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Research Paper

Prime Labeling of Some new Classes of Graphs

Mathematics

KEYWORDS : Prime labeling, splitting graph, Corona Cn and $k_{1,2}$

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ABSTRACT

Graph labeling is an important area of research in Graph theory. There are many kinds of graph labeling such as Graceful labeling, Magic labeling, Prime labeling, and other different labeling techniques. In this paper the Prime labeling of certain classes of graphs are discussed. The splitting graph of a graph G corona of Cycle graph Cn and A stepwise algorithm is given to prove that both these classes of graphs satisfy prime labeling.

1. INTRODUCTION

Labeling of a graph G is an assignment of integers either to the vertices or edges or both subject to certain conditions [2,3]. A dynamic survey on graph labeling is regularly updated by Gallian [1] and it is published in electronic journal of combinatorics. The different kinds of the graphs are studied by us are found in [5, 7, 8]. A Graph G = G(V,E) with V vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not to exceeding V such that the labels of each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a prime graph [6].

Definition 1.1

Let $G = (V(G), E(G))$ be a graph with p vertices. A bijection $f : V(G) \to \{1, 2, \ldots, p\}$ is called a prime labeling if for each edge e = uv, gcd(f (u), $f(v)$) = 1. A graph which admits prime labeling is called a prime graph.

The notion of a prime labeling was introduced by Roger Entringer and was discussed in a paper by (Tout, A,1982, p.365-368). Many researchers have studied the prime graphs. For e.g. in (Fu, H., 1994, p.181-186) have proved that path p_n on n vertices is a prime graph. In (Deretsky, T., 1991, p.359-369) have proved that the cycle *Cn* on *n* vertices is a prime

Definition 1.2: The graph corona of Cn and $k_{1,2}$ is obtained from a cycle Cn by introducing 2 new pendant edges at each vertex of cycle.

Definition 1.3 : The splitting graph of a graph *G* is obtained by adding to each vertex V a new vertex V' such that V' is adjacent to every vertex that is adjacent to V in *G*, i.e. $N(V) = N(V')$ The resultant graph is denoted by $S'(G)$

MAINRESULTS

Theorem:2.1 Corona $C_n(n \ge 3)$ and $K_{1,2}$ is Prime

Proof: In C_n $n = 3$ We have labeled 1,2,3

When Pendent vertex joined with Even number 2 We labeled Pendent vertex with odd number 5,7

When pendent vertex joined with odd number 3 we labeled pendent vertex with even number 4,8

And remaining number 6&9 join with 1

In C_n (When n=6,7,8,9,10,11) We have labeled 1,2,3,.........11,12 Removing 6

In C_n (When n=12,13,.......17)wehave labeled1,2,3,4,..........17,18,19

Removing 6,12Generally in C_n the integers are occurs in order other then multiple of 6 so in this patter n

- For C_n n=2 join with odd number satisfying the condition of prime labels
- For C_n n=3 join with even number 4,8 satisfying the condition of prime labeling
- Rest of number join with 1So Corona $C_n(n \ge 3)$ and $K_{1,2}$ is Prime

Theorem:2.2 Theorem: If $n \ge 4$ even integers and $n \in n$ then disjoint union of Gear graph G_{n1} and p_{n2} is prime

Proof: Let $V(G_{n1}) = \{c, u_{i,} v_i / 1 \le i \le n_1\}$

$$E(G_{n1}) = \{cu_{i,} u_i v_i / 1 \le i \le n_1\} \cup \{v_i v_{i+1} / 1 \le i \le n -1 \cup \{v_{n1} v_1\}$$

Let $w_{1,} w_{2,} w_{3,} \ldots\ldots\ldots w_{n2,}$ be the consecutive vertices of p_{n2}

