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Unification of Four Fundamental Forces of Nature Developing 7-Dimensional Metric on The Basis Of New Concept of Time

Physics

KEYWORDS : 7-d space-time continuum, 4-time components, changing parameter, Schwarzs-child-like solution, weakon, outward energy or hot energy, inward energy or cold energy.

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ABSTRACT

Simply present here a new concept of time on the basis of the four fundamental forces of nature. This gives us naturally a line element or metric in a simple way with simple mathematics and extended for 7-dimensional space-time continuum, where 3 usual space components and 4 time components instead of 1. For generalisation introduced here (3+4) dimensions in metric . Using Einstein's original field equation , for gravitational field of an isolated particle in empty space, the simplest solution was first given by Schwarzschild on the basis of 4-d space-time metric, but in here extended our view to the referred 7-d space-time continuum. The solution give us some new interesting results means naturally the four forces comes to the picture in the metric equation and gives the density of neutron stars etc. which agrees with other results.

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1. Introduction

It is likely that we will have a hard time in answering in a simple way, the fundamental question, 'what is time?'[1]. In philosophical and fundamental [2] discussions of special theory of relativity [3] it is claimed that relativistic time does not have the same status as time in classical physics: relativistic time is more strongly connected with physical phenomena than its classical counterpart. In 1905, Einstein unified [4] the concept of space and time using Lorentz transformation [5-8]. In 1916 publishing 'The general theory of relativity' Einstein showed that Newtonian gravity was a manifestation of the curvature of the space- time manifold. This audacious concept of a dynamic space-time led to spectacular advances in cosmology; predicting on the one hand an expansion of the universe and on the other hand predicting the 3K remnant of a big bang which signalled the 'birth' of the universe some 10^{10} years ago.

For flat space-time the line element according to relativity is,

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2 \qquad (1)$$

The equation (1) can be rewritten in most general form as,

$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + (dx^4)^2 \qquad (2)$$

Whereas $(-c^2dt^2) = (dx^4)^2$

The interval ds in general theory of relativity for Riemannian space is,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \qquad (\mu,\nu = 1,2,3,4) \qquad (3)$$

Einstein's original field equations representing the law of gravitation in empty space [5-9] is

$$R_{\mu\nu} = 0 \qquad (4)$$

Whereas $R_{\mu\nu}$ is known as the 4-d Ricci tensor.

The solution of above equations merely consists of finding the line element for empty space surrounding a gravitating point particle, which ultimately correspond to the field of an isolated particle continuously at rest at the origin. In the absence of any mass point the space-time would be flat so that the 4-d line element in spherical polar co-ordinates would be expressed as

$$ds^2 = -dr^2 - r^2dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2 + c^2dt^2 \qquad (5)$$

The presence of the mass point would modify the line element. However since mass is static and isolated, the line element would be spatially spherically symmetric about the point mass and is static.

The most general form of such a four dimensional line element considering the velocity c is taken to be unity in order to use as astronomical unit and may be expressed as,

$$ds^2 = -e^{\lambda} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + e^{\nu} dt^2 \qquad (6)$$

Where λ and ν are functions of *r* only; since for spherically symmetric isolated particle the field will depend on *r* alone and not on θ and ϕ.

Finally the line element due to static, isolated gravitating mass point is found

$$ds^2 = -\left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + \left(1 - \frac{2m}{r}\right) dt^2 \qquad (7)$$

The Schwarzschild [5-7, 10] is seen to have that the solution becomes singular at *r=0*; but this singularity also occurs in Newton's (classical) theory. Again solution becomes singular at *r=2m*.

After Einstein's November 1915 announcement of his general theory of relativity, physicists initiated efforts to generalize it an attempts to develop a unified field theory of the gravitational and electro-magnetic force. In 1918, Hermann Weyl [5, 11, 12] used an intuitively appealing version of non-Riemannian geometry to embed the entirely of electrodynamics into the affine connection of general relativity. Then in 1919 the Theodor Kaluza came up with another idea that employed ordinary Riemannian geometry but with 5-dimensions (1 time and 4 spaces). Einstein famously lauded Weyl's theory, but quickly withdrew his support when he discovered that the theory was not physical.

In 1926 the Swedish physicist Oskar Klein [13] came up with some major improvements to Kaluza's theory, at which time it became universally known as Kaluza-Klein theory. But the theory languished for decades until the early advent of string theory in the 1970s, when serious interest in extra dimensions experienced resurgence.

