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Research Paper

A Fuzzy Approach to Find the Shortest Path Using Pascal's Triangle Graded Mean

MATHEMATICS

KEYWORDS : Fuzzy Distance, Graded Mean, Pascal's Triangle, Fuzzy Number.

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ABSTRACT

This paper deals with an investigation of finding the Fuzzy Shortest Path in the Network using Pascal's Triangle Graded Mean Integration Representation. Here the trapezoidal fuzzy numbers are taken as edge weights and the optimum shortest path was found by comparing the fuzzy distance among all adjacent nodes using fuzzy order relation. Finally we arrive at the smallest fuzzy distance which gives the fuzzy shortest path for the given network by giving suitable numerical example.

1. INTRODUCTION

One of the most common fundamental problems in network theory is finding the shortest paths in a network. Shortest-Path Problems play an important role in routing messages efficiently in network. This shortest path problem determines the distance of the shortest route between a source node and a destination node in a transportation network. The numbers associated with the edges of networks may represent characteristics other than lengths, and we may want the optimum path, where optimum path can be found by using different criteria. The fuzzy shortest path problem was first analyzed using Floyd's algorithm by Dubois and Prade [3]. Here we consider as numerical example, the fuzzy shortest path problem as a directed graph where the edge weights of the network are trapezoidal fuzzy numbers as uncertain; which means that it is either imprecise or unknown. Shinkoh Okada and Mitsuo Gen [7] introduced fuzzy shortest problem with the help of Dijkstra's method in the year 1994.

In 1998, Lotfi.A.Zadeh [12] introduced the concept of fuzzy set theory in the year 1965. Chen and Hsieh [9,10] proposed fuzzy graded mean integration representation. In this proposed method the graded mean integration of Pascal's Triangle was introduced for computing fuzzy shortest path with the help of fuzzy distance and trapezoidal fuzzy numbers. This paper consists of five sections: First section is the introduction part, Second section defines the methodologies used in this paper, Third section explains the algorithm or the working rule of this method, identifying shortest path by giving suitable numerical example in the Fourth section and finally the conclusion based on our study is given in the fifth section.

2. METHODOLOGIES

In this paper, we use the following methodologies to find the optimum shortest path.

2.1 Representation of Generalized (Trapezoidal) Fuzzy Number

In general, a generalized fuzzy number A is described at any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions:

- μ_A is a continuous mapping from R to [0,1],
- $\mu_A(x)=0, -\infty \prec x \leq c$,
- $\mu_A(x) = L(x)$ is strictly increasing on [c,a]
- $\mu_A(x) = w, a \leq x \leq b$,
- $\mu_A(x) = R(x)$ is strictly decreasing on [b,d],
- $\mu_A(x) = 0, d \leq x \prec \infty$ Where $0 \prec w \leq 1$ and a, b, c and d are real numbers.

We denote this type of generalized fuzzy number as $A = (c,a,b,d;w)_R$. When w=1, we denote this type of generalized fuzzy number as $A=(c,a,b,d)_R$. When L(x), R(x) are straight lines, then A is Trapezoidal fuzzy number. We denote it as (c, a, b, d).

2.2 Graded Mean Integration Representation

In 1998, Chen and Hsieh [9,10] proposed graded mean integration representation for representing generalized fuzzy number. Suppose L^{-1}, R^{-1} are inverse functions of L and R respectively, and the graded mean h-level value of generalized fuzzy number $A=(c,a,b,d$ is $h[L^{-1}(h)+R^{-1}(h)]/2$. Then the graded mean integration representation of generalized fuzzy number based on the integral value of graded mean h-level is

$$P(A) = \frac{\int_0^w h\left(\frac{L^{-1}(h)+R^{-1}(h)}{2}\right)dh}{\int_0^w h\,dh},$$

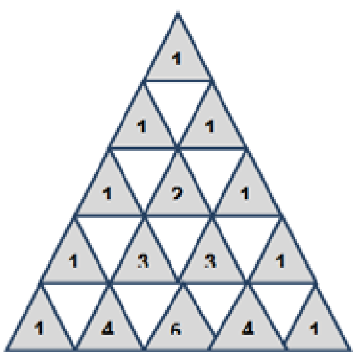
$$P(A) = \frac{c+2a+2b+d}{6}$$

2.3 Pascal's Triangle Graded Mean Approach

The Graded Mean Integration Representation for generalized fuzzy number by Chen and Hsieh [9,10].But the present approach is very simple was of analyzing fuzzy variables to get the optimum shortest path. This procedure is taken from the following Pascal's triangle. We take the coefficients of fuzzy variables as Pascal's triangle numbers.

