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An Optimal Repair Replacement Model for A Repairable System Using Geometric Process Exposing to Weibull Failure Law

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ABSTRACT

This paper applies the geometric process repair model exposing to Weibull failure law to a bivariate replacement policy for a two unit identical component cold standby repairable system with one repairman. It assumed that the successive working times forms decreasing geometric process, while the successive repair times forms an increasing geometric process and both the process are exposing to Weibull failure law. Under these assumptions, an expression for the long-run average cost per unit time is derived and determines optimal number of failures such that the long run average cost per unit time is minimized.

1. Introduction

Most repair replacement models discussed earlier are mainly concentrated on the study of the model for a one-component system and it is usually assumed that the system after repair is 'as good as new'. But however, it is not always true for a deteriorating system. That is, the system after repair cannot be 'as good as new'. Under this assumption, a minimal repair model, in which the minimal repair does not change the age of the system. It was first proposed by Barlow and Hunter [1]. Lot of research works have been carried out by Phelps [8], Block et al [2] and others along perfect repair and minimal repair, however a more reasonable model is the geometric repair model.

Lam [6,7] studied two kinds of replacement policy one based on the working age T of the system and the other based on the failure number N of the system. The explicit expressions of the long-run average loss per unit time under these two kinds of policy can be evaluated and the corresponding optimal replacement policies T* and N* can be found analytically. Stadje and Zuckerman [10] developed a general monotone process repair model to generalize Lam's work. Under some mild conditions, they showed the optimal policy N* was better than the optimal policies T*. Zhang [13] generalize Lam's work by a bivariate replacement policy (T,N). Under which the system is replaced at the working age T or at the time of the N^{th} failure, which ever occurs first and showed that the optimal policy (T,N)* is better than the optimal policy N*. Other replacement policies under the geometric process repair model are reported by Stadge and Zuckerman [11], Stanley [12], Leung and Lee [7], Zhang et al [16] and Zhang [13,14].

The researchers mentioned above pointed to one-component repairable system. However, in practice for improving the reliability of the system, the standby techniques are generally used. Zhang [15] applied the geometric process repair model to two identical-component cold standby repairable system with one repairman and assumed that each component after repair is not 'as good as new'. He considered a policy N based on the number of repairs of component 1 and by using the geometric process determined the optimal policy N* such that the long-run expected reward per unit time is maximized.

The purpose of this paper is to apply the geometric process repair model exposing to Weibull failure law to a bivariate replacement policy for a two unit identical component cold standby repairable system with one repairman. It assumed that the successive working times forms decreasing geometric process, while the successive repair times forms an increasing geometric process and both the process are exposing to Weibull failure law. Under these assumptions, an expression for the long-run average cost per unit time is derived and determine optimal number of failures such that the long run average cost per unit time is minimized.

2. The Model

In this section, we develop a model for a bivariate replacement policy (T,N) and an optimal replacement policy N for two identical component cold standby repairable system with one repairman using geometric process and exposing to Weibull failure law, under the following assumptions.

ASSUMPSTIONS

1. At the beginning, both components in the system are good. Component 1 is in working state while component 2 is in cold standby state.

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2. If the working component fails immediately it is repaired and at the same time the standby one begins to work. As soon as the repair of the failed one is completed it either begins to work again or becomes cold standby. If one fails and the other is still under repair then the system breaks down. When system breaks down, immediately it is replaced and replacement time is negligible.

3. Let $X_n^{(i)}$ *and* $Y_n^{(i)}$, i=1,2,; n=1,2,3,…. are all independent and denotes working time and repair time respectively.

4. Let the sequence $\left\{X_n^{(i)}, n=1,2,...\right\}$ form a decreasing geometric process, exposing to decreasing Weibull failure law with parameter a $\geq$ 1.

5. Let the sequence $\left\{Y_n^{(i)}, n=1,2,...\right\}$ form an increasing geometric process, exposing to an increasing Weibull failure law with parameter 0<b<1.

6. Let the time interval between the completion of the (n-1)th repair and the nth repair on component I is called nth cycle of component i, for i=1,2 and n=1,2,3…., and the inter arrival time between two consecutive repair times is called a renewal cycle.

7. A component in the system can't produce working reward during cold standby state and no cost is incurred during waiting period for repair.

8. The repair cost of two components are both C_r, the working cost of two components are both C_w, and the replacement cost of the system is C.