Later Einstein developed the electrodynamics and established the equation for the gravitational field of a charged particle or an electron [7] was given by

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{4\pi\varepsilon^2}{r^2}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + \left(1 - \frac{2m}{r} + \frac{4\pi\varepsilon^2}{r^2}\right) dt^2 \qquad (8)$$

Where m and ε are the mass and charge of the particle. *r* is the distance between two particle. At every point lying outside the electron, *(m/r)* is of the order 10^{-40} or smaller. After Einstein the unification process [14, 15] is also going on but still the unification of the 4-forces unsolved.

The purpose of this article is simply to introduce the basic concept of time looking into the extra dimensions of space-time continuum. Therefore in this work we put here justification for 7-d space-time continuums where 3 space and 4 time components on the basis of the four fundamental forces of nature viz electro-magnetic (e-m), strong, weak and gravitational forces.

Another purpose of this article is simply to solve Einstein's field equations for the gravitational field of an isolated particle on the basis of 7-dimensional metric similar to that of Schwarzschild in 4-dimesional. Taking new idea of time [16, 17] and looking in to the extra dimensions of space-time continuum, developed a 7-dimensional metric and able to unified the 4-fundamental forces.

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2. Assumptions

Change and evolution are fundamental aspects of the universe. Change and evolution is the description of the medium to experience the time i.e. time shows itself as a parameter of processes of change [2, 16]. If something were unchanging, we would not have any experience of time in relation to that. It would be like a static picture. In the other hand time can be understood when changes occur in the material universe.

Physics has already discovered four fundamental forces of nature viz e-m, strong, weak and gravitational forces whereby our universe is governed. Our universe is constantly being acted upon by these four fundamental forces of nature. Changes occur in four different ways co-responding to four forces. For different ways of change determined the four components of time.

The changes occur in the material universe at the speed of the mediator particle in four fundamental interactions. Because mediator particles playing main role in these four fundamental interactions.

Now our assumptions are

(i) Time shows itself as a parameter of processes of change.

(ii) The changes occur in the material universe due to the interactions of four fundamental forces of nature. Therefore four ways of change determined the four components of time.

(iii) The rate of changes is equal to the speed of the mediator particles in respective fundamental interactions of nature.

3. Mathematical Formulation

Let we consider our assumption (i) that time shows itself is a parameter of process of change and changing occurs with a resultant speed [16] due to the four fundamental forces of nature.

Now we consider for simplicity one particular fundamental force which is e-m force and is responsible for changes in nature especially in molecular and atomic level. Due to this e-m interaction the changes occur with a constant speed in nature. But in e-m interaction photon plays a vital role with the speed *c* as mediator particle and is responsible for changes in nature. This gives us that the changing speed is nothing but the speed of photon *c* , the speed of light.

$$\frac{d\vec{R}}{dt} = \vec{c} \qquad (9)$$

Where $\vec{R}$ is position co-ordinate in 3-d space and $\vec{R} = \vec{R}(x, y, z)$. Here *t* represents time and c is a constant but it represent speed of changes occur due to particular fundamental force e-m, means the speed of light.

Squaring equation (9) the interval in 3-d between two neighbouring event is,

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2 \qquad (10)$$

Equation (10) is nothing but the Lorentz expression using by Einstein. And this expression must be independent of the transformation from one system to another. Hence the expression (10) is invariant for Lorentz transformation.

Considering assumption (ii) that changes occur in our world in four different ways co-responding to four different fundamental forces of nature. Hence time experienced in four ways, means time has 4-components. Like space time gives us $\tau(t^1, t^2, t^3, t^4)$ and t^1, t^2, t^3, t^4 are independent time components as changes occur due to the strong, e-m, weak and gravitational forces of nature respectively.

Since $\tau = \tau(t^1, t^2, t^3, t^4)$ therefore,

$$d\tau = \frac{\partial \tau}{\partial t^1} dt^1 + \frac{\partial \tau}{\partial t^2} dt^2 + \frac{\partial \tau}{\partial t^3} dt^3 + \frac{\partial \tau}{\partial t^4} dt^4$$

$$d\tau = \hat{l} dt^1 + \hat{m} dt^2 + \hat{n} dt^3 + \hat{o} dt^4 \quad (11)$$

As $\frac{\partial \tau}{\partial t^1} = \hat{l}$, $\frac{\partial \tau}{\partial t^2} = \hat{m}$, $\frac{\partial \tau}{\partial t^3} = \hat{n}$, and $\frac{\partial \tau}{\partial t^4} = \hat{o}$ are unit time in their respective time axis.