Then we just add and divide by the total of Pascal's number and we call it as Pascal's Triangle Graded Mean Approach.

Figure: 1 PASCAL'S TRIANGLE



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The following are the Pascal's triangular approach:

Let $A=(a_1,a_2,a_3,a_4)$ and $B=(b_1,b_2,b_3,b_4)$ are two trapezoidal fuzzy numbers then we can take the coefficient of fuzzy numbers from Pascal's triangles and apply the approach we get the following formula:

$$P(A)=\frac{a_1+3a_2+3a_3+a_4}{8};$$
$$P(B)=\frac{b_1+3b_2+3b_3+b_4}{8}$$

The coefficients of a_1,a_2,a_3,a_4 and b_1,b_2,b_3,b_4 are 1, 3, 3, 1.This approach can be extended for n-dimensional Pascal's Triangular fuzzy order also.

2.4 Order Relation

For fuzzy order relation [2], let us consider two trapezoidal fuzzy numbers

$$A=(a_1,a_2,a_3,a_4)\; B=(b_1,b_2,b_3,b_4)$$

Then A ≤ B iff the following inequalities holds: (i) $a_1 \le b_1$ (ii) $a_2 \le b_2$ (iii) $a_3 \le b_3$ (iv) $a_4 \le b_4$.

2.5 Fuzzy Distance

Let $A=(a_1,a_2,a_3,a_4)$ and $B=(b_1,b_2,b_3,b_4)$ are two trapezoidal fuzzy numbers and their Graded Mean Integration Representation are P (A), P (B) respectively. Assume

$$S_i=(a_i-P(A)+b_i-P(B))/2,\ i=1,2,3,4;$$
$$C_i=|P(A)-P(B)|+S_i,\ i=1,2,3,4;$$

Then the fuzzy distance of A, B is $c=(c_1,c_2,c_3,c_4)$.

3. ALGORITHM

Step: 1	START with source/initial node S=(0,0,0,0).Then compute Fuzzy Distance
Step: 2	Compute Pascal's Triangle Graded Mean Value for each adjacent vertex from the current vertex P(A)= $\frac{a+3b+3c+d}{8}$, where A=(a, b, c ,d) is the edge weight of the node(i , j)
Step: 3	Compute Fuzzy Distance between two edges using $S_i=(a_i-P(A)+b_i-P(B))/2,\ i=1,2,$ $C_i=\|P(A)-P(B)\|+S_i,\ i=1,2,3,4;$ And for each adjacent node the fuzzy distance of A, B is $c=(c_1,c_2,c_3,c_4)$ for each adjacent node.
Step: 4	Compare fuzzy distance among all adjacent nodes using order relation $a_i \le a_j, b_i \le b_j, c_i \le c_j, d_i \le d_j$
Step: 5	Repeat until the smallest fuzzy distance is obtained and the destination node is reached.
Step: 6	STOP. Finally such smallest fuzzy distance gives the fuzzy shortest path for the given network.

4. Numerical Example

A Numerical Example is given as a Directed Graph which contains 11 nodes and 16 edges to explain this algorithm. To start with Let us assume the distance S= (0, 0, 0, 0).Here from Node 1 the adjacent nodes are 2,3,4,5.

