Characteristics of Petersen Graph in Path Matrix with Algebraic Graph Theory.

Mathematics

KEYWORDS : Walk –path- Petersen graph –path matrix –adjacency matrix-incidence matrix.

Dr.G.Nirmala Principal, Govt Arts Collage Melur, Mudurai (D.T)

M.Murugan Research Scholar Department of Mathematics, Periyar Maniyammai University Vallam Thanjavur.

ABSTRACT

In this work basic concepts of algebraic graph theory and its properties are reviewed and extended to the related concept of path matrix in Petersen graph and its properties.

The relation between path matrix and incidence matrix introduced in Petersen graph.

INTRODUCTION:

Algebraic graph theory can be viewed as an extension to the graph theory in which algebraic methods are applied to graph theory problem.

Structural path analysis are applied to studies in the field of Industrials ecology. Application of structural path analysis enriches the interpretation in a life cycle. Assessment study by decomposing a single numerical measure

We can also track energy flows and material flows. Which go through the individual paths. Path-based matrix decomposition analysis to additively decompose an input –output matrix into the contribution of each path based on structural path analysis. Path matrix is used to communication and transportation network.

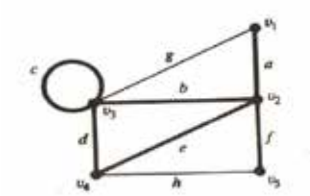
This paper deals with Petersen graph and to properties with path matrix and finding the different path matrix in a Petersen graph Relating to the path matrix with incidence matrix in a Petersen graph is dealt with.

Definition-1:

A walk is defined as a finite alternating sequence of vertices and edges beginning and ending with vertices.

No edges appears more than once in a walk. A walk is also referred to as an edge train(or) chain

The set of vertices and edges constituting a given walk in a graph **G** is clearly a subgraph of **G**

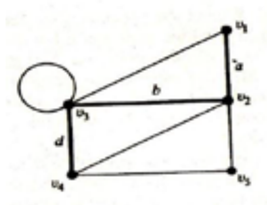


An Open Walk

Definition-2:

An open walk in which no vertex appears more than once is called path (or) a simple path (or) an elementary path.

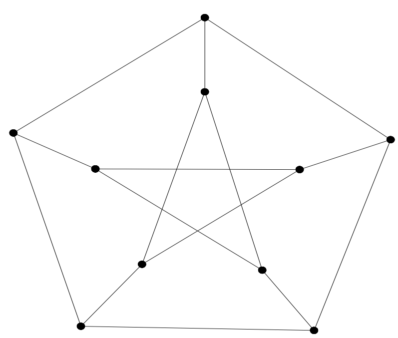
The number of edges in a path is called the length of the path. The terminal vertices of a path are of degree one and the rest of vertices. are of degree two.



A Path of Length Three

Definition-3:

Petersen graph is a 3- regular graph of 10 vertices and 15 edges



Definition-4:

Let **G** be a graph with "n" vertices "q" edges and no self loops. The incidence matrix **I(G)** is given by

I (G) = (aij) nxq

$$a_{ij} = \begin{cases} 1 & \text{if jth edge incident on ith vertex} \\ 0 & \text{otherwise} \end{cases}$$

Definition-5:

A path matrix is defined for a specific pair of Vertices in graph say (x,y). and its denoted by P(x,y).

let “q” be the number of different paths between the Vertices x and y.

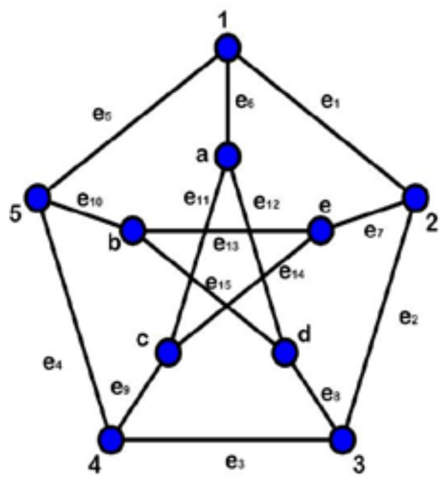
let “m” be the number of edges.