In generalization the equation (9) can be written as,

$$\frac{d\vec{R}}{d\tau} = \vec{C} \quad (12)$$

Here $\vec{C}$ is changing parameter due to the 4-fundamental forces of nature.

Putting equation (11) and the value of $d\vec{R}$ in equation (12) we get,

$$\hat{i} dx + \hat{j} dy + \hat{k} dz = \vec{C}\hat{l} dt^1 + \vec{C}\hat{m} dt^2 + \vec{C}\hat{n} dt^3 + \vec{C}\hat{o} dt^4 \quad (13)$$

In r. h. s. of equation (13), $\hat{l}\vec{C} = \vec{c}_1, \hat{m}\vec{C} = \vec{c}_2, \hat{n}\vec{C} = \vec{c}_3$ and $\hat{o}\vec{C} = \vec{c}_4$ (14)

Whereas we consider c_1, c_2, c_3, c_4 are changing units in their respective axis i.e. the speed of mediator particles known as photon, gluon, weakon ($w^{\pm}$ and z^0) and graviton (?) of e-m, strong, weak, gravitational forces of nature respectively. Hence equation (16) becomes

$$\hat{i} dx + \hat{j} dy + \hat{k} dz = \vec{c}_1 dt^1 + \vec{c}_2 dt^2 + \vec{c}_3 dt^3 + \vec{c}_4 dt^4 \quad (15)$$

Therefore the interval between two neighbouring event is

$$ds^2 = dx^2 + dy^2 + dz^2 - c_1{}^2 (dt^1)^2 - c_2{}^2 (dt^2)^2 - c_3{}^2 (dt^3) - c_4{}^2 (dt^4) \quad (16)$$

Like equation [10] this expression must be independent of the transformation from one system to another. Hence the expression (16) is invariant for Lorentz transformation.

Again now consider the assumptions (i), (ii) and (iii) which gives that the space time continuum has (3+4) dimensions, 3 space and 4 time co-ordinates. So the interval between two neighbouring events can be considered in 7-d co-ordinate system is,

$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + (dx^4)^2 + (dx^5)^2 + (dx^6)^2 + (dx^7)^2 \quad (17)$$

Now the equation (17) comparing with equation (16) we get,

$$(dx^4)^2 = -c_1{}^2 (dt^1)^2 , \ (dx^5)^2 = -c_2{}^2 (dt^2)^2 , \ (dx^6)^2 = -c_3{}^2 (dt^3)^2 , \ (dx^7)^2 = -c_4{}^2 (dt^4)^2 \quad (18)$$

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In spherical polar co-ordinates the line element in equation (16) for flat space time is given by

$$ds^2 = -dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + {c_1}^2 (dt^1)^2 + {c_2}^2 (dt^2)^2 + {c_3}^2 (dt^3)^2 + {c_4}^2 (dt^4)^2 \qquad (19)$$

Considering $c_1 = c_2 = c_3 = c_4 = c = 1$ in the equation (19) becomes,

$$ds^2 = -dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + (dt^1)^2 + (dt^2)^2 + (dt^3)^2 + (dt^4)^2 \qquad (20)$$

A line element equivalent to this line element can be expressible as

$$ds^2 = -e^{\lambda} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + e^{\nu_1} (dt^1)^2 + e^{\nu_2} (dt^2)^2 + e^{\nu_3} (dt^3)^2 + e^{\nu_4} (dt^4)^2 \qquad (21)$$

Where λ and ν are functions of *r* only; since for spherically symmetric isolated particle the field will depend on *r* alone and not on θ and ϕ.

Since the gravitational field (i.e. the disturbance from flat-space time) due to a particular diminishes indefinitely as we go to an infinite distance, therefore line element (21) must reduce to line element (20) at an infinite distance from the particle.

Hence at $r \to \infty; \lambda = \nu_1 = \nu_2 = \nu_3 = \nu_4 = 0$.

