Research Paper

Volume : 4 | Issue : 5 | May 2015 • ISSN No 2277 - 8179

Rotation of Fluid by Using New Field Equations and Isotropic Line Element

Physics

KEYWORDS : Newtonian approximation, Christoffel symbols,Newtonian potential

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ABSTRACT

In the case of pure rotation, Newtonian approximation of Einstein's theory is not possible because of the off diagonal term ' 'in the metric, the field equation makes the field non Newtonian. The new field equations can easily explain rotation since field and hydrodynamic equations are approximately same as Newtonian. In the present investigation these field equations are applied in the problem of Rotation of a star (fluid with pressure).

INTRODUCTION: In the case of radial motion of a star pressure gradient and gravitational field are both in radial direction the fluid will be accelerated only radially according to Newton's theory. For rotation the resultant of the two forces must be perpendicular to the axis of rotation according to Newton's theory. In the case of pure rotation, Newtonian approximation of Einstein's theory is not possiblebecause of the off diagonal term' $2\omega r^2 Sin^2\phi \cdot d\phi \cdot dt$ 'in the metric, the field equation makes the field non Newtonian.Let us consider the metric in isotropic[1]

$$ds^2 = -e^{\mu}\left(dr^2 + r^2 d\theta^2 + r^2 Sin^2\theta \cdot d\phi^2\right) + e^{\nu} dt^2 + 2\omega r^2 Sin^2\theta \cdot d\phi \cdot dt \;.......(1)$$

where $\mu = \mu(r,\theta)$ and $\nu = \nu(r,\theta)$

In Cartesian system above line element takes the form:

$$ds^2 = -e^{\mu}(dx^2 + dy^2 + dz^2) + e^{\nu} dt^2 - 2\omega y dx dt + 2\omega x dy dt \;...........(2)$$

In this case $\mu = \mu(x, y, z)$ and $\nu = \nu(x, y, z)$

$g_{11} = -e^{\mu} = g_{22} = g_{33}$ $g_{44} = e^{\nu}$ $g_{14} = -\omega y$ $g_{41} = -\omega y$

$g_{24} = \omega x$ $g_{42} = \omega x$

$$g_{\mu\nu} = \begin{matrix} -e^{\mu} & 0 & 0 & -\omega y \\ 0 & -e^{\mu} & 0 & \omega x \\ 0 & 0 & -e^{\mu} & 0 \\ -\omega y & \omega x & 0 & e^{\nu} \end{matrix} \qquad g^{\mu\nu} = \begin{matrix} -e^{-\mu} & 0 & 0 & -\omega y \\ 0 & -e^{-\mu} & 0\,- & \omega x \\ 0 & 0 & -e^{\mu} & 0 \\ -\omega y & \omega x & 0 & e^{-\nu} \end{matrix}$$

Since $\omega x \approx 10^{-6} \approx \omega y$ and space is almost flat one can verify that

$\left|g_{\mu\nu}\right| \left|g^{\mu\nu}\right| \cong |\,1\,|$ where $\left|g_{\mu\nu}\right|$ is the metrics of $g_{\mu\nu}$.

It is easy to show that

$$\because\; g \approx g_{11} \cdot g_{22} \cdot g_{33} \cdot g_{44}$$

$$\therefore\; g \approx \left(-e^{\mu}\right)\left(-e^{\mu}\right)\left(-e^{\mu}\right)\left(e^{\nu}\right)$$

$$\therefore\ g = -e^{3\mu+\nu}$$

$$\therefore\ \sqrt{-g} = e^{\frac{3\mu+\nu}{2}}$$

Christoffel symbols: $\{\mu\nu, \alpha\} = \frac{1}{2} g^{\alpha\lambda}\left(\frac{\partial g_{\mu\lambda}}{\partial x^{\nu}} + \frac{\partial g_{\nu\lambda}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\lambda}}\right)$

We use the following symbols.