In the next section, we discuss an optimal solution for a bivariate replacement policy (T,N) and an optimal replacement policy N, based on the above assumptions.

3. Optimal Solution

In this section, we develop an optimal solution for a bivariate replacement policy (T,N) under which we replace the system when working age of component 1 reaches T and number of failure of the component 1 reaches N and an optimal replacement policy N under which we replace the system when number of failures of the component 1 reaches N for cold standby repairable system using geometric process and exposing to Weibull failure law.

Let C(T,N) be the long-run average cost per unit time under the policy (T,N). Thus according to the renewal reward theorem of Ross [54], the long run average cost per unit time is:

$$C(T,N) = \frac{\text{The expected cost incurred in a renewal cycle}}{\text{The expected length of a renewal cycle}}. \tag{3.1}$$

Let L be the length of a renewal cycle of the system under policy (T.N). Then

$$L = L_1 I_{\{U_N > T\}} + L_2 I_{\{U_N \leq T\}} \tag{3.2}$$

Where

$$L_1 = T + \sum_{n=1}^{k} Y_n^{(1)} + \sum_{n=2}^{k}\left(Y_{n-1}^{(2)} - X_n^{(1)}\right) I_{\left\{Y_{n-1}^{(2)} - X_n^{(1)} > 0\right\}} + \sum_{n=1}^{k}\left(X_n^{(2)} - Y_n^{(1)}\right) I_{\left\{X_n^{(2)} - Y_n^{(1)} > 0\right\}} \tag{3.3}$$

where T= total working time under policy T, second term refers repair time, third term refers waiting length of repair while the fourth term refers cold standby time of component 1 and

$$L_2 = \sum_{n=1}^{N} X_n^{(1)} + \sum_{n=1}^{N-1} Y_{n-1}^{(1)} + \sum_{n=2}^{N}\left(Y_{n-1}^{(2)} - X_n^{(1)}\right) I_{\left\{Y_{n-1}^{(2)} - X_n^{(1)} > 0\right\}} + \sum_{n=1}^{N-1}\left(X_n^{(2)} - Y_n^{(1)}\right) I_{\left\{X_n^{(2)} - Y_n^{(1)} > 0\right\}} + X_N^{(2)}, \tag{3.4}$$

where the first, second, third, fourth and fifth terms are respectively working time, repair time, waiting time for repair, cold standby time of component 1 under policy N and working time of component 2 under policy N, while

I is the indicator function such that

I_A = 1, if event A occurs

= 0, if event a doesn't occurs.

Now the expected length of a renewal cycle L can be evaluated as follows:

$$E(L) = E\left[L_1 I_{\{U_N > T\}}\right] + E\left[L_2 I_{\{U_N \le T\}}\right]. \tag{3.5}$$

Let $E\left[L_1 I_{\{U_N > T\}}\right] = E\left[L_1\right].E\left[I_{\ U_N > T}\right]$

$$= \left[T + \sum_{n=1}^{k} Y_n^{(1)} + \sum_{n=2}^{k}\left(Y_{n-1}^{(2)} - X_n^{(1)}\right) I_{\left\{Y_{n-1}^{(2)} - X_n^{(1)} > 0\right\}} + \sum_{n=1}^{k}\left(X_n^{(2)} - Y_n^{(1)}\right) I_{\left\{X_n^{(2)} - Y_n^{(1)} > 0\right\}}\right] E\left[I_{\ U_N > T}\right]$$

$$= E[T]E\left[I_{\ \{U_N > T\}}\right] + E\left[\sum_{n=2}^{k}\left(Y_{n-1}^{(2)} - X_n^{(1)}\right) I_{\left\{Y_{n-1}^{(2)} - X_n^{(1)} > 0\right\}}\right] E I_{\{U_N > T\}}$$

$$+ E\left[\sum_{n=1}^{k}\left(X_n^{(2)} - Y_n^{(1)}\right) I_{\left\{X_n^{(2)} - Y_n^{(1)} > 0\right\}}\right] E\left[I_{\ \{U_N > T\}}\right] + E\left[\sum_{N=1}^{K} Y_n^{(1)}\right] E\left[I_{\ \{U_N > T\}}\right]. \tag{3.6}$$