In Riemannian space of *n* dimensions the line element for the distance between two neighbouring points x and $(x + dx)$ is

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \qquad (22)$$

Here $\mu, \nu = 1,2,3 .... n$

Considering $\mu, \nu = 1,2,3,4,5,6 \ \& \ 7$ in equation (22) the co-ordinates are $x^1 = r, x^2 = \theta, x^3 = \phi, x^4 = t^1, x^5 = t^2, x^6 = t^3 \ \& \ x^7 = t^4$;

For $\mu, \nu = 1,2,3,4,5,6 \ \& \ 7$ in equation (22) Comparing with (21) ,

$$g_{\mu\nu} = \begin{bmatrix} -e^{\lambda} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -r^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -r^2 \sin^2 \theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\nu_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{\nu_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\nu_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & e^{\nu_4} \end{bmatrix} \qquad (23)$$

This gives

$$\left. \begin{array}{l} g_{11} = -e^{\lambda}, g_{22} = -r^2, g_{33} = -r^2 \sin^2 \theta, g_{44} = e^{\nu_1} \\ g_{55} = e^{\nu_2}, g_{66} = e^{\nu_3}, g_{77} = e^{\nu_4} \ \& \ g_{\mu\nu} = 0 \ for \mu \neq \nu \end{array} \right\} \qquad (24)$$

And the determinant of $g_{\mu\nu}$ is

$$g = \left| g_{\mu\nu} \right| = -e^{\lambda + \nu_1 + \nu_2 + \nu_3 + \nu_4} r^4 \sin^2 \theta \qquad (25)$$

Also $g^{\mu\mu}=\frac{1}{g_{\mu\mu}}$ and $g^{\mu\nu}=0$ for $\mu\neq 0$, this gives us,

$$\left.\begin{array}{l} g^{11}=-e^{-\lambda}, g^{22}=-\frac{1}{r^2}, g^{33}=-\frac{1}{r^2\sin^2\theta}, g^{44}=e^{\nu_1} \\ g^{55}=e^{\nu_2}, g^{66}=e^{\nu_3}, g^{77}=e^{-\nu_4}\ \& \ g^{12}=g^{23}=.....=0, as \mu\neq\nu \end{array}\right\} \tag{26}$$

Now the equation (25) can be written as

$$|g|=-e^{\lambda+\nu_1+\nu_2+\nu_3+\nu_4}r^4\sin^2\theta$$

Therefore, $\sqrt{|g|}=-e^{\frac{1}{2}(\nu_1+\nu_2+\nu_3+\nu_4)}r^2\sin\theta$

$$\left.\begin{array}{l} \therefore \log\sqrt{|g|}=\frac{1}{2}(\lambda+\nu_1+\nu_2+\nu_3+\nu_4)+2\log r+\log\sin\theta \\ and \frac{\partial}{\partial r}\left(\log\sqrt{|g|}\right)=\frac{1}{2}\frac{\partial}{\partial r}(\lambda+\nu_1+\nu_2+\nu_3+\nu_4)+\frac{2}{r} \\ again \frac{\partial^2}{\partial r^2}\left(\log\sqrt{|g|}\right)=\frac{1}{2}\frac{\partial^2}{\partial r^2}(\lambda+\nu_1+\nu_2+\nu_3+\nu_4)-\frac{2}{r^2} \\ \frac{\partial}{\partial\theta}\left(\log\sqrt{|g|}\right)=\cot\theta, \frac{\partial^2}{\partial\theta^2}\left(\log\sqrt{|g|}\right)=-\cos ec^2\theta, \frac{\partial}{\partial\phi}\left(\log\sqrt{|g|}\right)=0 \\ \frac{\partial}{\partial x^4}\left(\log\sqrt{|g|}\right)=\frac{\partial}{\partial x^5}\left(\log\sqrt{|g|}\right)=\frac{\partial}{\partial x^6}\left(\log\sqrt{|g|}\right)=\frac{\partial}{\partial x^7}\left(\log\sqrt{|g|}\right)=0 \end{array}\right\} \tag{27}$$

If μ,ν,σ are different suffixes we can now easily get the Christofell's 3-index symbols in the following possible cases

$$\left.\begin{array}{l} \Gamma^{\mu}_{\mu\mu}=\frac{1}{2}g^{\mu\mu}\frac{\partial g_{\mu\mu}}{\partial x^{\mu}}=\frac{1}{2}\frac{\partial(\log g_{\mu\mu})}{\partial x^{\mu}};\Gamma^{\nu}_{\mu\mu}=-\frac{1}{2}g^{\nu\nu}\frac{\partial g_{\mu\mu}}{\partial x^{\nu}} \\ \Gamma^{\nu}_{\mu\nu}=\frac{1}{2}g^{\nu\nu}\frac{\partial g_{\nu\nu}}{\partial x^{\mu}}=\frac{1}{2}\frac{\partial(\log_{\nu\nu})}{\partial x^{\mu}};\Gamma^{\sigma}_{\mu\nu}=0 \end{array}\right\} \tag{28}$$