$$\mu_x = \frac{\partial\mu}{\partial x},\ \nu_x = \frac{\partial\nu}{\partial x},\ \mu_y = \frac{\partial\mu}{\partial y},\ \nu_y = \frac{\partial\nu}{\partial y},\ \mu_z = \frac{\partial\mu}{\partial z},\ \nu_z = \frac{\partial\nu}{\partial z}$$

$$\mu_{xx} = \frac{\partial^2\mu}{\partial x^2},\ \nu_{xx} = \frac{\partial^2\nu}{\partial x^2},\ \mu_{yy} = \frac{\partial^2\mu}{\partial y^2},\ \nu_{yy} = \frac{\partial^2\nu}{\partial y^2},\ \mu_{zz} = \frac{\partial^2\mu}{\partial z^2}, \nu_{xx} = \frac{\partial^2\nu}{\partial z^2}$$

Christoffel symbols having non- zero values are the following

$$\{11,1\} = \frac{1}{2}\cdot\mu_x$$

$$\{11,2\} = -\frac{1}{2}\cdot\mu_y$$

$$\{11,3\} = -\frac{1}{2}\cdot\mu_z$$

$$\{11,4\} = \frac{\omega}{2}\left(y\mu_x + x\mu_y\right)$$

$$\{12,1\} = \frac{1}{2}\mu_y = \{21,1\}$$

$$\{12,2\} = \frac{1}{2}\mu_x = \{21,2\}$$

$$\{13,1\} = \frac{1}{2}\mu_z = \{31,1\}$$

$$\{13,4\} = \frac{\omega y}{2}\mu_z$$

$$\{14,1\} = -\frac{\omega y}{2}\nu_x = \{41,1\}$$

$$\{14,2\} = -\omega + \frac{\omega x}{2}\nu_x = \{41,2\}$$

$$\{21,1\} = \frac{1}{2}\mu_y$$

$$\{21,2\} = \frac{1}{2}\cdot\mu_x = \{12,2\}$$

$$\{22,1\} = -\frac{1}{2}\mu_x$$

$$\{22,2\} = \frac{1}{2}\mu_y$$

$$\{22,3\} = -\frac{1}{2}\mu_z$$

$$\{22,4\} = -\frac{\omega}{2}\left(y\mu_x + x\mu_y\right)$$

$$\{23,2\} = \frac{1}{2}\mu_z = \{32,2\}$$

$$\{23,3\} = \frac{1}{2}\mu_y = \{32,3\}$$

$$\{23,4\} = -\frac{\omega x}{2}\mu_z$$

$$\{24,1\} = \omega - \frac{\omega y}{2}\nu_y = \{42,1\}$$

$$\{24,2\} = \frac{\omega x}{2}\nu_y = \{42,2\}$$

$$\{24,4\} = \frac{1}{2}\nu_y + \omega^2 y = \{42,4\}$$

$$\{31,1\} = \frac{1}{2}\mu_z = \{13,1\}$$

$$\{32,2\} = \frac{1}{2}\mu_z = \{23,2\}$$

$$\{33,2\} = -\frac{1}{2}\mu_y$$

$$\{33,3\} = \frac{1}{2}\mu_z$$

$$\{33,4\} = -\frac{\omega}{2}\left(y\mu_x - x\mu_y\right)$$

$$\{34,1\} = -\frac{\omega y}{2}\nu_z = \{43,1\}$$

$$\{34,2\} = \frac{\omega x}{2}\nu_z = \{43,2\}$$

$$\{34,4\} = \frac{1}{2}\nu_z = \{43,4\}$$

$$\{32,3\} = \frac{1}{2}\mu_y = \{23,3\}$$

$$\{33,1\} = -\frac{1}{2}\mu_x$$

$$\{31,3\} = \frac{1}{2}\mu_x = \{13,3\}$$

$$\{41,4\} = \frac{1}{2}\nu_x + \omega^2 x = \{14,4\}$$

$$\{42,4\} = \frac{1}{2}\nu_y + \omega^2 y$$

$$\{44,2\} = \frac{1}{2}\nu_y$$

$$\{44,3\} = \frac{1}{2}\nu_z$$

$$\{44,4\} = -\frac{\omega}{2}\left(x\nu_y - y\nu_x\right)$$

$$\{43,4\} = \frac{1}{2}\nu_z$$

$$\{44,1\} = \frac{1}{2}\nu_x$$

Rest of the Christoffelsymbols vanish.