This can be evaluated by partly using the following lemma

Lemma : Let $U_N = \sum_{i=1}^{n} X_i^{(1)} = U_n + W_{N-n},\ n = 1, 2, ...N-1$ then the expectation of indicator function $I\{U_n \le T < U_N\}$ is given by

$$I\{U_n < T < U_N\} = \int_0^T \bar{F}_{N-n}(a^n(T-t)dF_n(t) = F_n(T) - F_N(T), \text{ where n=1,2,....N-1.}$$

Where T and N are respectively the working age and the failure number of component 1 (See Leung [33]).

$$E\left[TI_{\{U_N>T\}}\right] = T.E\left[I_{\{U_N>T\}}\right] = T\bar{F}_N(T) \tag{3.7}$$

$$E\left[\sum_{n=2}^{K}\left[\left(Y_{n-1}^{(2)} - X_n^{(1)}\right)I_{\left\{Y_{n-1}^{(2)}-X_n^{(1)}>0\right\}}\right]\right]E\left[I_{\{UN>T\}}\right]$$

$$= \sum_{n=2}^{N-1}\int_0^{\infty} ug_n(u)du\left[F_n(T) - F_N(T)\right]$$

$$\sum_{n=2}^{N-1}\int_0^{\infty} ug_n(u)du\ F_n(T) - \sum_{n=2}^{N-1}\int_0^{\infty} ug_n(u)du\ F_N(T). \tag{3.8}$$

Similarly

$$E\left[\sum_{n=1}^{K} Y_n^{(1)} I_{\{U_N>T\}}\right] = \sum_{n=1}^{N-1} E\left(Y_n^{(1)}\right)F_n(T) - \sum_{n=1}^{N-1} E\left[Y_n^{(1)}\right]F_N(T). \tag{3.9}$$

$$\sum_{n=1}^{K} E\left[\left(X_n^{(2)} - Y_n^{(1)}\right)I_{\{U_N>T\}}\right] = \sum_{n=1}^{N-1} E\left(X_n^{(2)} - Y_n^{(1)}\right)E\left[I_{\{U_N>T\}}\right].$$

$$= \sum_{n=1}^{N-1}\int_0^{\infty} vg_n(v)dv\left[F_n(T) - F_N(T)\right]$$

$$= \sum_{n=1}^{N-1}\int_0^{\infty} vg_n(v)dv\ F_n(T) - \sum_{n=1}^{N-1}\int_0^{\infty} vg_n(v)dv\ F_N(T) \tag{3.10}$$

Let $E\left[L_2 I_{\{U_N \le T\}}\right] = E\left[\sum_{n=1}^{N} Y_n^{(1)} + \sum_{n=1}^{N-1} Y_n^{(1)} + \sum_{n=2}^{N}\left(Y_{n-1}^{(2)} - X_n^{(1)}\right)I_{\left\{Y_{n-1}^{(2)}-X_n^{(1)}\right\}} + X_N^{(2)}\right.$

$$\left. + \sum_{n=1}^{N-1}\left(X_n^{(2)} - Y_n^{(1)}\right)I_{\left\{Y_n^{(2)}-X_n^{(1)}>0\right\}}\right].E\left[I_{\{U_N \le T\}}\right] \tag{3.11}$$

This also can be evaluated by partly as follows:

$$E\left[\sum_{n=1}^{N} X_n^{(1)} I_{\{U_N<T\}}\right] = \sum_{n=1}^{N} E\left[X_n^{(1)}\right]F_N(T) \tag{3.12}$$

$$E\left[\sum_{n=1}^{N-1} Y_n^{(1)} I_{\{U_N \le T\}}\right] = \sum_{n=1}^{N-1} E\left[Y_n^{(1)}\right] F_N(T) \tag{3.13}$$

$$E\left[\sum_{n=2}^{N}\left(Y_{n-1}^{(2)} - X_n^{(1)}\right) I_{\left\{Y_{n-1}^{(2)} - X_n^{(1)} > 0\right\}}\right] . E\left[I_{\{U_N < T\}}\right] = \sum_{n=2}^{N}\int_0^\infty u g_n(u) du \; F_N(T) \tag{3.14}$$

$$E\left[\sum_{n=1}^{N-1}\left(X_n^{(2)} - Y_n^{(1)}\right) I_{\left\{X_n^{(2)} - Y_n^{(1)} > 0\right\}}\right] . E\left[I_{\{U_N \le T\}}\right] = \sum_{n=1}^{N-1}\int_0^\infty v g_n(v) dv \; F_N(T) \tag{3.15}$$

$$E\left[X_N^{(2)} I_{\left\{U_N \le T\right\}}\right] = E\left[X_N^{(2)}\right] . F_N(T) \tag{3.16}$$

