Research Paper

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INVENTORY MODEL FOR DETERIORATING ITEMS WITH STOCK DEPENDENT DEMAND UNDER PARTIAL BACKLOGGING AND VARIABLE SELLING PRICE

Statistics

KEYWORDS : Inventory, Stock dependent demand, Deterioration, Variable selling price, Partial backlogging

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ABSTRACT

A Weibull deteriorating items inventory model with variable selling price and stock dependent demand is developed. Holding cost is linear function of time. Shortages are allowed and partially backlogged. Numerical example is considered and sensitivity analysis is also carried out for parameters.

INTRODUCTION:

In past few decays deteriorating items inventory models were widely studied. An inventory model with constant rate deterioration was developed by Ghare and Schrader [5]. The model was further extended by considering variable rate of deterioration by Covert and Philip [2]. Shah [15] further extended the model by considering shortages. The related work are found in (Nahmias [12], Raffat [14], Goyal and Giri [6], Mandal [8]), Mishra et al. [9]).

Burewell [1] developed economic lot size model for price dependent demand under quantity and freight discounts. Mukhopadhyay et al. [11] developed an inventory model for deteriorating items with a price-dependent demand rate. Teng and Chang [16] considered the economic production quantity model for deteriorating items with stock level and selling price dependent demand. Wang et al. [19] considered the problem of determining the optimal replenishment policy for deteriorating items with variable selling price under stock dependent demand. Karmakar and Choudhury [7] made a review of different inventory models with shortages of different types for deteriorating items with different demand patterns. A deterministic inventory model when deterioration rate is time proportional was developed by Patra et al. [13]. Demand rate was taken as a nonlinear function of selling price, deterioration rate, inventory holding cost and ordering cost were all functions of time. Tripathy and Mishra [18] dealt with development of an inventory model when the deterioration rate follows Weibull two parameter distribution, demand rate is a function of selling price and holding cost is time dependent. Dye [3] developed an optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging inventory model. Dye et al. [4] obtained an optimal selling price and lot size model with a varying rate of deterioration and exponential partial backlogging. Teng et al. [17] has given a comparison between two pricing and lot sizing models with partial backlogging and deteriorated items. Yang [20] developed the inventory model to allow for selling price as well as purchasing cost to change from one replenishment cycle to another during a finite time horizon. Mathew [10] developed an inventory model for deteriorating items with mixture of Weibull rate of decay and demand as function of both selling price and time.

In this paper an inventory model for stock dependent demand, time varying holding cost and variable selling price is developed. Shortages are allowed and partially backlogged. To illustrate the model, numerical example is provided. Sensitivity analysis also carried out for major parameters.

NOTATIONS AND ASSUMPTIONS:

The following notations are used here:

NOTATIONS:

The following notations are used for the development of the model:

$$D(t) = \begin{cases} a+bI(t), & I(t) > 0 \\ a, & I(t) \le 0, \end{cases}$$

A : Ordering cost per order

c : Unit purchasing cost per item

c_2 : Shortage cost per unit

c_3 : Cost of lost sales per unit

HC : Holding cost per unit time is a linear function of time t ($x+yt$, $x>0$, $0<y<1$)

SC : Shortage cost

DC : deterioration cost

MC : Manufacturing cost

SR : Sales Revenue

I(t) : Inventory level at any instant of time t, $0 \le t \le T$

Q_1 : Inventory level initially

Q_2 : Shortage of inventory

Q : Order quantity

T : Cycle length

α : Scale parametrs ($0 < \alpha < 1$)

β : Shape parameter ($\beta > 0$)

δ : Lost sales

π : Total profit

ASSUMPTIONS:

The following assumptions are used in the development of the model:

- The demand of the product is declining as a function of inventory level I(t).
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and partially backlogged.
- The deteriorated units can neither be repaired nor replaced during the cycle time.
- The deterioration of the items follows a Weibull deterioration with parameter α and β.
- The variable selling price S(t) is a function of demand,

  i.e. $S(t) = S_0 - \rho\left(D(t)\right)$

  where S_0, ρ, a and b are positive constants.

