Research Paper

Calculus of firing distance of a bullet by measuring the depth of penetration during autopsy.

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ABSTRACT
In this paper we would put a mathematic relation between the displacement(x_2) of penetration of the lunched bullet inside the victim and the displacement(x_1) in air before impact. The cardinal achievement of the article is that we use the physical definition of the inertial own mass of the matter as defined in the paper published by the author in 2014 under the title; physics of the giant atom[5].
From the definition we can state that if a projectile stroke a defined object with a force more than the binding force between its physical units (like macromolecules of the soft tissue) then the localized stroke zone would lose its physical relative inertia of the mother object and would be related to the inertial mass of the striking projectile in the form of conservation of kinetic energy as;
$E_p - E_b = (m + \delta m) v^2$ Where v is the velocity after strike and the subscripts p and b means projectile and binding respectively and where m and δm means mass of the projectile and the incremental mass of the stroked zone respectively.
Since the tissue has some degree of elasticity and consequently each type of tissue has defined tensile strength and defined ultimate elongation, so the dot product of these physical quantities gives the defined toughness energy density of the tissue. The cardinal idea of our work is that: since toughness energy brings each incremental section area (of an elastic rode) in ultimate elongation meaning that offering each section enough energy to break, simultaneously the bullet throughout its pass brings each cross section of the tissue at break
This means that the energy lost to do cavitations is equivalent to the toughness energy density time the volume of the permanent cavitations. This means that the bullet throughout its pass succeeds to bring the successive section areas of the tissue at break but without elongation. Since we have two unknowns; the stoppage energy of the bullet inside the victim and the displacement before striking the body, so we would divide our work into two indistinctive sections: where in each one we would put an equation to relate each unknown with the other so the two equations would solve the problem and finally we can define the displacement before impact.

Introduction

The two major forces decelerating velocity of the bullet are gravity and drag.

Gravity is a vector affecting the trajectory of the motion rather than affecting the scalar speed especially on the short range and if the bullet passes horizontally (perpendicular with gravity vector)

On these two conditions drag is the considerable affecting force. Our work studies the effect of drag on the bullet kinetic energy.

Discussion

1-Defining the impact velocity

The bullet throughout its pass continuously loses kinetic energy to do: break the binding bonds of molecules of air (or the macromolecules of tissue) and to carry and push these broken molecules (or tissue). Since the volume of the bullet have to equal the poured broken volume of the stroked medium so the pushed mass is

Proportional with; the density of the medium σ, the pushed area a and the displaced length[3]

$$m = c \, \sigma \, a \, \Delta x$$

Where c is a coefficient measuring the efficiency of the bullet to do the action.

$$m \,/\delta t = c \, \sigma \, a \, v$$

$$m \, \delta v/ \, \delta t = - \tfrac{1}{2} c \, \sigma \, a \, v^2 = F_d$$

$$m \, v \, \delta v/\delta x = - \tfrac{1}{2} c \, \sigma \, a \, v^2$$

$\int \delta v/v = - k \int \delta x$ where the integration on the left side from the initial velocity v_o to the impact velocity $v(x1)$ and the integration on the right side from x = o to x = x_o

$$\ln v - \ln v_o = - k \, x$$

$$v \,/v_o = e^{- kx} \quad \text{(in air)}$$

 (1)

Let us call k_1 and x_1 for fighting in air and let k_2 and x_2 for penetration in the victim tissue.

Let $v_{(x1)}$ and $v_{(x2)}$ refer to velocity of impact and the final velocity (= V_f = final velocity inside the tissue) respectively.

$$V_f \div \{v(x1) - t\} = e^{- (k2 \, x2)} \quad \text{(inside the victim)}$$
(2)

From (1) and (2)

$$V_f \div (v_o - t) = e^{-(k_1 x_1 + k_2 x_2)}$$
(3)

$$\ln\{(v_o - t) \div v_f\} = k_1 x_1 + k_2 x_2$$

Where t is a quantity of energy lost due to a cause other than the pushing effect, and we will study it under studding toughness of the tissue.