FIELD EQUATIONS ACCORDING TO EINSTEIN'S THEORY:

Einstein's equationis $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi T_{\mu\nu}$ ……………………….(3)

$R_{\mu\nu}$ and $T_{\mu\nu}$ can be found using following equations.

$$R_{\mu\nu} = \{\mu\sigma, \alpha\}\{\alpha\nu, \sigma\} - \{\mu\nu, \alpha\}\{\alpha\sigma, \sigma\} + \frac{\partial\{\mu\sigma, \sigma\}}{\partial x^{\nu}} - \frac{\partial\{\mu\nu, \sigma\}}{\partial x^{\sigma}} \text{ ………..(4)}$$

$R_{\mu\nu} \approx \frac{\partial\{\mu\sigma, \sigma\}}{\partial x^{\nu}} - \frac{\partial\{\mu\nu, \sigma\}}{\partial x^{\sigma}}$ (Neglecting non-linear terms because they are $\approx 10^{-36}$ except where they contain $\omega^2 \approx 10^{-30}$)

$$T^{\mu\nu} = (P + \rho)\frac{dx^{\mu}}{ds} \cdot \frac{dx^{\nu}}{ds} - g^{\mu\nu} P \text{ ……………….(5)}$$

Applying approximations field equations can be found. For approximation order of the various quantities at the surface of the earth are as follows[2]

$\mu \approx \nu \approx 10^{-9}$ $\omega_1 \approx \omega \approx 10^{-15}$ $\mu_x \approx \nu_x \approx 10^{-18}$ $x \approx y \approx 10^{9}$ $\mu_{xx} \approx \nu_{xx} \approx 10^{-27}$
$\rho \approx 10^{-27}$

It can be seen that Christoffell symbols can be equal to ω (i.e. of the order of 10^{-15}) for earth or otherwise without ω, Christoffell symbols without ω will be of the order of 10^{-18} for earth.

When we take product of Christoffell symbols the product will be of the order of ω^2 if both Christofell symbols are equal to ω. But $\omega^2 = 10^{-30}$ and $\rho = 10^{-27}$. So ω^2 has to be taken into consideration. Now if none of the Christofell symbols contain ω then their product will be of the order of 10^{-36} so negligible. The interesting case will be when one of the Christoffell symbols is ω and other Christoffell symbols does not contain ω the product will be of the order of $10^{-15} x 10^{-18} = 10^{-33}$. But 10^{-33} is 10^{-6}

times less than the main term ρ and Clairaut's theorem has been verified experimentally up to 1 in 10^{6}. So terms of the order $10^{-6}(\rho)$ or 10^{-33} are negligible.

With these approximations we get the following equations (approximately)

$$R_{11} = \frac{1}{2}\left(\nabla^2\mu + \mu_{xx} + \nu_{xx}\right) + \omega^2 - \frac{\omega}{2}\left(\omega_1 x\mu_x + y\dot{\mu}_x - \omega_1 y\mu_y + x\dot{\mu}_y\right)$$

$$R_{22} = \frac{1}{2}\left(\nabla^2\mu + \mu_{yy} + \nu_{yy}\right) + \omega^2 + \frac{\omega}{2}\left(\dot{y}\mu_x + y\dot{\mu}_x + \dot{x}\mu_y + x\dot{\mu}_y\right)$$

$$R_{33} \approx \frac{1}{2}\left(\nabla^2\mu + \mu_{zz} + \nu_{zz}\right) + \frac{\omega}{2}\left(\dot{y}\mu_x + y\dot{\mu}_x - \dot{x}\mu_y - x\dot{\mu}_y\right)$$

$$R_{44} \approx -\frac{1}{2}\nabla^2\nu - \frac{\omega}{2}\left(y\dot{\nu}_x + \nu_x\dot{y}\right) + \frac{\omega}{2}\left(x\dot{\nu}_y + \nu_y\dot{x}\right) - 2\omega^2$$

where $\dot{x} = -\omega_1 y$ and $\dot{y} = \omega_1 x$ where ω_1 is the angular velocity of pure rotation, different from ω in the metric, which is just a constant in the metric.