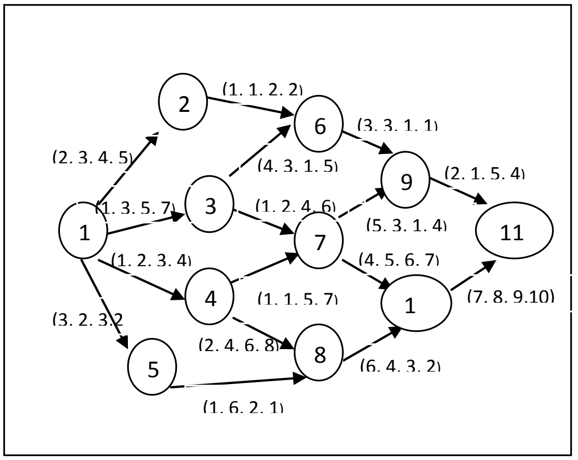


Figure: 2 NETWORK CONNECTION

4.1 TABLE OF CALCULATIONS

Node (1,2)
P(A)=P(1,1) = (0,0,0,0) = 0/8 =0
P(B)=P(1,2) = (2,3,4,5) =(2+9+12+5)/8=9.5
S_1=(0-0+2-9.5)/2= - 3.75
S_2=(0-0+3-9.5)/2= - 3.25
S_3=(0-0+4-9.5)/2= - 2.75
S_4=(0-0+5-9.5)/2= - 2.25
C_1= |0-9.5 |+(-3.75) = 5.75
C_2= |0-9.5 |+(-3.25) = 6.25
C_3= |0-9.5 |+(-2.75) = 6.75
C_4= |0-9.5 |+(-2.25) = 7.25
C=(5.75,6.25,6.75,7.25)

Node (1,3)
P(A)=P(1,1) = (0,0,0,0) = 0/8 =0
P(B)=P(1,3) = (1,3,5,7) =(1+9+15+7)/8=4
S_1=(0-0+1-4)/2= - 1.5
S_2=(0-0+3-4)/2= -0.5
S_3=(0-0+5-4)/2= 0.5
S_4=(0-0+7-4)/2= 1.5
C_1= |0-4|+(- 1.5) = 2.5

C_2= |0-4|+(- 0.5) = 3.5
C_3= |0-4|+(0.5) = 4.5
C_4= |0-4|+(1.5) = 5.5
C=(2.5,3.5,4.5,5.5)

Node (1,4)
P(A)=P(1,1) = (0,0,0,0) = 0/8 =0
P(B)=P(1,4) = (1,2,3,4) =(1+6+9+4)/8=2.5
S_1=(0-0+1-2.5)/2= - 0.75
S_2=(0-0+2-2.5)/2= - 0.25
S_3=(0-0+3-2.5)/2= 0.25
S_4=(0-0+4-2.5)/2= 0.75
C_1= |0–2.5|+(- 0.75) = 1.75
C_2= |0-2.5|+(- 0.25) = 2.25
C_3= |0-2.5|+(0.25) = 2.75
C_4= |0-2.5|+(0.75) = 3.25
C=(1.75,2.25,2.75,3.25)

Node (1,5)
P(A)=P(1,1) = (0,0,0,0) = 0/8 =0
P(B)=P(1,5) = (3,2,3,2) = (3+6+9+2)/8 =2.5
S_1=(0-0+3-2.5)/2= 0.25
S_2=(0-0+2-2.5)/2= - 0.25
S_3=(0-0+3-2.5)/2= 0.25
S_4=(0-0+2-2.5)/2= - 0.25
C_1= |0–2.5|+(0.25) = 2.75
C_2= |0–2.5|+(- 0.25) = 2.25
C_3= |0–2.5|+(0.25) = 2.75
C_4= |0–2.5|+(- 0.25) = 2.25
C=(2.75,2.25,2.75,2.25)

While Comparing fuzzy distances among adjacent edges (1, 2),(1,3),(1,4) and (1,5). We get (1, 4) is the smallest among them according to order relation and from node 4, node 7 and 8 are the two adjacent nodes.