2- Defining k

From above then,

$$k = \tfrac{1}{2} c \, \sigma \, a / \, m$$

$c = c_g \, m \div d^2 B.$ **where;**

cg is a dimensionless predetermined factor of G1 model used by the manufacture(13), (-for a rounded head bullet like 9mm ×19mm and weighting 124 grain = 8 gram- it equals o.517)

d is the diameter of the projectile (caliper of the bullet), a = the cross section area = $\pi (d/2)^2$ and B is the dimensionless ballistic coefficient.

From above; k = o.2σ/B.

For the same bullet mentioned above, the empirical studies[8] showed that B = o.13 and consequently;

From the above relation, k = 1.54σ   (4)

To be adequate with estimated ballistic coefficient we have to use the measurement units in pound and inch. Taking the mean value of the human body density = $1/29.6$ Ib per inch^3. So from above;

$K_1 = 1/1.55\times10^4$ (where density of air –in room temperature and pressure- equals 4.13×10^{-5} Ib/inch^3)

$K_2 = 1/19.2$

This means that we can determine- by equation (3) - the distance of firing if we determined the final velocity inside the tissue. This is the next discussion.

3- Bullet energy at stoppage inside the victim body;

The bullet throughout its pass loses continuously kinetic energy to do evacuation (anti-toughness) and to push these evacuated tissues (conservation of kinetic energy and of momentum)

A- Toughness energy
Suppose we had a rode of a matter has some degree of elasticity. The rode is fixed in an end and pulled from the other one in z- direction where there would be coetaneous elongation proportion with the stress – F / A. The total elongation could be expressed as;

$\Delta z = \Sigma\Delta_i = n\Delta$ where Δ is the elongation in each cross section area and n is the number of cross section areas. If we increased the stress till elongation at break then this ultimate stress (which in this case is called tensile) times the ultimate elongation is the toughness energy density.

Although elongation at break means cut in one cross section area of the rode yet this doesn't mean that the gained energy was localized only in this cut area. Each of the gained potential energy and the elongation Δz are distributed equivalently among the cross section areas as; $\Delta E_i = \Delta E_1 = \Delta E_2$. Where $\Sigma E_i = \Delta E$

The cut occurs at the weakest area defined by the rode anisotropy. We have to notice that at ultimate elongation the gained potential energy is wasted in the cut section area to break the binding bonds in z- direction while the equivalent gained potential energies in all the other cross section areas are converted into kinetic energy where rebound and recoil occurs.

Simultaneously the bullet does the same action but with three different points. When the bullet passes in the tissue (or in the air) it breaks the binding bonds along its pass. So,

No elongation in the matter instead there is displacement of the bullet (elongation in the pass $v\ \delta t$) and this is the first different point. No cut in one area instead there is pass in all cross section areas (penetration) which means break down of the binding bonds in all the cross section areas along the pass of the bullet (z- direction) and this is the second different point.

The third different point although is not significant yet it appears from the following discussion;

Suppose that a projectile was penetrating a muscle in z- direction and each cross section of the stroked muscle has a volume equal x y δz. If the projectile had a constant diameter along its length then the gradient penetration of the projectile would lead to break of the bonds among the macromolecules along z- direction only. The number of the cut areas = n = z ÷ δz where z is the depth of the penetrated tissue and δz is the length of the macro-

molecule.

But the bullet has a variable diameter along its head. If the length of the curve of the bullet head = H and the length (from the base till the tip) of the head = h, then there would be additional cut pieces within xy level = N.

If the number z of broken macromolecules in a tissue area (having the same area of the bullet head base) = z = xy ÷ δx δy where δx δy is the surface area of the macromolecule.

Number of the broken macromolecules in the radius of this area = N

= (x ÷ δx) (h÷ H)

Where; H and h are the length of the curve of bullet head and the length of head (the distance from the base till the tip) respectively.

The total broken macromolecules = z +(x h ÷ δx H)

Since δy is measured in nanometer[13] and y is measured in millimeter so the additional number approaches zero

The total number = z which means that the broken macromolecules in the level of xy is so small that it is not effective in the number broken in z direction.

This discussion means that the bullet to do permanent cavity loses energy equals tensile strength of the defined tissue times its ultimate elongation times the volume of the cavity.