Hence we get the following independent non-vanishing Christofell's 3-index system

$$\left.\begin{array}{l} \Gamma^1_{11}=\frac{1}{2}\frac{\partial\lambda}{\partial r};\Gamma^1_{22}=-re^{-\lambda};\Gamma^1_{33}=-re^{-\lambda}\sin^2\theta; \\ \Gamma^1_{44}=\frac{1}{2}e^{\nu_1-\lambda}\frac{\partial\nu_1}{\partial r};\Gamma^1_{55}=\frac{1}{2}e^{\nu_2-\lambda}\frac{\partial\nu_2}{\partial r};\Gamma^1_{66}=\frac{1}{2}e^{\nu_3-\lambda}\frac{\partial\nu_3}{\partial r};\Gamma^1_{77}=\frac{1}{2}e^{\nu_4-\lambda}\frac{\partial\nu_4}{\partial r} \\ \Gamma^2_{33}=-\sin\theta\cos\theta;\Gamma^2_{12}=\Gamma^3_{13}=\frac{1}{r};\Gamma^3_{23}=\cot\theta \\ \Gamma^4_{14}=\frac{1}{2}\frac{\partial\nu_1}{\partial r};\Gamma^5_{15}=\frac{1}{2}\frac{\partial\nu_2}{\partial r};\Gamma^6_{16}=\frac{1}{2}\frac{\partial\nu_3}{\partial r};\Gamma^7_{17}=\frac{1}{2}\frac{\partial\nu_4}{\partial r} \\ \Gamma^{\alpha}_{\beta\gamma}=0, for \alpha\neq\beta\neq\gamma \end{array}\right\} \tag{29}$$

We have the Ricci tensor

$$R_{\mu\nu}=\frac{\partial}{\partial x^{\nu}}\Gamma^{\beta}_{\mu\beta}-\frac{\partial}{\partial x^{\beta}}\Gamma^{\beta}_{\mu\nu}+\Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\alpha\nu}-\Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta}$$

$$R_{11}=\frac{1}{2}\frac{\partial^2}{\partial r^2}(v_1+v_2+v_3+v_4)+\frac{1}{4}\left\{\left(\frac{\partial v_1}{\partial r}\right)^2+\left(\frac{\partial v_2}{\partial r}\right)^2+\left(\frac{\partial v_3}{\partial r}\right)^2+\left(\frac{\partial v_4}{\partial r}\right)^2\right\}$$
$$-\frac{1}{4}\frac{\partial\lambda}{\partial r}\frac{\partial}{\partial r}(v_1+v_2+v_3+v_4)-\frac{1}{r}\frac{\partial\lambda}{\partial r} \tag{30}$$

$$R_{22}=\left[e^{-\lambda}\left\{1+\frac{1}{2}r\frac{\partial}{\partial r}(v_1+v_2+v_3+v_4)\right\}-1\right] \tag{31}$$

$$R_{33}=e^{-\lambda}\left[\left\{1+\frac{1}{2}r\frac{\partial}{\partial r}(v_1+v_2+v_3+v_4)-\frac{1}{2}r\frac{\partial\lambda}{\partial r}\right\}-1\right]\sin^2\theta \tag{32}$$

$$R_{44}=-\frac{1}{2}e^{v_1-\lambda}\left[\frac{\partial^2 v_1}{\partial r^2}+\frac{1}{2}\left(\frac{\partial v_1}{\partial r}\right)^2-\frac{1}{2}\frac{\partial v_1}{\partial r}\frac{\partial\lambda}{\partial r}+\frac{2}{r}\frac{\partial v_1}{\partial r}\right] \tag{33}$$

$$R_{55}=-\frac{1}{2}e^{v_2-\lambda}\left[\frac{\partial^2 v_2}{\partial r^2}+\frac{1}{2}\left(\frac{\partial v_2}{\partial r}\right)^2-\frac{1}{2}\frac{\partial v_2}{\partial r}\frac{\partial\lambda}{\partial r}+\frac{2}{r}\frac{\partial v_2}{\partial r}\right] \tag{34}$$

$$R_{66}=-\frac{1}{2}e^{v_3-\lambda}\left[\frac{\partial^2 v_3}{\partial r^2}+\frac{1}{2}\left(\frac{\partial v_3}{\partial r}\right)^2-\frac{1}{2}\frac{\partial v_3}{\partial r}\frac{\partial\lambda}{\partial r}+\frac{2}{r}\frac{\partial v_3}{\partial r}\right] \tag{35}$$

$$R_{77}=-\frac{1}{2}e^{v_4-\lambda}\left[\frac{\partial^2 v_4}{\partial r^2}+\frac{1}{2}\left(\frac{\partial v_4}{\partial r}\right)^2-\frac{1}{2}\frac{\partial v_4}{\partial r}\frac{\partial\lambda}{\partial r}+\frac{2}{r}\frac{\partial v_4}{\partial r}\right] \tag{36}$$

Obviously equation (32) is a mere repetition of equation (31).