Using equation (3.12) to (3.16), we have:

$$E\left[L_2 I_{\{U_N \le T\}}\right] = \sum_{n=1}^{N} E\left[X_n^{(1)}\right] F_N(T) + \sum_{n=2}^{N}\int_0^\infty u g_n(u) du \; F_N(T) + \sum_{n=1}^{N-1}\left[E\left(Y_n^{(1)}\right) + \int_0^\infty v g_n(v) dv\right] F_N(T) + E\left(X_N^{(2)}\right) F_N(T) . \tag{3.17}$$

Using equation (3.7) to (3.12), we have:

$$E\left[L_1 I_{\{U_N > T\}}\right] = T\overline{F}_N(T) + \sum_{n=1}^{N-1} E\left(Y_n^{(1)}\right)\left[F_n(T) - F_N(T)\right] + \sum_{n=2}^{N-1}\int_0^\infty u g_n(u) du \left[F_n(T) - F_N(T)\right] + \sum_{n=1}^{N-1}\int_0^\infty v g_n(v) dv \left[F_n(T) - F_N(T)\right]. \tag{3.18}$$

From equations (3.18) and (3.20), equation (3.5) becomes:

$$E[L] = T\overline{F}_N(T) + \sum_{n=1}^{N-1} E\left(Y_n^{(1)}\right) F_n(T) + \sum_{n=2}^{N-1}\int_0^\infty u g_n(u) du \; F_n(T) + \int_0^\infty u g_N(u) du F_N(T) + \sum_{n=1}^{N-1}\int_0^\infty v g_n(v) dv F_n(T) + E\left(X_N^{(2)}\right) F_N(T) + \sum_{n=1}^{N} E\left(X_n^{(1)}\right) F_N(T). \tag{3.19}$$

Since the failure number of component 1 reaches N, component 2 is in cold standby state in the N^{th} cycle or in repair state in the $(N-1)^{th}$ cycle.

Let S and $\overline{S}$ be such an event that component 2 is the cold standby state and repair state respectively when the working age of component 1 reaches T ie., under policy T. And the expectations of indicator functions Is and $I_{\overline{S}}$ are respectively given by:

$$E\left[Is\right]=P(S)=P;P(I_{\overline{S}})=P(\overline{S})=q;0<p<1$$

Thus, according to equations (3.1) and (3.19), we have :

$$C(T,N)=\left[C_rE\left[\left[\sum_{n=1}^{K}\left(Y_n^{(1)}+Y_n^{(2)}\right)I_{\{U_N>T\}}\right]I_S+\left[\left(\sum_{n=1}^{k}Y_n^{(1)}+Y_n^{(2)}-\overline{Y}_k^{(2)}\right)I_{\{U_N>T\}}\right]I_{\overline{S}}\right]\right.$$

$$+C_rE\left[\sum_{n=1}^{N-1}\left(Y_n^{(1)}+Y_n^{(2)}\right)I_{\{U_N\le T\}}\right]+C$$

$$\left.+C_wE\left[\left(T+\sum_{n=1}^{k}X_n^{(2)}\right)I_{\{U>T\}}+\left(\sum_{n=1}^{N}X_n^{(1)}+\sum_{n=1}^{N-1}X_n^{(2)}\right)I_{\{U_N\le T\}}\right]\right]\div E\left[L\right]$$

$$C(T,N)=\left\{2C_rE\left[\sum_{n=1}^{K}Y_n^{(1)}I_{\{U_N>T\}}\right](p+q)-C_rqE\left[\overline{Y}_K^{(2)}I_{\{U_N>T\}}\right]\right.$$

$$+2C_rE\left[\sum_{n=1}^{N-1}Y_n^{(1)}I_{\{U_N>T\}}\right]+C$$

$$-C_W\left[E\left(TI_{\{U_N>T\}}\right)+E\left(\sum_{n=1}^{K}X_n^{(2)}I_{\{U_N>T\}}\right)\right.$$

$$\left.\left.+E\left(2\sum_{n=1}^{N}X_n^{(1)}-X_n^{(2)}\right)I_{\{U_N>T\}}\right]\right\}\div E\left[L\right] \tag{3.20}$$

The C(T,N) which is a bivariate function of T and N.