During autopsy we can collect the toughness energies T of the different types of the tissues weighted by the depth L of the permanent of the cavity such that;

A ($T_1 L_1 + T_2 L_2 + T_i L_i$) = A$\Sigma\ T_i L_i$
(5)

Where; A is the width of the cavity.

The empirical studies[6][17] showed that the human muscle has tensile S = 15 ±3 N/c^2 and has ultimate elongation U about 150 percent so, the dot product gives toughness T = 22 j /c^3. It also showed that rabbit intestine and soft tissue has S = 0.03 N/c^2 and U = 50 percent[2][9].

So, toughness energy of muscles and of skin is hundreds times that of the other tissues so if the bullet stroke a bulk muscle like that of the arm before penetrating the body cavity we can consider only the toughness of the muscle Tm and neglect that of the other tissues. In this case we can rewrite the lost kinetic energy to do cavitations as;

$$E = A\ T_m\ L_m$$
Where; L_m is the depth of the bulk penetrated muscle.

The critical speed
It is the least speed required for the bullet to pierce the victim skin.

Suppose we have an elastic rode of human skin with cut section area A equal one cubic centimeter and a length z equal one centimeter. The energy need for this rode to elongate till cut (ultimate elongation) = (F ÷ A) (Δz ÷ z).

If a bullet with a caliper equal –as example- 9mm lunched towards a victim then the stroked skin area = π r^2 = π (9÷2)^2 = 0.63 c^2

The empirical studies showed that tensile of skin[1] = F ÷ A = 25 N per unit area and its ultimate elongation equal 25 percent hence its toughness energy equal the tensile times the ultimate elongation = (F ÷ A) (Δz ÷ z) = 6 j per cubic centimeter.

The energy need to cut this rode (with diameter = 9mm) or to penetrate it without elongation = T

= (F ÷A) (Δz ÷ z) A z = 6.25 ×o.63 = 4j = ½mv².

The empirical studies[7][12] showed that this critical speed (which is termed in the textbooks as the speed of the stoppage power) for the human skin equals 65 meter per second. This empirical magnitude could be concluded as follows;

The reverse of the cut process is to search for the maximum energy need for this defined area (o.63 c²) not to be pierced or by other meaning to vanish Δz.

Then, This maximum energy is equivalent to the least need kinetic energy for the skin to be penetrated = E_c = ½ mv_c^2 = {(F ÷A) (Δz ÷ z) (A z) }÷ Δz = 4×100 / 25 = 16j which is equivalent to the tensile of this defined surface area (o.63c²)..

Putting the mass m = 7.5 gram then v_c = 65 m/s.

We might refer here to that the magnitude of critical energy depends on the stroked section area (caliper of the bullet) but not depend on the depth of the tissue because of z in the above brackets vanish each other so no penetration..

B-The lost energy to push the evacuated tissues ;
Let us call the kinetic energy of the bullet at the inlet of the victim body as E_{x1} and at stoppage inside the body as $E_{(x1,x2)}$

If the target zone – relative to the bullet- has kinetic energy = 0 (at rest) and if it's mass at the level of action of toughness (molecule of air or macromolecule of tissue) = Δm

½ mv_b^2 + 0 = ½ (m + Δm) v_a^2

Where the subscripts b, and a, refers to before and after strike respectively.

This result is realized by conservation of momentum as follows;

i) In x- component (i direction);

mv_b i = mv_a i + Δm v_a i (where the vector is denoted by the bold letter)

0 = mv_a cosθ + Δmv_a cosϕ
(6)

Where, θ is the angle between x- coordinate and the bullet motion vector while ϕ is the angle between the same coordinate and direction of the motion of Δm after strike.