Einstein's field equations for empty space is, $R_{\mu\nu}=0$, therefore

$R_{11}=R_{22}=R_{33}=R_{44}=R_{55}=R_{66}=R_{77}=0$, Putting these values in above equations and adding equations (33),(34),(35) and (36) we get,

$$-\frac{1}{2}\frac{\partial^2}{\partial r^2}(v_1+v_2+v_3+v_4)-\frac{1}{4}\left\{\left(\frac{\partial v_1}{\partial r}\right)^2+\left(\frac{\partial v_2}{\partial r}\right)^2+\left(\frac{\partial v_3}{\partial r}\right)^2+\left(\frac{\partial v_4}{\partial r}\right)^2\right\}$$
$$+\frac{1}{4}\frac{\partial\lambda}{\partial r}\frac{\partial}{\partial r}(v_1+v_2+v_3+v_4)-\frac{1}{r}\frac{\partial}{\partial r}(v_1+v_2+v_3+v_4)=0 \tag{37}$$

Taking $R_{11}=0$, the equation (30) adding with (37) we get,

$$\frac{1}{r}\frac{\partial\lambda}{\partial r}+\frac{1}{r}\frac{\partial}{\partial r}(v_1+v_2+v_3+v_4)=0 \tag{38}$$

Integrating equation (41) we get,

$$-\lambda=(v_1+v_2+v_3+v_4) \tag{39}$$

Constant of integration which may be set equal to zero, without any loss of generality, since at $r\to\infty,\lambda=0$ and $v_1=v_2=v_3=v_4=0$. Hence,

$$e^{-\lambda}=e^{v_1+v_2+v_3+v_4}=e^{v_1}e^{v_2}e^{v_3}e^{v_4} \tag{40}$$

Now we are going to find out the value of $e^{\nu_1}, e^{\nu_2}, e^{\nu_3}$ & e^{ν_4} .

For our simplicity let us we consider e-m interaction and other forces are absent except gravitational. Because charge particle have definite mass, hence gravitational force is there, which behave as constraint force against e-m interaction, but for very small mass the gravitational force is very weak. Now considering e-m interaction only the line element is,

$$ds^2 = -e^{\lambda} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + e^{\nu_1} (dt^1)^2 \tag{41}$$

Solving like Schwarzschild we get, $-\lambda_1 = \nu_1$.

And finally we get the result, $e^{-\lambda_1} = e^{\nu_1} = 1 + \frac{B}{r}$ (42)

Here *B* is a constant. But according to Schwarzschild, $B = -2m$, and is done in order to facilitate the physical interpretation of m.

Einstein shows that the quantity g_{44} of the general theory in 4-d closely related to the gravitational potential ψ of the Newtonian theory.

i.e. $g_{44} = 1 + \frac{\psi}{c^2}$ (43)

Using this concept they found, $m = \frac{GM}{c^2}$ (44)

Classically after a long process Einstein found the equation (8) considering the potential g_{44} in equation (43) for both gravitational and e-m interaction. The ψ is written as,

$$\psi = -\frac{m}{r} + \frac{2\pi\varepsilon^2}{r^2} \tag{45}$$

Here *m* has the same meaning as equation (44) and ε^2 is charge for both particle. Thus Einstein tried to unify the gravitational interaction with the e-m interaction. Let us we consider the t^1 component of time which is for e-m interaction in our 7-d metric equation. Considering interaction for an electron and proton having charge –e and +e the e-m potential energy *V* is,

$$V = \frac{-e^2}{4\pi\varepsilon_0 r} = -\frac{\alpha\hbar c}{r} \tag{46}$$

$$\& \ \Phi = -\frac{GNm_p m_e}{r} = -\frac{\alpha_g \hbar c}{r} \tag{47}$$

Here $\Phi, \alpha, \alpha_g, \hbar, m_p, N$ & r are the gravitational potential, e-m coupling constant, gravitational coupling constant, Planck's constant, mass of proton, mass of electron, number of proton & distance between proton and electron respectively.

Since ψ is combination of e-m potential *V* and gravitational potential Φ, because these are related to t^1 component of time.