THE MATHEMATICAL MODEL AND ANALYSIS:

The figure describes the behaviour of inventory system.

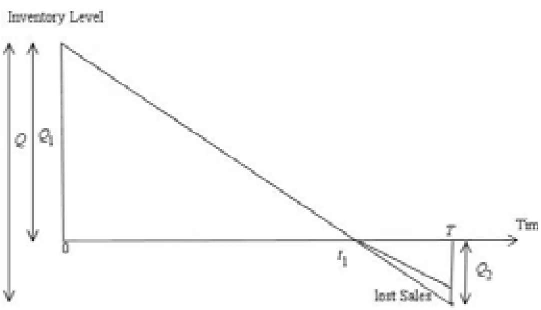

Figure 1

The inventory level of the product at time t over the period (0,T) can be represented by the differential equations

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -(a + bI(t)), \qquad 0 \le t \le t_1, \quad (1)$$

$$\frac{dI(t)}{dt} = -ae^{-\delta(T-t)}, \qquad t_1 \le t \le T, \quad (2)$$

with boundary conditions $I(0) = Q_1$, $I(t_1)=0$ and $I(T)=-Q_2$. The solutions of equations (1) and (2) using boundary conditions are given by:

$$I(t) = -a\left(t+\frac{\alpha}{(\beta+1)}t^{\beta+1}+\frac{1}{2}bt^2\right)+a\left(t_1+\frac{\alpha}{(\beta+1)}t_1^{\beta+1}+\frac{1}{2}bt_1^2\right) -\alpha t^{\beta}\left(-a\left(t+\frac{1}{2}bt^2\right)+a\left(t_1+\frac{1}{2}bt_1^2\right)\right) \quad (3)$$
$$t\left(-a\left(t+\frac{\alpha}{(\beta+1)}t^{\beta+1}+\frac{1}{2}bt^2\right)+a\left(t_1+\frac{\alpha}{(\beta+1)}t_1^{\beta+1}+\frac{1}{2}bt_1^2\right)\right)$$

$$I(t) = -a\left(t-\delta\left(\left(Tt-\frac{1}{2}t^2\right)\right)\right)+a\left(t_1-\delta\left(\left(Tt_1-\frac{1}{2}t_1^2\right)\right)\right) \quad (4)$$

(by neglecting higher powers of α and β)

Putting t=0 in equation (3) we get

$$Q_1 = a\left(t_1+\frac{\alpha}{(\beta+1)}t_1^{\beta+1}+\frac{1}{2}bt_1^2\right). \quad (5)$$

Putting t = T in equation (4) we get

$$Q_2 = a\left(T-\frac{1}{2}\delta T^2\right)-a\left(t_1-\delta\left(\left(Tt_1-\frac{1}{2}t_1^2\right)\right)\right), \quad (6)$$

And the order quantity is

$$Q = a\left(\frac{\alpha}{(\beta+1)}t_1^{\beta+1}+\frac{1}{2}bt_1^2+T-\frac{1}{2}\delta T^2+\delta Tt_1-\frac{1}{2}\delta t_1^2\right). \quad (7)$$

Based on the assumptions and descriptions of the model, the total relevant profit, include the following elements:

(i) Ordering cost (OC) = A (8)

(ii) $HC = \int_0^{t_1}(x+yt)I(t)dt$

$$= -\frac{1}{8}\frac{at_1^2}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)}\left(\begin{array}{l} -8\alpha t_1^{\beta}\left(\begin{array}{l}\left(-2yt_1-2x\right)\beta^2 \\ +\left(-14x+ybt_1^2-11yt_1\right)\beta \\ -24x+3ybt_1^2-12yt_1\end{array}\right) \\ +(\beta+2)(\beta+3)(\beta+4) \\ \left(\begin{array}{l}4\alpha t_1^{\beta}\left(\frac{2}{3}ybt_1^2+\left(-y+xb\right)t_1-2x\right) \\ +(\beta+1)\left(\begin{array}{l}\frac{8}{15}yb^2t_1^3+\left(-\frac{1}{3}y+xb\right)bt_1^2 \\ +\left(-\frac{4}{3}y-\frac{4}{3}xb\right)t_1-4x\end{array}\right)\end{array}\right)\end{array}\right) \quad (9)$$