Where $\overline{Y}_n^{(2)}$ is excess repair time of component 2 in Kth cycle. When the working age of component 1 reaches T, the component 2 is either the repair state in the Kth cycle or cold standby state in the (K+1)th cycle. Thus

$$\overline{Y}_K^{(2)}=Y_K^{(2)}-\left(T-\sum_{n=1}^{K}X_n^{(1)}\right)$$

$$=Y_K^{(2)}+\sum_{n=1}^{K}X_n^{(1)}-T \tag{3.21}$$

To evaluate the expected value of excess repair time, we determine the following probability mass function of K.

$$P\left[K \geq k\right] = P\left[\sum_{n=1}^{K} X_n^{(1)} < T\right] = F_K\left(T\right) \quad (3.22)$$

Where $F_K\left(T\right)$ is distribution function of $\sum_{n=1}^{K} X_n^{(1)}$. Now

$$P\left[K = k\right] = P\left[K \geq k\right] - P\left[K \geq k+1\right]$$

From equation (3.22), we have:

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$$P\left[K = k\right] = F_K\left(T\right) - F_{K+1}\left(T\right) \quad (3.23)$$

From equation (3.21), we have:

$$E\left[Y_k^{(2)} I_{\{U_N > T\}}\right] = E\left[\left(Y_k^{(2)} + \sum_{n=1}^{K} X_n^{(1)} - T\right) I_{\{U_N > T\}}\right] \quad (3.24)$$

The equation (3.24) can be evaluated by partly.

By definition of conditional expectation:

$$E\left[Y_k^{(2)} I_{\{U_N > T\}}\right] = E\left[E\left(Y_k^{(2)} I_{\{U_N > T\}}\right) / K\right]$$

$$= \sum_{n=0}^{N} E\left[Y_k^{(2)} I_{\{U_N > T\}} / K = n\right] P\left(K = n\right)$$

$$= \sum_{n=0}^{N} E\left[Y_k^{(2)} I_{\{U_N > T\}}\right] F_n\left(T\right) - F_{n+1}\left(T\right)$$

$$= \overline{F}_N\left(T\right) \sum_{n=1}^{N} E\left(Y_k^{(2)}\right)\left[F_n\left(T\right) - F_{n+1}\left(T\right)\right] \quad (3.25)$$

$$E\left[\sum_{n=1}^{K} X_n^{(1)} I_{\{U_N > T\}}\right] = \sum_{n=1}^{N-1} E\left(X_n^{(1)}\right)\left[F_n\left(T\right) - F_N\left(T\right)\right] \quad (3.26)$$

According to equations (3.25) and (3.26), we have

$$E\left[\overline{Y}_K^{(2)} I_{\{U_N > T\}}\right] = \overline{F}_N\left(T\right) \sum_{n=1}^{N-1} E\left(Y_K^{(2)}\right)\left[F_n\left(T\right) - F_{n+1}\left(T\right)\right] +$$

$$\sum_{n=1}^{N-1} E\left(X_n^{(1)}\right)\left[F_n(T)-F_N(T)\right]-T\overline{F}_N(T) \tag{3.27}$$

Thus, according to equations (3.7) to (3.27), equation (3.20) becomes:

$$C(T,N)=\Bigg\{2C_r\sum_{n=1}^{N-1}E\left(Y_n^{(1)}\right)F_n(T)+C_rq\overline{F}_N(T)\sum_{n=1}^{N-1}\left(Y_n^{(2)}\right)\left[F_{n+1}(T)-F_n(T)\right]$$

$$+C+C_rq\left[\sum_{n=1}^{N-1}E\left(X_n^{(1)}\right)F_N(T)-\sum_{n=1}^{N-1}E\left(X_n^{(1)}\right)F_n(T)+T\overline{F}_N(T)\right]$$

$$-C_w\Bigg[T\overline{F}_n(T)+\sum_{n=1}^{N-1}E\left(X_n^{(2)}\right)F_n(T)-\sum_{n=1}^{N-1}E\left(X_n^{(2)}\right)F_N(T)$$

$$+\left(2\sum_{n=1}^{N}E\left(X_n^{(1)}\right)-E\left(X_n^{(2)}\right)\right)F_N(T)\Bigg]\Bigg\}\div E[L]. \tag{3.28}$$