In hydrogen atom the electron is bounded by the e-m potential, when kinetic energy of the electron will greater than the Potential Energy then the electron leave the nucleus. This means when energy supplied to the electron, the electron always have the outward tendency from the centre of nucleus. Such type of energy considered as outward energy or cold energy. But gravitational force always attract towards the centre of nucleus. Hence such type of energy considered as inward energy or cold energy. Gravitational force always behaves just opposite to e-m force or behaves as a constraint force. To balance the motion of electron Kinetic Energy of the electron must be equal to the P. E. of the electron. Hence we consider the P. E. just using the opposite sign. Hence we can write,

$$\psi = -\frac{GNm_p m_e}{r} + \frac{\alpha\hbar c}{r} \quad (48)$$

Therefore force, $\frac{\partial \psi}{\partial r} = \frac{GNm_p m_e}{r^2} - \frac{\alpha\hbar c}{r^2}$ (49)

Now we consider the time component t^1 the equation (42) becomes,

$$g_{44} = e^{-\lambda_1} = e^{\nu_1} = 1 - \frac{2m}{r} \quad (50)$$

But, $g_{44} = 1 + \frac{\psi}{c_1^2}$ (51)

Putting the equation (50) in (51) we get,

$\psi = -\frac{mc_1^2}{r}$ and $\frac{\partial \psi}{\partial r} = \frac{mc_1^2}{r^2}$ (52)

Hence, $m = \frac{GNm_p m_e}{c_1^2} - \frac{\alpha\hbar c}{c_1^2}$ (53)

$$\therefore e^{-\lambda_1} = e^{\nu_1} = \left(1 - \frac{2GNm_p m_e}{rc_1^2} + \frac{2\alpha\hbar c}{rc_1^2}\right) \quad (54)$$

Now considering the time component t^2 for strong interaction ψ becomes,

$$\psi = -\frac{GNm_q m_q}{r} + \frac{\alpha_s \hbar c}{r} \quad (55)$$

Here α_s is the coupling constant of strong force, m_q and N are the mass of quark and number of quark. Then we get,

$$e^{-\lambda_2} = e^{\nu_2} = \left(1 - \frac{2GNm_q m_q}{rc_2^2} + \frac{2\alpha_s \hbar c}{rc_2^2}\right) \quad (56)$$

Similarly for time components t^3 & t^4 for weak & gravitational force we get,

$$e^{-\lambda_3} = e^{\nu_3} = \left(1 - \frac{2GNm_q m_q}{rc_3^2} + \frac{2\alpha_w \hbar c}{rc_3^2}\right) \quad (57)$$

& $e^{-\lambda_4} = e^{\nu_4} = \left(1 - \frac{2GM}{rc_4^2}\right)$ (58)

Here α_w & M represents weak coupling constant and one mass is $M(=Nm_q)$ and other mass is unit mass. Using equations (54), (56), (57), (57) and (39) we get,

$$-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)=(\nu_1+\nu_2+\nu_3+\nu_4) \quad (59)$$

Therefore the final metric becomes,

$$ds^2=-\left[\left(1-\frac{2GNm_pm_e}{rc_1^{\,2}}+\frac{2\alpha\hbar c}{rc_1^{\,2}}\right)\left(1-\frac{2Gm_qm_q}{rc_2^{\,2}}+\frac{2(2/3)\alpha_s\hbar c}{rc_2^{\,2}}\right)\left(1-\frac{2Gm_qm_q}{rc_3^{\,2}}+\frac{2\alpha_w\hbar c}{rc_3^{\,2}}\right)\left(1-\frac{2GM}{rc_4^{\,2}}\right)\right]^{-1}dr^2$$

$$-r^2d\theta^2-r^2\sin^2\theta d\phi^2+\left(1-\frac{2GNm_pm_e}{rc_1^{\,2}}+\frac{2\alpha\hbar c}{rc_1^{\,2}}\right)(dt^1)^2+\left(1-\frac{2GNm_qm_q}{rc_2^{\,2}}+\frac{2(2/3)\alpha_s\hbar c}{rc_2^{\,2}}\right)(dt^2)^2$$

$$+\left(1-\frac{2GNm_qm_q}{rc_3^{\,2}}+\frac{2\alpha_w\hbar c}{rc_3^{\,2}}\right)(dt^3)^2+\left(1-\frac{2GM}{rc_4^{\,2}}\right)(dt^4)^2$$

(60)

4. Conclusion

In r. h. s. of the equation (60) in the time component t^1, the coefficient term is $\left(1-\frac{2GNm_pm_e}{rc_1^{\,2}}+\frac{2\alpha\hbar c}{rc_1^{\,2}}\right)$.