(iii) Deterioration cost:

$$DC = c\left[Q_1-\int_0^{t_1}D(t)dt\right] = c\left[Q_1-\int_0^{t_1}(a+bI(t))dt\right]$$

$$= \frac{1}{8}\frac{ac}{(\beta+1)(\beta+2)(\beta+3)}\left(\left(\begin{array}{l}-8b^2\alpha(\beta+2)t_1^{\beta+3}-8\alpha(\beta+2)(\beta+3)(bt_1-1)t_1^{\beta+1} \\ +b\left(\begin{array}{l}8\alpha t_1^{\beta+2}\left((bt_1+1)\beta+2bt_1+3\right) \\ +(\beta+3)\left(\begin{array}{l}8\alpha t_1^{\beta+2}+ \\ \left(4\alpha t_1^{\beta}+(\beta+1)(bt_1-\frac{4}{3})\right)b(\beta+2)t_1^3\end{array}\right)\end{array}\right)\end{array}\right)\right) \quad (10)$$

(iv) Manufacturing cost is given by

$$MC = cQ = ca\left(\begin{array}{l}\frac{\alpha}{(\beta+1)}t_1^{\beta+1}+\frac{1}{2}bt_1^2+T \\ -\frac{1}{2}\delta T^2+\delta Tt_1-\frac{1}{2}\delta t_1^2\end{array}\right). \quad (11)$$

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(v) Shortage cost is given by

$$SC = -c_2\left[\int_{t_1}^{T} I(t)dt\right] = -c_2\left[\int_{t_1}^{T} a(t_1 - t)dt\right]$$

$$= -c_2 a\begin{pmatrix} -\frac{1}{6}\delta\left(T^3 - t_1^3\right) - \frac{1}{2}\left(1-\delta T\right)\left(T^2 - t_1^2\right) \\ +\left(t_1 - \delta\left(Tt_1 - \frac{1}{2}t_1^2\right)\right)\left(T - t_1\right)\end{pmatrix} \quad (12)$$

(vi) $LS = c_3\left[\int_{t_1}^{T} a\left(1 - \left(1 - \delta\left(T-t\right)\right)\right)dt\right]$

$$= c_3 a\left(-\frac{1}{2}\delta\left(T^2 - t_1^2\right) + \delta T\left(T - t_1\right)\right). \quad (13)$$

(vii) $SR = \int_0^T S(t)D(t)dt = \int_0^{t_1} S(t)D(t)dt + \int_{t_1}^{T} S(t)D(t)dt$

$$= \int_0^{t_1}\left(S_0 - \rho(a+bI(t)\right)(a+bI(t))dt + \int_{t_1}^{T} a\left(S_0 - \rho a\right)dt$$

$$= -\frac{a}{(\beta+1)^2(\beta+2)(\beta+3)(\beta+4)}\begin{pmatrix} \frac{1}{3}\rho b^2\left(3 - 3bt_1 + b^2t_1^2\right)\alpha^2(\beta+2)(\beta+3)(\beta+4)t_1^{2\beta+3}a \\ +\frac{2}{15}b\alpha t_1^{\beta+2} \\ \begin{pmatrix} -15\rho b\alpha\begin{pmatrix}\left(2 - 2t_1 b\right)\beta^2 \\ +\left(14 - 11bt_1 + b^2t_1^2\right)\beta \\ +24 - 12bt_1 + 3b^2t_1^2\end{pmatrix}at_1^{\beta+1} \\ +(\beta+1)(\beta+21)(\beta+3)(\beta+4) \\ \left(-\frac{15}{2}S_0 + 15\rho a + \frac{15}{4}bt_1S_0 - \frac{5}{2}\rho ab^3t_1^3 + \rho ab^4t_1^4\right)\end{pmatrix}\end{pmatrix}$$