Δm/m = − cosθ/cosϕ

Since Δm/m = Δ →0

0 = − cos (90 + ϕ)/cosϕ = −sinϕ/cosϕ = tanϕ

ϕ = 0

θ = π/2

ii-) y- component (j direction)

mv_b j = mv_a j + Δmv_a j

mv_b sin90 = mv_a sinθ + Δmv_a sinϕ

mv_b = mv_a cosϕ + Δmv_a sinϕ
 (7)

From (6) the x- component is in the form;

0 = − mv_a sinΔ + Δm cosϕ
(8)

Squaring (7) and (8) then adding to each other → $m^2v_b^2$ = v_a^2 (m^2 + Δm^2)

= v_a^2 (m + Δm) (m − Δm)

Multiplying the left side by 1 = (m/m) = {(m − Δm)/m} gives;

½ mv_b^2 = ½ (m + Δm)v_a^2

This form realizes conservation of kinetic energy in each cross section with infinitesimal length and infinitesimal mass along the pass of the bullet. Allover the complete pass the conservation is realized on the form of;

½ mv_b^2 = ½ (m + $\Sigma \Delta m$) v_a^2

Now we will try to estimate the total pushed mass $\Sigma \Delta m$. It is simply the mean density of the human body tissue times the cross section area of the bullet times the depth of penetration. But we have to introduce the effect of Yaw. For simplification and as approximation let us study the effect of Yaw on the round head, 9mm×1.9mm, bullet (which is characterized by the comparatively small effect of Yaw[7]). The empirical studies[8] showed that Yaw appears effectively in the middle third of the penetration depth, so if we supposed –as example- that the depth equals 51 c then we can consider that Yaw appears effectively at the second 17 c. The deviation angle of Yaw appears at entrance as 1degree where it increases gradually and dramatically especially at the second third. In the first third the angle begins at 1 degree and increases till it reach to about 15 degree at the end of the first third. In the second third it increases gradually till it reaches at the end of this third to 180 degree. The last third it return the phase of the first one. Now we have to define the pushing face of the bullet. It is simply equals the base of the bullet =A = {π (0.9 ÷ 2)²} = o.63 c².But Yaw affects greatly the pushing surface.

In the middle third; if sin90 and sin 15 are the maximum and minimum sin respectively then;

A_1 = π R l {(sin90 + sin15) ÷ 2} where R = r ÷ 1.05 = o.45 ÷ 1.05 = 4.2c, such that the factor 1.1 is to compensate for the gradient decrease in the radii of the bullet head.

Putting l = 1.9, then;
A_1 = 2.5× o.625 = 1.56 c². Now we have two pushing faces as;

A_t = A_1 + A = 1.56 + o.63 = 2.2 c². Then; The pushed volume of the tissue in the middle third of the pass = v_1 = 2.2 × 17 = 38 c³. In the other two thirds;

A_1 = 2.5 sin7.5 = o.4 c²

A_t = o.4 + o.63 = 1 c². Then the pushed volume = v_2 = 1×34 = 34c³. The total volume = v = v_1 + v_2 = 73c³.

Since the mean density of the missile (the led and the copper cover)of the studied bullet = 10 times the mean density of the

human body then; $m + \Delta m = b = 7.3 + 1 = 8.3$ times the mass of the missile.

The effect of Yaw on the energy lost due to toughness if the depth of the stroke muscle = 8 c and where the angle reaches within this depth to 10 degree is as follows;

$A = o.63 + (2.5sin5) = o.82 \ c^2$. Then;

$T = 22 \times o.82 \times 8 = 142$ j.

Now we can define the required function as;

If a projectile stroke an object with a force more than the inter macromolecules binding force then the projectile would loss kinetic energy equal the binding energy of the penetrated zone as discussed under the toughness energy T. The remainder kinetic energy R of the bullet would gradually reduced due to the load of the pushed evacuated mass as discussed under conservation of kinetic energy .So; if the energy lost to destroy the binding energy allover the volume = T and if

$E(x_1) = E_1$ then; $E_1 - T = R$ and the final kinetic energy $E_{(x1,x2)} = (R \div b)$ where $(m + \sum \Delta m) \div m = b_{(x2)} = b$

Let the kinetic energy of the bullet at the inlet = $E_{(x1)} = E_1$ and the remainder kinetic energy of the bullet after its stoppage inside the victim body = $E_{(x1,x2)} = E_2$

Then, $(E_1 - T) \div b = R \div b = E_2$
$\qquad$ (9)

From equation (1);

$E_1 = E_o \ e^{-2k_1x_1}$
$\qquad$ (10)

From (9) and (10);