Node (4,7)
P(A)=P(1,4) = (1,2,3,4) =(1+6+9+4)/8=2.5
P(B)=P(4,7) = (1,1,5,7) =(1+3+15+7)/8=3.25
S_1=(1-2.5+1-3.25)/2= - 3.75
S_2=(2-2.5+1-3.25)/2= -2.75
S_3=(3-2.5+5-3.25)/2= 2.25
S_4=(4-2.5+7-3.25)/2= 5.25
C_1= |2.5– 3.25|+(- 3.75) = - 3
C_2= |2.5 – 3.25|+(-2.75) = - 2
C_3= |2.5 – 3.25|+ (2.25) = - 1.5
C_4= |2.5 – 3.25|+ (5.25) = - 4.5
C = (- 3, - 2 , -1.5 , - 4.5)

Node (4,8)
P(A)=P(1,4) = (1,2,3,4) =(1+6+9+4)/8 =2.5
P(B)=P(1,4) = (2,4,6,8) =(2+12+18+8)/8 =5
S_1=(1-2.5+2-5)/2= -2.25
S_2=(2-2.5+4-5)/2= - 0.75
S_3=(3-2.5+6-5)/2= 0.75
S_4=(4-2.5+8-5)/2= 2.25
C_1= | 2.5-5|+(-2.25) =0.25
C_2= |2.5-5|+(- 0.75) = 1.75
C_3= |2.5-5|+ (0.75) = 3.25
C_4= |2.5-5|+ (2.25) = 4.75
C = (0.25,1.75,3.25,4.75)

While Comparing fuzzy distances among adjacent edges (4, 7) and (4, 8) using order relation, we get (4, 7) is the smallest among them and from node 4 and node 7 the adjacent nodes are nodes 9 and 10.

Node (7,9)
P(A)=P(4,7) = (1,1,5,7) =(1+3+15+7)/8 =3.25
P(B)=P(7,9) = (5,3,1,4) =(5+9+3+4)/8 =2.625
S_2=(1-3.25+3-2.625)/2= - 0.9375
S_3=(5-3.25+1-2.625)/2= 0.0625

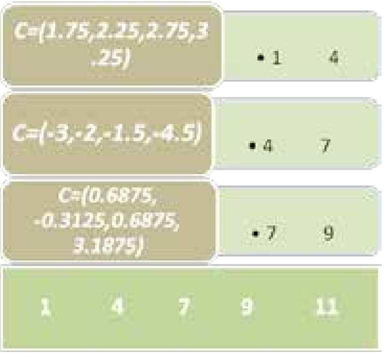
S_4=(7-3.25+4-2.625)/2= 2.5625
C_1= |3.25–2.625|+(0.0625) = 0.6875
C_2= |3.25-2.625|+(- 0.9375) = 0.3125
C_3= |3.25-2.625|+(0.0625) = 0.6875
C_4= |3.25-2.625|+(2.5625) = 3.1875
C=(0.6875,-.3125,0.6875,3.1875)

Therefore by comparing the fuzzy distance (7, 9) and (7, 10) we get (7, 9) is the smallest among them, according to order relation. From Node 9 the only adjacent node is 11. Then choose the edge (9, 11) and 11 is the destination node. Now the process is terminated and we get the fuzzy shortest path as 1-4-7-9-11. This method is simple and it is an alternative to the fuzzy graded mean approach given by Chen [9].

5. CONCLUSION

Node (7,10)
P(A)=P(4,7) = (1,1,5,7) =(1+3+15+7)/8 =3.25
P(B)=P(7,9) = (4,5,6,7) =(4+15+18+7)/8 =5.5
S_1=(1-3.25+4-5.5)/2= -1.875
S_2=(1-3.25+5-5.5)/2= - 1.375
S_3=(5-3.25+6-5.5)/2= 1.125
S_4=(7-3.25+7-5.5)/2= 2.625
C_1= |3.25–5.5|+(-1.875) = 0.375
C_2= |3.25-5.5|+(- 1. 375) = 0.875
C_3= |3.25-5.5|+(1.125) = 3.375
C_4= |3.25-5.5|+(2.5625) = 4.875
C=(0.375,0.875,3.375,4.875)

This paper gives the solution for finding the shortest path using Pascal's Triangle Graded Mean approach along with the trapezoidal fuzzy number. This method is an alternative for fuzzy graded mean approach and the calculations of this method are simpler and it can be extended to any number of nodes.



Concluded Figure: 3

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