Research Paper

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Use of Game Theory for Solving minimization type connected Graph Problem

Mathematics

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ABSTRACT

A simple but interesting method is developed to know the minimum number of Primary Schools which has to newly built up and to select their appropriate locations from the network of the population centers of certain remote region such that any population center is to be covered by at least one these Primary Schools within distance of three Kilometers. This type of connected Graph problem is solved by game theory method.

Introduction:

Education in our country, in India**,** is provided by the public sector as well as the private sector, with control and funding coming from three levels: Central Government, State Governments and local authority. Under various articles of the Indian Constitution, free and compulsory education is provided as a fundamental right to children between the ages of 6 and 14. But it is a huge task for the administrators who are responsible for implementing '**Sarva Shiksha Abhiyan'** (In English, **The Education for All Movement**). To select locations in remote places and to confirm that every remote village is covered by at least one Primary School within certain distance, it becomes necessary to introduce optimization techniques for solution of such problems. We think of a mathematical optimization model for solving the above problem. In this paper, we consider a small model problem and it can be generalized in the similar way for large problem.

As an example, we consider that there are eleven population centers **A** to **K** in some region of our country. These centers are connected by road network as shown in **Figure-I** and distances between the populations center are given in kilometer. Assume also that the Primary Schools have to be situated at different locations in the region in such a way that no child has to travel more than three kilometers from his/her residence for getting admission to the nearest Primary School. We have to find out minimum number of Primary Schools and their prime locations in the region in such a way that every child can go to a Primary School within a distance of three kilometers. This type of connected and weighted Graph problems is generally solved by the method of minimum spinning tree. Kruscal and Prim algorithms are well known methods for finding spanning trees and other methods [**Deo (2002), West (2002), Wilson (2013)**] are also available in the literature. But when the network is large and weight of edges also large numbers, above methods may create problem for finding solution using personal computer. Here, we are going to introduce a new but simple method by which we can solve such basic problems of our country.

2. Solution Method:

A new interesting method is introduced here for the solution of the above problem using two persons zero sum game model as used in the Game Theory [**Taha (2004)**]. The distance (in kilometer) between population centers, are shown in the adjacency matrix in **Table-I**. In this new method, we replace the distance between the population centers by the value one, if the distance between them is less than or equal to three kilometers; and replace the distance by value zero, if the distance is greater than three kilometer or when two population centers are not directly connected by a road. This method not only gives us the minimum number of Primary Schools but also some important information about the location of the Primary Schools within the region. The newly created adjacency matrix by zeros and ones of the given problem, is shown in **Table-II**. We may now think that this adjacency matrix as the pay-off matrix [H. A. **Taha (2004)]** for the player **X** (shown at the left side of the Table-II) and the second player **Y** is shown at the top of the **Table-II**. Using the rule of row dominance as used in the Game theory for two-person zero sum game, remove rows corresponding to **F, H, I, J** and **K** respectively and the game reduces to a game as given in **Table-III**. Using column dominance (as done in the Game Theory), remove columns **C**, **D, E, H and I** respectively**.** After removing above columns, the pay off matrix reduces to the game, given in **Table-IV**. After rearranging columns of **Table-IV**, we get **Table-V**.

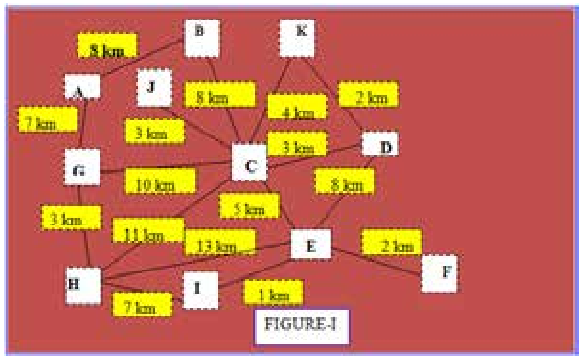

FIGURE-I

	A	B	C	D	E	F	G	H	I	J	K
A	0	8	-	-	-	-	7	-	-	-	-
B	8	0	8	-	-	-	-	-	-	-	-
C	-	8	0	3	5	-	10	11	-	3	4
D	-	-	3	0	8	-	-	-	-	-	2
E	-	-	5	8	0	2	-	13	1	-	-
F	-	-	-	-	2	0	-	-	-	-	-
G	7	-	10	-	-	-	0	3	-	-	-
H	-	-	11	-	13	-	3	0	7	-	-
I	-	-	-	-	1	-	-	7	0	-	-
J	-	-	3	-	-	-	-	-	-	0	-
K	-	-	4	2	-	-	-	-	-	-	0

TABLE-I: Adjacency Matrix. Dashes denote that no direct connection between the population centers.