$E_2 = (E_o \div be^{2k_1x_1}) - (T \div b)$
$\qquad$ (11)

From (2);

$E_2 = (E_1 - T) \div e^{2k_2x_2}$
$\qquad$ (12)

From (12) and (10);

$E_2 = (E_o \div e^{2k_1x_1 + 2k_2x_2}) - (T \div e^{2k_2x_2})$
$\qquad$ (13)

From (11) and (13);

$E_2 = (E_o \div e^{2k_1x_1 + 2k_2x_2}) - (T \div e^{2k_2x_2}) = (E_o \div be^{2k_1x_1}) - (T \div b)$
$\qquad$ (14)

By dividing on T;

$E_2 = (E_o \div T \ e^{2k_1x_1 + 2k_2x_2}) - (1 \div e^{2k_2x_2}) = (E_o \div T \ be^{2k_1x_1}) - (1 \div b)$
(15)

Where $E_o \div T$ is a ratio of energies so it is dimensionless and b is ratio of masses so it is also dimensionless. Each of x_1 and x_2 is in inches to compensate with k which depends on the ballistic coefficient whose magnitudes calculated using the inch.

Equation (14) gives the following **trivial solution;**

$b = e^{2k_2x_2}$
(16)

Actually this result gives the mathematic meaning of the mass (and the volume) of the destroyed tissue and gives –also- the physical meaning of Yaw and its effect on the destroyed tissue. The volume V of the broken tissue of the permanent cavity could be estimated directly as; $V = (b - 1) \div \sigma$ where σ is the mean density of the human body.

A case study

A bullet stoke a victim where the missile entered from the bulk muscle of the arm to outer from the interior side of the same arm and then to enter again to the chest without strike the bone where it passed obliquely after penetrating the lungs to pass though the liver and finally to be impacted behind the liver in front of the muscle of the back with a total penetrating depth = 51 c. We measured the depth of the muscle of the arm and chest where it was about 8 c. After autopsy we examined the missile where it had 9mm caliber, 1.9mm length and mass = 8 gram.

Since the energy lost due to toughness depends mainly on toughness of the muscle so, toughness equals toughness density times the cross section area of the bullet time the penetrated depth of the muscle then;

$T = 142$ j.

Putting;

b = 8.3 (as estimated in the same studied case), and if the muzzle energy[8] E_o = 435 joule, then We can define E_2 as follows;

Since The critical kinetic energy of the bullet to pass the muscle of the back = E_c =

$T_m \ A = 17 \times 0.82 = 13.9$ j (where tensile of the muscle = 15 ± 3 N.)

Since E_2 have to be equal or less than the critical energy then matching in equation (13) gives x_1 equal at least 100m.

We suggested that the shooter aimed the muzzle barrel towards the head of the victim to compensate for the drop trajectory of the bullet. Since the 9mm bullet drop after 100m about half a meter so the bullet stroked the arm of the victim.

Checking the result of equation (15);

Put $x_2 = 51c$ in the exponential; $e^{2k_2x_2} = 8.3$
(17)

Comparing the above magnitude with that of b then,

$e^{2k_2x_2} = b$. So we can define b –directly- from x_2.

Actually the empirical studies[7] on the bullet of our case showed that it loses about 1.3 of its initial speed after displacing 100 m which is fit with our calculus. Also many empirical[12] studies showed that the same bullet (9mm) at speed of entrance on the range of 250 m/s (like our case) can penetrate the body victim (or the simultaneous prepared gelatin) for about 51c which is –also- fit with our results.

Conclusion;

The paper suggests a new theory to explain the reduced velocity of the bullet versus displacement graphs. The theory depends on dealing the medium (molecules of air or macromolecules of the tissue) in which the bullet passes as a matter with some degree of elasticity having tensile (ultimate stress). Permanent cavitations –on this view- means break down (of the inter-particles binding bonds) but without elongation. The bullet to break and overcome the binding forces loses energy. This provides the first term of the lost energy E_1 and gives the physical interpretation

of the permanent cavity. The second term E_2 arises from that these broken particles are load, carried and pushed by the bullet into it's besides throughout its pass. Since the bullet kinetic energy and its momentum before and after loading have to be conserved so this provides the second term of the lost energy and gives the mathematic interpretation of the temporary cavitations through the effect of the broken pushed tissue into the besides of the bullet pass.