$$\nabla^2\mu + \mu_{xx} + \nu_{xx} \approx -8\pi\rho_0 - 2\omega^2$$

$$\nabla^2\mu + \mu_{yy} + \nu_{yy} \approx -8\pi\rho_0 - 2\omega^2$$

$$\nabla^2\mu + \mu_{zz} + \nu_{zz} \approx -8\pi\rho_0$$

$$\nabla^2\nu \approx 8\pi\rho_0 - 4\omega^2$$

Adding first three equations we get

$4\nabla^2\mu + \nabla^2\nu = -24\pi\rho - 4\omega^2$ Substituting the fourth equation in this equation we get

$$4\nabla^2\mu + 8\pi\rho = -24\pi\rho - 8\omega^2$$

$$4\nabla^2\mu = -32\pi\rho - 8\omega^2$$

$\nabla^2\mu = -8\pi\rho - 2\omega^2$ ……(6) This equation differs from Newton's theory.

$\nabla^2\nu = 8\pi\rho - 4\omega^2$ ………..(7) This equation also differs from Newton's theory.

ROTATION ON THE BASIS OF 'NEW FIELD EQUATIONS:

Einstein's field equation $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi T_{\mu\nu}$ shows dependence of motion on the field and field on the motion. Einstein's equation has field on one side and motion on the other side. The new field equation[3] is given as

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 4\pi\rho_0 g^{\mu\nu} + \eta^{\mu\nu} \text{ where } \eta^{\mu\nu} = 4\pi P\left[4\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} - g^{\mu\nu}\right]$$

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 4\pi\rho_0 g^{\mu\nu} + 4\pi P\left[4\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} - g^{\mu\nu}\right] \quad \ldots\ldots\ldots\ldots(8)$$

The field depends very slightly on motion through $\eta_{\mu\nu}$. The field depends on ρ_0 and is approximately Newtonian. The motion or hydrodynamics is given by a separate equation $T^{ik}_{,k} = 0$ ……..(9) gives hydrodynamics which is approximated to Newtonian hydrodynamics.

We can work with a diagonal metric of the form prescribed by Eddington[2] i.e.

$$-(1 + 2\Omega)(dx^2 + dy^2 + dz^2) + (1 - 2\Omega)dt^2 = ds^2 \quad \ldots\ldots\ldots(10)$$

where Ω is approximately Newtonian potential for rotating fluid. We now put approximate Newtonian potential for Ω in the above line element and we can get Newtonian approximation of the field. Further there is Newtonian approximation of hydrodynamic equation as proved by Eddington.

In the RHS of Einstein's equation we get ω_1 the angular velocity. Hence it is not possible to solve the Einstein's field equations even approximately, because of $T_{\mu\nu}$'s in RHS.

Multiplying equation (8) by $g_{\mu\nu}$ we get

$$R^{\mu\nu} g_{\mu\nu} - \frac{1}{2} g^{\mu\nu} g_{\mu\nu} R = 4\pi\rho_0 g^{\mu\nu} g_{\mu\nu} + 4\pi P \left[4 g_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} - g^{\mu\nu} g_{\mu\nu} \right]$$

$$R - \frac{1}{2} \cdot 4 \cdot R = 4\pi\rho_0 \cdot 4 + 4\pi P [4 \cdot 1 - 4] \because g_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = 1$$

$$R = -16\pi\rho_0$$

Now the 4-4 equation gives

$$R_4^4 - \frac{1}{2} g_4^4 R = 4\pi\rho_0 g_4^4 + \eta_4^4$$

$$\eta_4^4 = g_{4\alpha} \eta^{4\alpha} = g_{44} \eta^{44} = g_{44} 4\pi P \left[4 g_{44} \frac{dx^4}{ds} \frac{dx^4}{ds} - g^{44} \right]$$

$$= 4\pi P \left[4 g_{44} \left(\frac{dt}{ds} \right)^2 - g^{44} g_{44} \right]$$

$$= 4\pi P [4 \cdot 1 - 1] \because g_{44} \left(\frac{dt}{ds} \right)^2 = 1$$

$$= 12\pi P$$

$$R_{44} - \frac{1}{2} g_{44} R = 4\pi\rho_0 g_{44} + \eta_{44}$$

$$R_{44} + g_{44} 8\pi\rho_0 = -4\pi\rho_0 + \eta_{44}$$

$R_{44} \approx -4\pi\rho_0$, which is approximately Newtonian.