The path matrix p(x,y)

$$P(x,y) = (p_{ij})$$

$$p_{ij} = \begin{cases} 1 & \text{if } j^{th} \text{ edge incident on } i^{th} \text{ vertex} \\ 0 & \text{otherwise} \end{cases}$$

Path matrix of the petersen graph



The different paths between the vertices 1 and 5

$P_1 = \{e_5\}$

$P_2 = \{e_6, e_{11}, e_9, e_4,\}$

$P_3 = \{e_6, e_{11}, e_9, e_3, e_8, e_{15}, e_{10}\}$

$P4 = \{e_6, e_{11}, e_9, e_3, e_2, e_7, e_{13}, e_{10}\}$

$P_5 = \{e_6, e_{11}, e_{14}, e_{13}, e_{10}\}$

$P_6 = \{e_6, e_{11}, e_{14}, e_{13}, e_{15}, e_8, e_3, e_4\}$

$P_7 = \{e_6, e_{11}, e_{14}, e_7, e_2, e_3, e_4\}$

$P_8 = \{e_6, e_{12}, e_{15}, e_{10}\}$

$P_9 = \{e_6, e_{12}, e_8, e_3, e_4\}$

$P_{10} = \{e_6, e_{12}, e_8, e_2, e_7, e_{14}, e_9, e_4\}$

$P_{11} = \{e_6, e_{12}, e_8, e_2, e_7, e_{13}, e_{10}\}$

$P_{12} = \{e_6, e_{12}, e_{15}, e_{13}, e_{14}, e_9, e_4\}$

$P_{13} = \{e_6, e_{12}, e_{15}, e_{13}, e_7, e_2, e_3, e_4\}$

$P_{14} = \{e_1, e_2, e_3, e_4\}$

$P_{15} = \{e_1, e_7, e_{13}, e_{10}\}$

$P_{16} = \{e_1, e_7, e_{13}, e_{15}, e_{12}, e_{11}, e_9, e_4\}$

$P_{17} = \{e_1, e_7, e_{13}, e_{15}, e_8, e_3, e_4\}$

$P_{18} = \{e_1, e_7, e_{14}, e_9, e_4\}$

$P_{19} = \{e_1, e_7, e_{14}, e_{11}, e_{12}, e_{15}, e_{10}\}$

$P_{20} = \{e_1, e_7, e_{14}, e_{11}, e_{12}, e_8, e_3, e_4\}$

$P_{21} = \{e_1, e_2, e_8, e_{15}, e_{10}\}$

$P_{22} = \{e_1, e_2, e_8, e_{15}, e_{13}, e_{14}, e_9, e_4\}$

$P_{23} = \{e_1, e_2, e_8, e_{12}, e_{11}, e_9, e_4\}$

$P_{24} = \{e_1, e_2, e_8, e_{12}, e_{11}, e_{14}, e_{13}, e_{10}\}$

$P_{25} = \{e_1, e_2, e_3, e_9, e_{11}, e_{12}, e_{15}, e_{10}\}$

$P_{26} = \{e_1, e_2, e_3, e_9, e_{14}, e_{13}, e_{10}\}$

The path matrix of the Petersen graph with vertex 1 and 5 Whose order is 26 x 15 is denote by P(1,5)

$$P(1,5) = \begin{array}{cc}
 & \begin{array}{ccccccccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} & e_{11} & e_{12} & e_{13} & e_{14} & e_{15} \end{array} \\
\begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \\ p_{10} \\ p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{15} \\ p_{16} \\ p_{17} \\ p_{18} \\ p_{19} \\ p_{20} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{25} \\ p_{26} \end{array} &
\begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\
1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\
1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0
\end{pmatrix}
\end{array}$$

26 x 15

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RESULTS

1. A column of all zeros corresponds to an edge that does not lie in any path between 1 and 5.

   $\therefore$ In a Petersen graph all the edges contains any path between the vertex 1 and 5
2. A column of all one corresponds to an edge that lie in every path between the vertex.

$\therefore$ In a Petersen graph no column contains all ones

3. There is no row with all zeros.
4. The ring sum of any two rows in P(1,5) corresponds to a cycle (or) an edge disjoint union of cycles

Theorem 1

If the column of the incidence matrix I and the path matrix P(u,v) of a connected graph are arranged in the same order then under the product (mod 2)

$$I\, p^T (u,v) = M$$

Where M is a matrix having ones in rows u and v the zeros in the remaining n-2 rows

Proof:

Let G be the connected graph and let v_k=u and $v_i = v_t$ be any two vertices of G

Let I (G) be the incidence matrix p(u,v) be the path matrix of (u,v) in G

Now for any vertex v_i in G and for any u-v path p_j in G either $v_i \in p_j$ (or) $v_i \notin p_j$

If $v_i \notin p_j$ than there is no edge of p_j which is incident on v_i.