$$-\frac{1}{4}\frac{a^2\rho b^2\alpha^2t_1^{2\beta+3}}{\left(\frac{1}{2}+\beta\right)\left(\frac{3}{2}+\beta\right)\left(\frac{5}{2}+\beta\right)(\beta+1)^2(\beta+2)}\begin{pmatrix}\left(5 - 2bt_1 + b^2t_1^2\right)\beta^3 \\ +\left(\frac{51}{2} - \frac{15}{2}bt_1 + 5b^2t_1^2\right)\beta^2 \\ +\left(\frac{77}{2} - \frac{25}{2}bt_1 + \frac{15}{2}b^2t_1^2\right)\beta + 15 + 3b^2t_1^2\end{pmatrix}$$

$$-\frac{2ab\alpha t_1^{2\beta+2}}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)}\begin{pmatrix}\left(-2a\rho + S_0\right)\beta^4 \\ +\left(-36a\rho - 3a\rho bt_1 - a\rho b^2t_1^2 + 3a\rho b^3t_1^3 + a\rho b^4t_1^4 + 18S_0\right)\beta^3 \\ +\left(-45a\rho bt_1 - 238a\rho - 18a\rho b^2t_1^2 + 31a\rho b^3t_1^3 + 8a\rho b^4t_1^4 + 119S_0\right)\beta^2 \\ +\left(-684a\rho - 222a\rho bt_1 - 107a\rho b^2t_1^2 + 86a\rho b^3t_1^3 + 15a\rho b^4t_1^4 + 342S_0\right)\beta \\ -720a\rho - 360a\rho bt_1 - 210a\rho b^2t_1^2 + 48a\rho b^3t_1^3 + 360S_0\end{pmatrix}$$

$$-a\begin{pmatrix}\frac{2}{105}a\rho b^6t_1^7 - \frac{1}{30}a\rho b^5t_1^6 - \frac{1}{15}a\rho b^4t_1^5 + \frac{1}{8}S_0b^3t_1^4 \\ +\frac{2}{3}\left(a\rho - \frac{1}{4}S_0\right)b^3t_1^3 + \left(a\rho - \frac{1}{2}S_0\right)bt_1^2 + T\left(a\rho - S_0\right)\end{pmatrix} \quad (14)$$

The total profit per unit time is given by

$$\pi(t_1,T) = \frac{SR - A - MC - DC - HC - SC - LS}{T} \quad (15)$$

Putting values from equations (8) to (14) in equation (15), we get the average profit.

The optimal value of $t_1 = t_1^*$ and $T = T^*$ (say), which maximizes profit $\pi(t_1,T)$ can be obtained by differentiating equation (15) with respect to t_1 and T and equate it to zero

$$\text{i.e. } \frac{\partial\pi(t_1,T)}{\partial T} = 0, \quad \frac{\partial\pi(t_1,T)}{\partial t_1} = 0, \quad (16)$$

provided it satisfies the condition

$$\frac{\partial^2\pi(t_1,T)}{\partial T^2} < 0, \quad \frac{\partial^2\pi(t_1,T)}{\partial t_1^2} < 0 \text{ and}$$

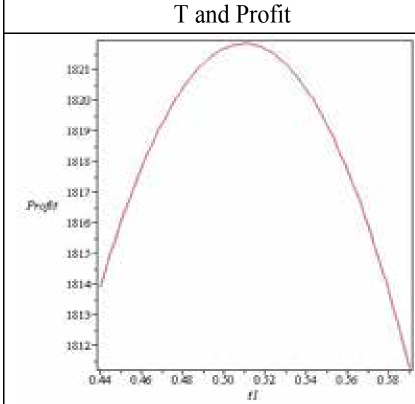
$$\left[\frac{\partial^2\pi_1(t_1,T)}{\partial T^2}\right]\left[\frac{\partial^2\pi_1(t_1,T)}{\partial t_1^2}\right] - \left[\frac{\partial^2\pi_1(t_1,T)}{\partial T\partial t_1}\right]^2 > 0. \quad (17)$$

NUMERICAL EXAMPLES:

Considering A= Rs.250, a = 600, b=0.05, c=Rs. 5, c_2 =Rs. 3, S_0= Rs. 15, c_3=Rs. 2, α=0.01, β =2, x = Rs. 1.7, y=0.05, ρ=0.01, δ=0.06, in appropriate units. The optimal values of t_1^* = 0.5098, T*=0.8593, Q* = 517.5447 and Profit π* = Rs. 1821.8561