Now 1-1 equation

$$\eta^{11} = 4\pi P \left[4 \frac{dx^1}{ds} \frac{dx^1}{ds} - g^{11} \right] \text{and } R_{11} - \frac{1}{2} g_{11} R = 4\pi\rho_0 g_{11} + \eta_{11}$$

The RHS of the equation will be of the order of $4\pi\rho_0$ because P is very small as compared to ρ so that in approximation for η_μ^ν can be neglected. This will give

$$R_{11} - \frac{1}{2} g_{11} R = 4\pi\rho_0 g_{11}$$

$$R_{11} - \frac{1}{2} g_{11}\left(-16\pi\rho_0\right) = 4\pi\rho_0 g_{11}$$

$$R_{11} = -4\pi\rho_0 \because g_{11} = -(1 + 2\Omega) \approx -1$$

$$-\frac{1}{2}\nabla^2 g_{11} = -\frac{1}{2}\nabla^2(1 + 2\Omega)(-1)$$

$\approx \nabla^2\Omega \approx -4\pi\rho_0$ This is approximately Newtonian if Ω is Newtonian potential.

Similarly we can find out R_{22}, R_{33} and show that they satisfy the condition of Newtonian approximation.

RESULT AND DISCUSSION: In Einstein's theory rotation is impossible without the cross term $2\omega r^2 \text{Sin}^2\theta \cdot d\phi dt$ or Cartesian equivalent $2\omega y dx dt - 2\omega x dy dt$. Hence the field is not Newtonian and differs from Newtonian field by a factor of $\frac{\omega^2}{\rho}$ which is of the order of Newtonian ellipticity. On the other hand, with new field equation we can work with a diagonal metric of the form prescribed by Eddington[3] i.e. $-(1 + 2\Omega)(dx^2 + dy^2 + dz^2) + (1 - 2\Omega)dt^2 = ds^2$ where Ω is approximately Newtonian potential for rotating fluid. We now put approximate Newtonian potential for Ω in the above line element and we can get Newtonian approximation of the field. Further there is Newtonian approximation of hydrodynamic equation $T^{ik}_{,k} = 0$ as proved by Eddington. The new field equations, both field and hydrodynamics have Newtonian approximation and hence the shape of the rotating fluid will also have Newtonian approximation.

In the RHS of Einstein's equation we get ω_1 the angular velocity. Hence it is not possible to solve the Einstein's field equations even approximately because of $T_{\mu\nu}$'s in RHS. Hence if we use $T^{ik}_{,k} = 0$, as a separate equation and get the solutions.

ω_1 is the actual angular velocity and ω is just a parameter in the line element. For the sake of completeness and for the sake of convenience of the reader we are reproducing in the following.

The equations $T^{ik}_{,k} = 0$ are given below. (Using approximations)

$$\frac{\partial P}{\partial x} + \rho\frac{\nu_x}{2} + 2\rho\omega\omega_1 x - \rho\omega_1^2 x = 0 \qquad (\mu = 1)$$

$$\frac{\partial P}{\partial y} + \rho\frac{\nu_y}{2} + 2\rho\omega\omega_1 y - \rho\omega_1^2 y = 0 \qquad (\mu = 2)$$

$$\frac{\partial P}{\partial z} + \rho\frac{\nu_z}{2} = 0 \qquad (\mu = 3)$$

$$\frac{\partial P/\partial x}{\partial P/\partial y} = \frac{\mu_x}{\mu_y} = \frac{\nu_x}{\nu_y} = \frac{x}{y}$$

Adding above equations

$$\left[\frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy + \frac{\partial P}{\partial z}dz\right] + \frac{\rho}{2}\left[\frac{\partial \nu}{\partial x}dx + \frac{\partial \nu}{\partial y}dy + \frac{\partial \nu}{\partial z}dz\right]$$

$$+ 2\rho\omega\omega_1(xdx + ydy) - \rho\omega_1^2(xdx + ydy) = 0$$

Integrating above equation

$$P + \frac{\rho}{2}\nu + 2\rho\omega\overline{\omega}_1\left[\frac{x^2 + y^2}{2}\right] = C$$