PLAYER: Y

		A	B	C	D	E	F	G	H	I	J	K
AYER:X	A	1	0	0	0	0	0	0	0	0	0	0
	B	0	1	0	0	0	0	0	0	0	0	0
	C	0	0	1	1	0	0	0	0	0	1	0
	D	0	0	1	1	0	0	0	0	0	0	1
	E	0	0	0	0	1	1	0	0	1	0	0
	F	0	0	0	0	1	1	0	0	0	0	0
	G	0	0	0	0	0	0	1	1	0	0	0
	H	0	0	0	0	0	0	1	1	0	0	0
	I	0	0	0	0	1	0	0	0	1	0	0
	J	0	0	1	0	0	0	0	0	0	1	0
	K	0	0	0	1	0	0	0	0	0	0	1

TABLE-II: Adjacency matrix is converted to the binary matrix.

		PLAYER: Y										
		A	B	C	D	E	F	G	H	I	J	K
PLAYER: X	A	1	0	0	0	0	0	0	0	0	0	0
	B	0	1	0	0	0	0	0	0	0	0	0
	C	0	0	1	1	0	0	0	0	0	1	0
	D	0	0	1	1	0	0	0	0	0	0	1
	E	0	0	0	0	1	1	0	0	1	0	0
	G	0	0	0	0	0	0	1	1	0	0	0

TABLE-III: Using dominance removed rows corresponding to F, H, I, J and K

		PLAYER: Y					
		A	B	F	G	J	K
PLAYER: X	A	1	0	0	0	0	0
	B	0	1	0	0	0	0
	C	0	0	0	0	1	0
	D	0	0	0	0	0	1
	E	0	0	1	0	0	0
	G	0	0	0	1	0	0

TABLE-IV: Using dominance removed columns C, D, E, H and I.

		PLAYER: Y					
		A	B	J	K	F	G
PLAYER: X	A	1	0	0	0	0	0
	B	0	1	0	0	0	0
	C	0	0	1	0	0	0
	D	0	0	0	1	0	0
	E	0	0	0	0	1	0
	G	0	0	0	0	0	1

TABLE-V: Rearranging the irreducible form of the matrix.

esult and Discussion:

Table-V is the irreducible form of the game, that is, which cannot be reduced further. Order of the matrix, so obtained, gives us the minimum number of Primary Schools for the region. For our example it is six. Hence, **six Primary Schools** are needed for the region so that every child gets admitted in a School within the distance of three kilometers from his/her residence. It can be seen from **Table-V** that six sub-graphs are formed, namely, **{A}, {B}, {C, J}, {D, K}, {E, F} and {G}** and other population centers **I** and **H** which can be added at least in one of these sub-graphs. For large problem this can be done by searching and sorting method. Population center **I** becomes a new member of sub-graph **{E, F}** and population center **H** becomes a new member of the sub-group {G}. So, newly created sub-graphs are **{A}, {B}, {C, J}, {D, K}, {E, F, I} and {G, H}.** If a connected sub-graph, so obtained, is large then its center may be considered as the appropriate location for the Primary School for that sub-graph. In **Figure-II**, six appropriate locations of the Primary Schools are shown by arrows.

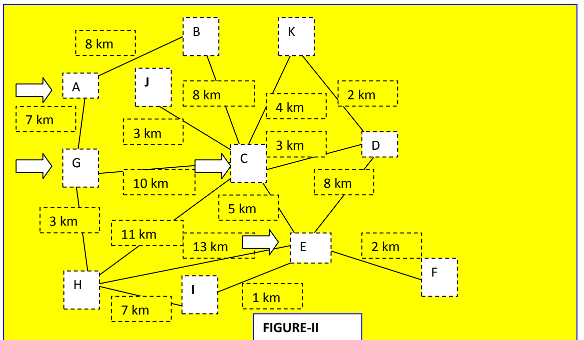

FIGURE-II

4. Conclusion:

Two persons zero-sum game is introduced here to solve minimization type connected and weighted graph problem. Novelty of this method is that it is not only gives us minimum number of the Primary Schools but also suitable locations of the Schools. This method may also be used for selecting suitable locations of Health Centers, locations of deep tube wells and their minimum numbers optimally.

REFERENCE

1. Deo, N., (2002), Graph Theory: with applications to Engineering and Computer Science, Prentice-Hall of India Private Limited, twenty-third printing, New Delhi. | 2. West, D. B., (2002), Introduction to Graph Theory, Prentice-Hall of India Private Limited, Second Edition (fourth printing), New Delhi . | 3. Wilson, R. J., (2013), Introduction to Graph Theory, Forth Edition, Pearson Education Ltd. | 4. Taha, H. A.,(2004), Operation Research: an introduction, Prentice-Hall of India Private Limited, Seventh Edition, New Delhi. | |