Due to the assumptions of the model, $X_n^{(i)}$ *and* $Y_n^{(i)}$ the two random variables, denoting working time and repair time of component i in the n^{th} cycle, form a decreasing GP exposing to decreasing Weibull failure law and an increasing GP exposing to an increasing Weibull failure law respectively. We have:

$$E\left(X_n^{(i)}\right)=\int_0^\infty x dF_n(x_n)=\frac{\eta_1\Gamma\left(1+\frac{1}{\beta_1}\right)}{a^{n-1}}=\frac{\lambda}{a^{n-1}}, where\ \lambda=\eta_1\Gamma\left(1+\frac{1}{\beta_1}\right), i=1,2 \tag{3.26}$$

$$E\left(Y_n^{(i)}\right)=\int_0^\infty x dG_n(x_n)=\frac{\eta_2\Gamma\left(1+\frac{1}{\beta_2}\right)}{b^{n-1}}=\frac{\mu}{b^{n-1}}, where\ \mu=\eta_2\Gamma\left(1+\frac{1}{\beta_2}\right), i=1,2. \tag{3.27}$$

$$E\left[\left(Y_{n-1}^{(2)}-X_n^{(1)}\right)I_{\left\{Y_{n-1}^{(2)}-X_n^{(1)}>0\right\}}\right]=\frac{\eta_2\Gamma\left(1+\frac{1}{\beta_2}\right)}{b^{n-2}}=\frac{\mu}{b^{n-2}}\ \text{where}\ \mu=\eta_2\Gamma\left(1+\frac{1}{\beta_2}\right) \tag{3.28}$$

$$E\left[\left(X_n^{(2)}-Y_n^{(1)}\right)I_{\left\{X_n^{(2)}-Y_n^{(1)}>0\right\}}\right]=\frac{\eta_1\Gamma\left(1+\frac{1}{\beta_1}\right)}{a^{n-1}}=\frac{\lambda}{a^{n-1}}\ where\ \lambda=\eta_1\Gamma\left(1+\frac{1}{\beta_1}\right). \tag{3.29}$$

Using equations (3.26) to (3.29), we have:

$$C(T,N)=\Bigg\{2C_r\sum_{N=1}^{n-1}\frac{\mu}{b^{n-1}}F_n(T)+C_rq\overline{F}_N(T)\sum_{n=1}^{N-1}\frac{\mu}{b^{n-1}}\left[F_{n+1}(T)-F_n(T)\right]$$

$$+C+C_r q\left[\sum_{n=1}^{N-1}\frac{\lambda}{a^{n-1}}\left(F_N(T)-F_n(T)\right)+T\overline{F}_N(T)\right]$$

$$-C_w\left[T\overline{F}_n(T)+\sum_{n=1}^{N-1}\frac{\lambda}{a^{n-1}}\left(F_n(T)-F_N(T)\right)+\left(2\sum_{n=1}^{N}\frac{\lambda}{a^{n-1}}-\frac{\lambda}{a^{N-1}}\right)F_N(T)\right]$$

$$\div E[L]\,. \tag{3.30}$$

Where

$$E[L\ ]=T\overline{F}_N(T)+\sum_{n=1}^{N-1}\frac{\mu}{b^{n-1}}F_n(T)+\sum_{n=2}^{N-1}\frac{\mu}{b^{n-2}}F_n(T)+\frac{\mu}{b^{n-2}}F_N(T)+\sum_{n=1}^{N-1}\frac{\lambda}{a^{n-1}}F_n(T)$$

$$+\frac{\lambda}{a^{N-1}}F_N(T)+\sum_{n=1}^{N}\frac{\lambda}{a^{n-1}}F_N(T), \tag{3.31}$$

which is a bivariate function of T and N.

Observations

i) When Nis fixed C(T,N) is a function of T i.e., if N=m, then

C(T,N) = C_m(T), for m=1,2,3,….., when N=1,2,3,….., m,….. we can find $C_1\left(T_1^*\right), C_2\left(T_2^*\right),\ldots\ldots, C_m\left(T_m^*\right),\ldots..$ respectively in such way that the corresponding long-run average costs are minimized.