Let G' be the disjoint union of G_{n1} and p_{n2}

Now we define $f : v \to \{1, 2, 3 \ldots\ldots |v|\}$ as follows

$f(c) = p$

$f(v_1) = 1$

$f(u_i) = 2i \quad i = 1, 2, 3 \ldots. n_1$

$f(v_i) = 2i + 1 \quad i = 2, 3 \ldots. n_1$

$f(w_j) = 2n_1 + 1 + j \quad j = 1, 2, 3 \ldots. n_2$

Now prove that f is a prime labeling

Then f is an injection function

For an arbitrary Edge e=ab we prove that

$\gcd(f(a), f(b)) = 1$

Now if e is an edge for the Gear graph G_{n1} there are following possibilities

$if\ e = cu_i'\ then\ \gcd(f(c), f(u_i')) = \gcd(1, 2i) = 1$

$if\ e = u_i v_i\ then \gcd(f(u_i), f(v_i)) = \gcd(2i, 2i+1) = 1$

$if\ e = v_i v_{i+1}\ then\ \gcd(f(v_i), f(v_{i+1})) = \gcd(2i+1, 2i+2) = 1$

$if\ e = v_n v_1\ then \gcd(f(v_n), f(v_1)) = \gcd(2n_1 + 1, 1) = 1$

Now if e is an edge of p_{n2} there are following possibilities

$e = w_j w_{j+1} = \gcd(f(w_j), f(w_{j+1})) = (2n_1 + 1 + j, 2n_1 + 2 + j)$
$= 1$

so each of the possibilities f admits prime labeling

so f is a prime graph

Theorem:2.3 Split graph $s(p_n)$ is prime

Let G be p_n The vertices of p_n are $v_1, v_2, v_3, ...v_n$ Then $S(G)$ has vertices $v_1, v_2, v_3, ...v_n, v_1', v_2', v_3', ...v_n'$

Now we define $f : V(S(G)) \to \{1, 2, 3......|v|\}$ as follows

$f(v_1) = 1$

$f(v_i) = 4i - 1 \quad i = 2, 3.....n \quad i \neq 1(\bmod 3)$

$f(v_i') = 2i \quad i = 1, 2, 3........n$

Now if e is an edge for the given graph $S(G)$ there are following possibilities

$if\ e = v_i v_{i+1} \gcd(f(v_i), f(v_{i+1})) = \gcd(4i - 1, 4i + 3) = 1$

$i \neq 1(\bmod 3)$

$if\ e = v_i v_{i+1}' \gcd\left(f(v_i), f\left(v_{i+1}'\right)\right) = \gcd(4i - 1, 2i + 2)$

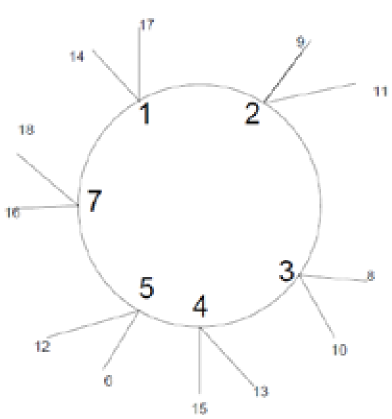
$= 1$

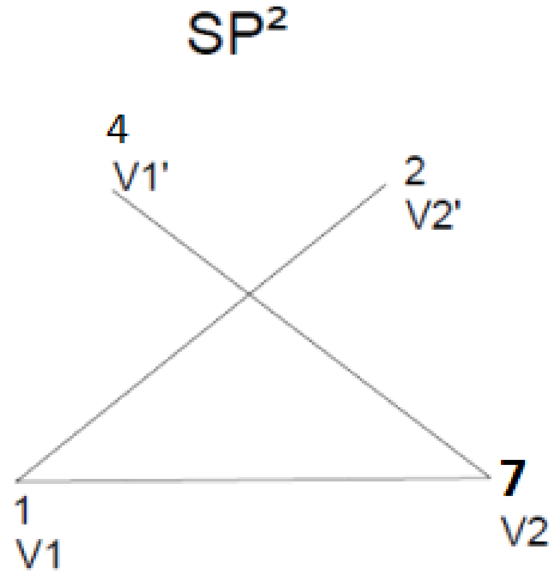
$e = v_i v_{i-1}' \gcd\left(f(v_i), f\left(v_{i-1}'\right)\right) = \gcd(4i - 1, 2i - 2) = 1$

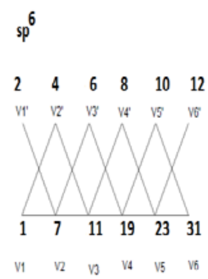
$i = 2, 3......n$

So each of the possibilities f admits prime labeling so f is a prime graph

Corona of c_5 and K_{12}







CONCIUSIONS:

Study of relative prime numbers is very interesting in the theory of numbers and it challenging to the investigate prime labeling of some families of graphs here we are investigate several results of some class of graph about prime labeling extending the study of other graph families is an open area of research.

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