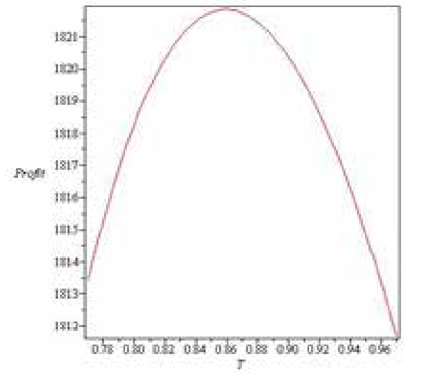
The second order condition given in equation (17) is also satisfied. The graphical representation of the concavity of the profit function is also given.

T and Profit



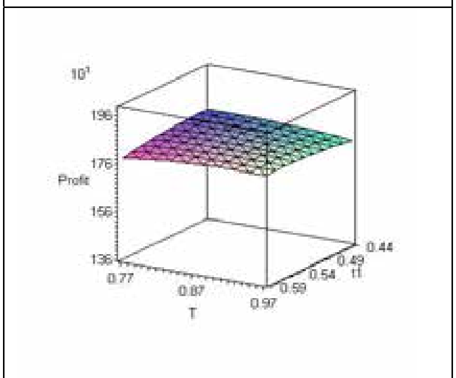
Graph 1

T and Profit



Graph 2

T and Profit



Graph 3

SENSITIVITY ANALYSIS:

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1
Sensitivity Analysis

Parameter	%	t_1	T	Profit	Q
a	+20%	0.4476	0.7716	1371.2525	557.0996
	+10%	0.4765	0.8125	1632.0948	537.9990
	-10%	0.5482	0.9135	1940.6712	495.4818
	-20%	0.5933	0.9775	1988.6767	471.6327
x	+20%	0.4544	0.8215	1793.4906	493.7591
	+10%	0.4870	0.8436	1810.3809	507.6596
	-10%	0.5423	0.8826	1838.2267	532.2058
	-20%	0.5817	0.9098	1856.3943	549.4116
ρ	+20%	0.4884	0.8445	1091.1497	508.2285
	+10%	0.4989	0.8517	1456.4104	512.7614
	-10%	0.5213	0.8673	2187.4973	522.5847
	-20%	0.5333	0.8757	2553.3455	527.8792

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	-20%	0.5333	0.8757	2553.3455	527.8792

α	+20%	0.5076	0.8575	1821.2225	516.4750
	+10%	0.5087	0.8584	1821.5384	517.0100
	-10%	0.5109	0.8602	1822.1755	518.0791
	-20%	0.5120	0.8611	1822.4966	518.6132
β	+20%	0.5121	0.8610	1822.8925	518.5238
	+10%	0.5110	0.8601	1822.4286	518.0019
	-10%	0.5084	0.8584	1821.1445	517.0344
	-20%	0.5070	0.8575	1820.2532	516.5390
δ	+20%	0.5082	0.8629	1823.7758	519.1590
	+10%	0.5090	0.8611	1822.8091	518.3552
	-10%	0.5106	0.8575	1820.9133	516.7274
	-20%	0.5114	0.8558	1819.9893	515.9624
A	+20%	0.5563	0.9403	1766.2890	566.5121
	+10%	0.5336	0.9007	1793.4478	542.5691
	-10%	0.4846	0.8157	1851.7064	491.1969
	-20%	0.4579	0.7695	1883.2467	463.2894

From the table we observe that as parameters a, A, x and ρ increases/ decreases, there is decrease/ increase in average total profit.

From the table we observe that with increase/ decrease in parameter α, there is corresponding small decrease/ increase in total profit.

From the table we observe that as parameters β and δ increases/ decreases, there is very slight increase/ decrease in average total profit.

6. CONCLUSION:

In this paper, we have developed an inventory model for deteriorating items with linear demand, partial backlogging under variable selling price. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding decrease/ increase in the value of profit.

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