If the initial kinetic energy of the bullet (at x = 0) = E_o then, the remainder energy

$E_R = E_o – (E_1 + E_2)$. Consequently vR is the velocity at stoppage inside the victim body. The paper succeeded to give the physical meaning of the critical speed vc of the bullet to pierce the outer surface of the victim body and its relation with v_R as; velocity of stoppage inside the victim body is equal or less than the critical speed. The forensic medicine expert is advised during autopsy to measure the site of the impacted bullet (to determine Ec), the depth of penetration along the bullet pass inside the dead body (to determine from equation 16 the magnitude of b) and -also- is advised to stress on the depth of the penetrated muscles and skin (to define E_2 and x_1). We studied a case and solved it mathematically and compared the results with the empirical studies where we found it fit. The author hopes scientific cooperation in this developing field.

REFERENCE   1- A.J. GallagERCOBI her, A. NI Anniadh. Dynamic tensile properties of human skin. Conference, 2012. | | 2- BP. Pereira, Pw Lueeecas, Ranking the fracture toughness of thin mammalian soft tissue using the scissors cutting test. J. Biomech. 1997 ,7, 30; 91-4. | | 3- Batchlor G. K, 1967 sec.3.5 pp. 156-164. | 4- C.E. Peter, C. L. Sebourn. Wound ballistics of unstable projectile. Part 1; C.E. E. Projectile Yaw growth and retardation. J. Trauma. 1996. | | | 5- E. Eldieb. Physics of the giant atom. I. J. of advanced research in physical sciences. Vol.1, issue5, sep. 2014, 27-42. | | 6- F. Huang, J. Rondall. Engineering of aligned skeletal muscle by micro pattering. 2010; 2 (1), 43-55 | | 7- Frenech, R. W. Callender,G. R. Ballistic characteristics of wounding agents.J. C. Beyer 1962, 91-141. | | 8- Gun data. Org. 9mm Ballistic Chart and coefficients (Luger), article posted; July, 20, 2012. | | 9- Hasegawa and Azuma. Tensile proparties of collagen rich sole tendon and intestinal smoth muscles. 1974. Peter, C. L. Sebourn. Wound ballistics of unstable projectile. Part 1; Projectile Yaw growth and retardation. J. Trauma. 1996. | | 10- Kozaburo Hayashi. Mechanical proparties of tissues and arterial walls. Toyonaka, Osaka 560-831, Japan. | | 11- K. Coley, NA Fleck,. The toughness of adipose tissue; measurments and physical basis. J. Biomech. 2010; 1823-6. | | 12- Macppherson,Duncan. Bullet pentration; Modeling the dynamics and the incapacitation resulting from wound trauma, 1994, Elsegunda, C A Ballestic publication. ISBN 0-9663577-0-4 | | 13- Mcshane, Kelley and Reno, Exterior ballistics. university Denver press,1953. | | 14- M.E. Nimni and R.D. Markness. Molecular structure and functions of collagen. CRC press, Boca Raton,FL,1988, pp.3-35 | | 15- Mike Nelsone. Bullet trajectory; fact and Myth. | M. J. Buehler, J. Mech, Behavior. Biomedicine of matter. 1,59 (2000) -14 | | 16- Miroshi Miyazaki, Kozaburo. Tensile tests of collagen fibers obtained from rabbit patellar tendons. J. Biomedical microdevices . March 1999, vol.2,issue 2, pp. 151-157 | | 17- M.L. Facler. Wound ballisticsl the management of assault rifle injuries. Mil. Med; 1990. | | 18- R.B. Martin, D.B. Burr, and N.A. Sharket. Skeletal tissue mech. S.V.N.G. 1998. | | 19- Toufic Azar, Vincent Hoyward. Estimation 0f the fraction toughness of soft tissue from needle insert.Centre of intelligent mechanics, MCGILL university, Montreal, Quebec, H3A2A7, Canada. | | 20- Y,C. Fung. Biomech. And proparties of living tissues. Springer- Verlag, Newyork, 2nd.