approach provides computationally simpler estimation strategy. The idea is to work off the joint distribution of the winning bid and the winner identity. Brendstrup and Paarsch (2006) have proposed a semi-nonparametric (SNP) estimation approach (developed by Gallant and Nychka (1987)) based on this likelihood. In the SNP approach, the unknown distribution function is replaced by a nonparametric approximation based on orthogonal polynomials.

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