Where 'C' is constant of integration and $\overline{\omega}_1 = \left(\omega_1 - \frac{\omega_1}{2\omega}\right)$

$$P + \frac{\rho}{2}\left[2\pi\rho - 4\pi\rho\frac{\omega_1}{\omega} - \frac{\omega^2}{2} + 2\omega\overline{\omega}_1\right]\left(x^2 + y^2\right) + \left[-\frac{\omega^2\rho}{2} + 4\pi\rho^2\frac{\omega_1}{\omega}\right]z^2 = C$$

The constant of integration can be determined by boundary conditions. At the equator, $z = 0$ and $x^2 + y^2 + z^2 = a^2$ similarly $P = 0$ at the boundary of the fluid.

$$\therefore C = \frac{\rho}{2}\left[2\pi\rho - 4\pi\rho\frac{\omega_1}{\omega} - \frac{\omega^2}{2} + 2\omega\overline{\omega}_1\right]a^2$$

$$P = \frac{\rho}{2}\left[2\pi\rho - 4\pi\rho\frac{\omega_1}{\omega} - \frac{\omega^2}{2} + 2\omega\overline{\omega}_1\right]\left[a^2 - \left(x^2 + y^2\right)\right] - \left[-\frac{\omega^2\rho}{2} + 4\pi\rho^2\frac{\omega_1}{\omega}\right]z^2$$

If pressure 'P' is assumed to be spherically symmetric then boundary surface is sphere, since $P = 0$ at the boundary (i.eat $r = a$).Equating coefficients of $\left(x^2 + y^2\right)$ and z^2 we get:

$$1 - 6\left[\frac{\omega_1}{\omega}\right] + \frac{\omega^2}{4\pi\rho} + \frac{\omega\omega_1}{\pi\rho} - \frac{\omega_1^2}{2\pi\rho} = 0$$

$$\approx 1 - 6\left[\frac{\omega_1}{\omega}\right] = 0$$

Exterior solutions are not the same in Einstein's and with new field equation. In Einstein's theory one gets the Kerr solution[5] as Exterior solution and with new field equation for rotation it is the same as Wyman's exterior solution[6]. When one puts Ω as Newtonian potential in the metric. For static sphere these are as given below.

$e^{\mu} = 1 + 2\Omega$ and $e^{\nu} = 1 - 2\Omega$

But for rotation we use Ω as Newtonian potential for rotating fluid.

For determining the shape of the fluid in Einstein theory Kelkaret al[2] used an ingenious method. They take 'ω' as parameter appearing in the interior in the '$g_{\mu\nu}$', but take 'ω_1' as the actual angular velocity of the fluid which appears in '$T_{\mu\nu}$'. Then they find the relationship between 'ω' and 'ω_1'for spherically symmetric solutions. They have taken '$d\phi dt$' term only in the interior metric, as without this term there is no rotation. All the '$T_{\mu\nu}$'s are zero in the exterior region, exterior solution has been determined on the basis of symmetry and boundary conditions. Symmetry will naturally depend upon the shaper of the fluid. If the fluid is spherical in shape then exterior solution is bound to be spherically symmetric.

CONCLUSION: Einstein's field equation shows dependence of motion on the field and field on the motion. Einstein's equation has field on one side and motion on the other side. In the new equation the field depends very slightly on motion through $\eta_{\mu\nu}$.The field depends on ρ_0 . The motion or hydrodynamics is given by a separate equation $T^{ik}_{,k} = 0$

There is no need of '$d\phi dt$' term in the theory of rotation as shown here. We have the simple line element $-(1+2\Omega)(dx^2+dy^2+dz^2)+(1-2\Omega)dt^2 = ds^2$ and this gives the field which can be approximated to Newton's field equation and the equation $T^{ik}_{,k} = 0$ approximates Newtonian hydrodynamics when Ω is Newtonian potential. So rotation of a star is explained in a very simple manner just like Newton giving same unique ellipticity of shape by Newton's theory and is also confirmed by observation.

We can use diagonal line element of the form given by Eddington and get Newtonian approximation for both the field equation and hydrodynamic equation and hence obtain observed ellipticity which is uniquely given by Newton's theory.

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