Since the total life time of the repairable system is limited, the minimum of the long-run average cost per unit time can be determined by comparing the values of $C_1\left(T_1^*\right), C_2\left(T_2^*\right),\ldots\ldots, C_m\left(T_m^*\right),\ldots..$, and it is denoted by

$$C(T,N)=C_n\left(T_n^*\right).$$

ii)

When
$$\left.\begin{array}{l} T\to\infty,\ \text{ then } L=L_2 and \\ \lim_{T\to\infty} I_{\{U_N>T\}}=0,\ \lim_{T\to\infty} I_{\{U_N\le T\}}=1 \\ \lim_{T\to\infty} F_n(T)=1,\ \lim_{T\to\infty} F_N(T)=1 \\ \lim_{T\to\infty}\overline{F}_N(T)=0 \end{array}\right\} \tag{3.32}$$

Based on the observation (ii), equation (3.30) becomes:

$$C(\infty,N)=C_2(N)=\frac{2C_r\sum_{n=1}^{N-1}\frac{\mu}{b^{n-1}}+C-C_w\left[2\sum_{n=1}^{N}\frac{\lambda}{a^{n-1}}-\frac{\lambda}{a^{N-1}}\right]}{E[L_2]}$$

$$=\frac{2C_r\mu\sum_{n=1}^{N-1}\frac{1}{b^{n-1}}+C-C_w\lambda\left[2\sum_{n=1}^{N}\frac{1}{a^{n-1}}-\frac{1}{a^{N-1}}\right]}{E[L_2]}$$

$$=\frac{2C_r\mu l_1+C-C_w\lambda\left[2l_2-\frac{1}{a^{N-1}}\right]}{E[L_2]}$$

From equation (3.17), we have:

Where $E[L_2\ ]=\sum_{n=1}^{N}\frac{\lambda}{a^{n-1}}+\sum_{n=1}^{N-1}\frac{\mu}{b^{n-1}}+\sum_{n=2}^{N}\frac{\mu}{b^{n-2}}+\sum_{n=1}^{N-1}\frac{\lambda}{a^{n-1}}+\frac{\lambda}{a^{N-1}}$

$$E[L_2]=2\mu l_1+2\lambda l_2$$

$$l_1=\sum_{n=1}^{N-1}\frac{1}{b^{n-1}},\ l_2=\sum_{n=1}^{N}\frac{1}{a^{n-1}},$$

$$\lambda=\eta_1\Gamma\left(1+\frac{1}{\beta_1}\right),$$

$$\mu=\eta_2\Gamma\left(1+\frac{1}{\beta_2}\right).$$

Which is average cost function of N, under policy N. i.e, as $T\to\infty$ $\mathrm{C}(\mathrm{T,N})=C_2(N)$ by the above analysis.

In the next section we provide numerical work to highlight the theoretical work. Using C(N) we determined an optimal replacement policy N* such that the long-run average cost is minimized analytically.

4. NUMERICAL RESULTS AND CONCLUSIONS

For given hypothetical values of a, b, Cw, C, Cr, λ, μ, η_1, β_1, η_2 and β_2 the optimal replacement policy N* is calculated as follows:

a=1.05, b=0.85, Cw=40, C=5000, Cr=40,

$\lambda=40,\ \mu$=4.43, η_1=20, $\eta_2=5,\ \beta_1=0.5,\ \beta_2=2$

TABLE 4.1: The long-run average cost C(N) against N

N	C(N)	N	C(N)
2	3.820591	27	19.856058
3	-7.77114	28	21.936447
4	-12.963908	29	23.877075

5	-15.603035	30	25.671776
6	-16.927744	31	27.318239
7	**-17.455545**	32	28.817526
8	-17.438393	33	30.1735
9	-17.012094	34	31.392231
10	-16.255558	35	32.481415
11	-15.21777	36	33.449837
12	-13.93144	37	34.306915
13	-12.42059	38	35.062309
14	-10.705065	39	35.725609
15	-8.803396	40	36.30611
16	-6.734647	41	36.812649
17	-4.51955	42	37.253487
18	-2.181118	43	37.636246
19	0.255208	44	37.967888
20	2.761806	45	38.254711
21	5.309461	46	38.502361
22	7.868212	47	38.715881
23	10.408365	48	38.899734
24	12.901563	49	39.057861
25	15.321796	50	39.193718
26	17.646273		

Conclusion:

From the tables 4.1 it was noticed that the C(7)= -17.4555 is the minimum of the long run expected cost per unit time of the system i.e, the optimal policy is N* = 7 and we should replace the system at the time of 7^{th} failure. Based on this understanding, it can be developed an optimal replacement policy for an improving system under certain conditions.

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