In this term to stop the e-m interaction by gravitational force, $c_1=c$,

$$-\frac{2GNm_pm_e}{rc^2}+\frac{2\alpha\hbar c}{rc^2}=0$$

$$\therefore Nm_p=M=\frac{\alpha\hbar c}{Gm_e} \quad (61)$$

Putting $\alpha=\frac{1}{137.05}$, $\hbar=1.0546\times10^{-27}$ erg second, $c=2.997925\times10^{10}$ cm/sec, $m_e=9.10953\times10^{-28}$ gm and $G=6.66\times10^{-8}$ dyne cm^2/gm^2 in equation (60) we get,

$M=3.802413\times10^{15}$ gm and the density $\rho=\frac{M}{(4/3)\pi r^3}=9.077571\times10^{14}\,gm/cm^3$. Here *r* is considered as unit distance. When in the equation (60), $M\geq3.802413\times10^{15}$ gm and the density $\rho\geq9.077571\times10^{14}\,gm/cm^3$ will stop the e-m interaction. This is critical mass and density. The electron merges into the proton and proton becomes a neutron. So the star becomes a neutron star. This density of neutron star approximately agrees with others results given by Nasa [18] which is 3×10^{14} gm/cm^3.

In r. h. s. of equation (60) in the time component t^2, considering $c_2=c_3=c_4=c$ the coefficient term is $\left(1-\frac{2GNm_qm_q}{rc^2}+\frac{2(2/3)\alpha_s\hbar c}{rc^2}\right)$. In this term to stop the strong interaction by gravitational force,

$$-\frac{2GNm_q m_q}{rc^2}+\frac{2(2/3)\alpha_s \hbar c}{rc^2}=0$$

$$\therefore Nm_q = M = \frac{(2/3)\alpha_s \hbar c}{Gm_q} \quad (62)$$

Putting $\alpha_s = 1$, (2/3) put in above term for quark-quark interaction, putting values of $\hbar$, c, $m_q = 8.91338\times 10^{-27}$ gm (bare mass [19-20] of lightest quark *u*=5 Mev/c^2, $1Mev = 1.782676\times 10^{-27}\,gm$) and G in equation (62) we get, $M = 3.550235\times 10^{16}$ gm and the density $\rho = \frac{M}{(4/3)\pi r^3} = 8.475543\times 10^{15}\,gm/cm^3$. This means that when the $M \ge 3.550235\times 10^{16}$ gm and the density $\rho \ge 8.475543\times 10^{15}\,gm/cm^3$ will stop the strong interaction & shows singularity i.e. the black hole character. This is critical mass and density. In this level no any other interaction will occur except gravitational.

In r. h. s. of equation (60) in the time component t^3, the coefficient term is $\left(1-\frac{2GNm_q m_q}{rc^2}+\frac{2\alpha_w \hbar c}{rc^2}\right)$. Putting $\alpha_w = 10^{-6}$ and values of other symbols as usual in weak term to stop weak interaction,

$M = Nm_q = \frac{\alpha_w \hbar c}{Gm_q} = 5.325886\times 10^{10}$ gm and $\rho = 1.271459\times 10^{10}\,gm/cm^3$.

Now the t^4 time component in equation (60), the coefficient term is $\left(1-\frac{2GM}{rc^2}\right)$. In this term when $2GM = rc^2$, shows singularity. Putting the density $\rho = 8.475543\times 10^{15}\,gm/cm^3$ in equation $2GM = rc^2$, as $M = \frac{4}{3}\pi r^3 \rho$, to stops all interactions except gravity, gives $r = 4.359534\times 10^5\,cm$, and we get $M = 2.941558 M_* \cong 2.94 M_*$. Here is $M_* (=10^{33}\,gm)$ represents the mass of sun. Therefore the mass of Neutron stars must be less than $2.94 M_*$ which is agrees with result given by reference [21].

In the equation $2GM = rc^2$ considering $M = \frac{4}{3}\pi r^3 \rho$, putting the density of universe $\rho = 9.9\times 10^{-30}\,gm/cm^3$ [22] gives the diameter of universe $d = 2.551151\times 10^{26}\,m$ which was same as given by reference [23] and mass $M = 8.606821\times 10^{52}\,kg$. Mass of our universe greater than the above mass *M* shows singularity nature. So mass of our universe is less than $8.606821\times 10^{52}\,kg \approx 10^{53}\,kg$ agrees with reference [24].

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