If $v_i \in p_j$ than either v_i is an intermediate vertex of P_j (or) $v_i=v_k$ (or) $V_i=V_t$

In case v_i is an intermediate vertex of P_j then there are exactly two edges of p_j which are incident on V_i

In case $V_i=V_k$ (or) V_t there is exactly one edge of p_j which is incident on V_i.

Now consider the i^{th} row of I and j^{th} row of P [which is the j^{th} column of P^T (u,v)]

As the edges are arranged in the same order.

The r^{th} entries. In these two rows are both non zero if and only if the edge e_r is incident on the i^{th} vertex Vi and is also on the j^{th} path Pj

Let

$$I\,P^T (u,v) = M = [m_{ij}]$$

We have

$$[I\,P^T]_{ij} = \sum_{r=1}^{m} [A]_{ir} \left[p^T\right]_{rj}$$

Therefore

$$m_{ij} = \sum_{r=1}^{m} a_{ir} P_{jr}$$

For each edge e_r of G we have one of the following cases.

(i) e_r is incident on Vi and $e_r \notin$ pj

Here $a_{ir} = 1$ $b_{jr} = 0$

ii. e_r is not incident on Vi and $e_r \in p_j$

Here $a_{ir} = 0$ $b_{jr} = 1$

iii. e_r is not incident on Vi and $e_r \notin p_j$

Here $a_{ir} = 0$ $b_{jr} = 0$

All these case imply that the i^{th} vertex v_i is not in j^{th} path p_j and we have

$M_{ij} = 0 \cong 0$ (mod 2).

iv. e_r is incident on v_i and $e_r \in p_j$

If v_i is an intermediate vertex of p_j then there are exactly two edges say e_r and e_t incident on v_i so that $a_{ir} = 1$ $a_{it} = 1$ $p_{jr} = 1$ $p_{jt} = 1$

Therefore $m_{ij} = 1+1 = 0$(mod 2).

If $V_i = V_k$ or V_t then the edge e_r is incident on either V_k or V_t so

$a_{kr} = 1$ $p_{jr} = 1$ or $a_{tr} = 1$ $p_{jr} = 1$

Thus $m_{kj} = \Sigma a_{ir} p_{jr} = 1.1 \tilde{=} 1$(mod 2)

$m_{tj} = \Sigma a_{ir} p_{jr} = 1.1 \tilde{=} 1$(mod 2)

Here $M=[m_{ij}]$ is a matrix such that under modulo 2.

$$m_{ij} = \begin{cases} 1 \text{ for } i = k, t \\ 0 \text{ otherwise} \end{cases}$$

$I\,P^T(u, v) = M$

Hence the theorem

In a Petersen graph

$$I.P^T(1,5) = \begin{array}{cc} & \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} & e_{11} & e_{12} & e_{13} & e_{14} & e_{15} \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ \underline{3} \\ .4 \\ 5 \\ a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0. & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}_{10\times 15} \end{array}$$

$e_1 \quad e_2 \quad e_3 \quad e_4 \quad e_5 \quad e_6 \quad e_7 \quad e_8 \quad e_9 \quad e_{10} \quad e_{11} \quad e_{12} \quad e_{13} \quad e_{14} \quad e_{15}$

$$\text{X} \begin{array}{cc} \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \\ p_{10} \\ p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{15} \\ p_{16} \\ p_{17} \\ p_{18} \\ p_{19} \\ p_{20} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{25} \\ p_{26} \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}^T \end{array}$$

26×15

$$I.P^T(1,5) = \begin{array}{cc} \begin{matrix} 1 \\ 2 \\ \underline{3} \\ .4 \\ 5 \\ a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

RESULT

1. If G is a Petersen graph then I(G) be the incidence matrix of a Petersen graph P(u, v) be the path matrix of the Petersen graph between the vertices

$$I\,P^T(u,v)=M=[m_{ij}]$$

Rank of IP^T (u, v)_in a Petersen graph

It is clear that all 2x2 determinant value is zero in $I\,P^T(u, v)$

Rank of $I\,P^T(u,v)$ is one

$$\rho[I\,P^T(u,v)]=1$$

Conclusion

Petersen graph is a special kind of graph path matrix is used to communication and transportation network. In addition using the relation of incidence matrix and path matrix of the Petersen graph incidence matrix and transpose of the path matrix that than the product (mod 2) are reviewed. Petersen graph $IP^T(u,v)$ rank can be found.Petersen graph has similarity with the ordinary graph properties of the path matrix.Petersen graph have 10 vertices and 15 edges the number of paths between the vertices is 10 C_2. The number of path between the vertices =45 ways

Total number different paths in a Petersen graph is 45x26=1170

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