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Research Paper

# DEA: a New Aproach to Mathematical Programming

Mathematics

**KEYWORDS :** Mathematical Programming, DEA, linear programming.

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## ABSTRACT

*Sowing seeds lead to their sprouting and then flowers come to them. Flowers fill the world with fragrance. If Mathematical Programming is a seed, then DEA is no doubt a beautiful flower spreading its fragrance in the different fields of mathematics. Data envelopment analysis (DEA) is a nonparametric method of mathematics which involve the estimation of production frontiers. It is used to empirically measure productive efficiency of decision making units (or DMUs). The paper describes DEA with a different way through linear programming.*

**Introduction**

Sowing seeds lead to their sprouting and then flowers come to them. Flowers fill the world with fragrance. If Mathematical Programming is a seed, then DEA is no doubt a beautiful flower spreading its fragrance in the different fields of mathematics.

1. **Mathematical programming**

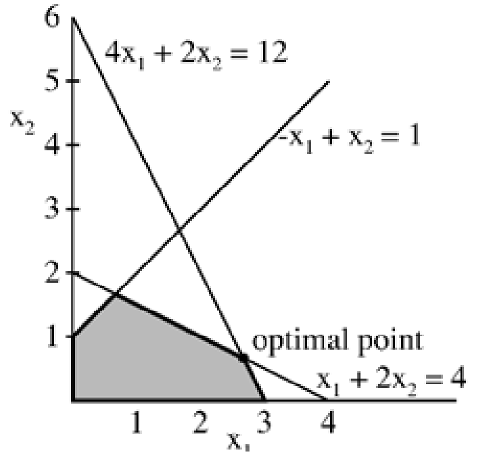
The Mathematical programming, especially its branch linear programming, is one of the best developed and most used branches of mathematics. It is the optimum allocation of limited resources, under a set of constraints depending upon the nature of the problem being studied. These constraints can be financial, organizational, or many other considerations. In broad terms, mathematical programming can be defined as a mathematical representation aimed at programming is the best possible allocation of limited resources. When the objective function and constraints include only linear functions, we have a linear-programming problem.

2. **Linear programming**

Since In 1947, George B. Dantzig, then part of a research group of the U.S. Air Force known as Project SCOOP (Scientific Computation Of Optimum Programs), developed the *simplex method* for solving the general linear-programming problem.

A linear programming problem may be defined as the problem of maximizing or minimizing a linear function subject to linear constraints. The constraints may be equalities or inequalities. Here is a simple example. Find numbers $x_1$ and $x_2$ that maximize the sum $x_1 + x_2$ subject to the constraints $10x_1 + 20x_2 \le 440$, $3x_1 + 6x_2 \le 36$, $-x_1 + x_2 \le 1$, and $x_1 \ge 0$, $x_2 \ge 0$.

In this problem there are two unknowns, and five constraints. The function to be maximized (or minimized) is called the *objective function*. Since there are only two variables, we can solve this problem by graphing the set of points in the plane that satisfies all the constraints (called the constraint set) and then finding which point of this set maximizes the value of the objective function. It is easy to see in general that the objective function, being linear, always takes on its maximum (or minimum) value at a corner point of the constraint set, provided the




constraint set is bounded.

The extraordinary computational efficiency and robustness of the simplex method, together with the availability of high-speed digital computers, have made linear programming the most powerful optimization method ever designed and the most widely applied in the business environment. Since then, many additional techniques have been developed, which relax the assumptions of the linearprogramming model and broaden the applications of the mathematical-programming approach.

Duality in linear programming is essentially a unifying theory that develops the relationships between a given linear program and another related linear program. The importance of duality is twofold. First, fully understanding the shadow-price interpretation of the optimal simplex multipliers can prove very useful in understanding the implications of a particular linear-programming model. Second, it is often possible to solve the related linear program with the shadow prices as the variables in place of, or in conjunction with, the original linear program, thereby taking advantage of some computational efficiencies.

The duality principles can be stated formally in general terms.

**Let the primal problem be:**
**Primal**
Maximize $z = \Sigma c_j x_j$

subject to $\Sigma a_{ij} x_j$ less than or equal to $b_i$, xj being non negative.

Associated with this primal problem there is a corresponding dual problem given by:

**Dual**
Minimize $v = \Sigma b_i y_i$

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subject to $\Sigma a_{ij} y_i$ greater than or equal to $b_j$, $y_i$ being non negative.

where $c_j$ runs over j = 1, 2, . . . , n,

and $y_i$ runs over i = 1, 2, . . . ,m.

## 3. Dea

**Data envelopment analysis (DEA)** is a nonparametric method of mathematics which involve the estimation of production frontiers. It is used to empirically measure productive efficiency of decision making units (or DMUs). The efficient DMUs, as defined by DEA, may not necessarily form a "production frontier", but rather lead to a "best-practice frontier" (Cook, Tone and Zhu, 2014). DEA is referred to as "balanced benchmarking" by Sherman and Zhu (2013). Data Envelopment Analysis (DEA) is an increasingly popular management tool. DEA is commonly used to evaluate the efficiency of a number of producers. A typical statistical approach is characterized as a central tendency approach and it evaluates producers relative to an average producer. In contrast, DEA compares each producer with only the "best" producers. By the way, in the DEA literature, a producer is usually referred to as a decision making unit or DMU.

In DEA, there are a number of producers. The production process for each producer is to take a set of inputs and produce a set of outputs. Each producer has a varying level of inputs and gives a varying level of outputs. For instance, consider a set of banks. Each bank has a certain number of tellers, a certain square footage of space, and a certain number of managers (the inputs). There are a number of measures of the output of a bank, including number of checks cashed, number of loan applications processed, and so on (the outputs). DEA attempts to determine which of the banks are most efficient. There are also parametric approaches which are used for the estimation of production frontiers. These require that the shape of the frontier be guessed beforehand by specifying a particular function relating output to input.

DEA is also regularly used to assess the efficiency of public and private organizations, e.g. hospitals, universities, police forces and sugar industries etc.

## 4. Conclusion

DEA has emerged as a new science in itself with vast applications. It can be applied to still hidden areas. By ploughing deeper one can rejuvenate it leading to other areas of research.

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