New Class of Generalized Closed Sets in Fuzzy Topological Spaces

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ABSTRACT *The aim of this paper is to introduce a new class of generalized closed sets namely πgr-closed sets fuzzy topological spaces. We discuss some of its basic properties and investigate the relationship between this type of closed sets and the other existing closed sets in fuzzy topological spaces.*

1. INTRODUCTION

Zadeh [14] introduce the concept of fuzzy sets. The study of fuzzy topology introduced by Chang [7], In 1995,Thakur S.S [12] introduce the concept of fuzzy generalized closed sets, Benchall S.S and Jenifer Karnel [2] introduce the concept of fuzzy b-closed sets, P.Sundram and M.Sekh John [11] introduce the concept of fuzzy w-closed sets, A. Hakeem and Othman [8] introduce the concept of α-closed sets, [10] introduce the concept of pre-closed sets,

In this paper, the concept of fuzzy π-generalized regular closed sets (briefly *fπgr*-closed sets) is introduced and studied some, properties are proved and their relations with different fuzzy sets in fuzzy topological spaces are investigation .

2. PRELIMINARIES

A family τof fuzzy sets of X is called a fuzzy topology on X if 0 and 1 belong to τ and τ is closed with respect to arbitrary union and finite intersection [7]. The members of τ are called fuzzy open sets and their complements are fuzzyclosed sets.

Throughout the present paper, (X, τ) or simply X mean fuzzy topological space (abbreviated as fts) on which no separation axioms are assumed unless otherwise mentioned. We denote and define the closure and interior for a fuzzy set λ by $cl(\lambda) = \wedge \{ \mu : \mu \geq \lambda, 1 - \mu \in \tau\}$ and $int(\lambda) = \vee \{ \mu : \mu \leq \lambda, \mu \in \tau\}$ respectively. This section contains some basic definition and preliminary results which will be needed in the sequel.

Definition 2.1. A fuzzy set λ of (X,τ) is called,
1. fuzzy semi open [13] (in short, fs-open) if $\lambda \leq cl(int(\lambda))$ and a fuzzy semi closed (in short, fs-closed) if $int(cl(\lambda)) \leq \lambda$.
2. fuzzy pre open [10] (in short, fp-open) if $\lambda \leq int(cl(\lambda))$ and a fuzzy pre-closed (in short, fp-closed) if $cl(int(\lambda)) \leq \lambda$.
3. fuzzy α-open [10] (in short, fα-open) if $\lambda \leq int(cl(int(\lambda)))$ and a fuzzy α-closed (in short, fα-closed) if $cl(int(cl (\lambda))) \leq \lambda$.
4. fuzzy semi pre-open [10] (in short, fsp-open) if $\lambda \leq cl(int(cl(\lambda)))$ and a fuzzy semi pre -closed (in short, fsp-closed) if $int(cl(int\lambda))) \leq \lambda$
5. fuzzy b-open [5] (in short, fb-open) $\lambda \leq int(cl(\lambda)) \vee cl(int(\lambda))$ an d fuzzy b-closed set (in short, fb-closed) $\lambda \geq int(cl(\lambda)) \wedge cl(int(\lambda))$.
6. fuzzy regular open set [10] (in short, fr-open) $\lambda = int(cl(\lambda))$and fuzzy regular-closed set(in short, fr-closed) $\lambda = cl(int(\lambda))$.
7. fuzzy generalized closed [13] (in short, fg-closed) if $cl(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is fuzzy open set in X.
8. fuzzy weakly-closed [13] (in short, fw-closed) if $cl(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is fuzzy semi open set in X.

Fuzzy πgr-Closed Set
Definition 3.1. A fuzzy set λ of a fuzzy topological spaces (X,τ) is called a fuzzy π-open set if λ is the finite union of fuzzy regular open of X.

Definition 3.2. A fuzzy set λ of a fuzzy topological spaces (X,τ) is called a fuzzy πgr-closed set if $rcl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy π-open of X.

Example 3.3. Consider X = {a,b} and τ = {1, 0, λ, μ, γ} is a fuzzy topology on X. The fuzzy sets are define as
$\lambda = \frac{0.3}{a} + \frac{0.4}{b}$, $\mu = \frac{0.7}{a} + \frac{0.9}{b}$, $\gamma = \frac{0.6}{a} + \frac{0.5}{b}$, fuzzy πgr-closed sets = {1, 0, μ}.

Theorem 3.4. Every fuzzy regular closed set in X is fuzzy πgr-closed in X.

Proof. Let λ be a fuzzy r-closed set in X. Let μ be a fuzzy π-open such that $\lambda \leq \mu$. Since λ is fuzzy r-closed, we have $rcl(\lambda) = \lambda \leq \mu$. Therefore $rcl(\lambda) \leq \mu$. Hence λ is fuzzy πgr-closed set in X

Remark 3.5. The converse of the theorem 3.4 is not true in general as shown the following example.

Example 3.6. Consider X = {a,b} and τ = {1, 0, λ, μ, γ} is a fuzzy topology

on X. The fuzzy sets are define as
$\lambda = \frac{0.3}{a} + \frac{0.5}{b}$, $\mu = \frac{0.7}{a} + \frac{0.6}{b}$, $\gamma = \frac{0.6}{a} + \frac{0.5}{b}$ fuzzy πgr-closed sets = {1, 0, μ}, fuzzy r-closed sets = {1, 0, λᶜ, γᶜ}. Hence the μ in fuzzy πgr-closed set need not be fuzzy r-closed set.

Theorem 3.7. Every fuzzy w-closed set in X is fuzzy πgr-closed in X.

Proof. Let λ be a fuzzy w-closed set in X. Let μ be a fuzzy π-open such that $\lambda \leq \mu$. Since λ is fuzzy w-closed, we have $wcl(\lambda) = \lambda \leq \mu$, such that $wcl(\lambda) \leq rcl(\lambda) \leq \mu$. Hence λ is fuzzy πgr-closed set in X.

Remark 3.8. The converse of the theorem 3.7 is not true in general as shown in the following example.

Example 3.9. Consider X = {a, b} and τ = {1, 0, λ, μ, γ} is a fuzzy topology on X. The fuzzy sets are define as
$\lambda = \frac{0.3}{a} + \frac{0.4}{b}$, $\mu = \frac{0.7}{a} + \frac{0.9}{b}$, $\gamma = \frac{0.6}{a} + \frac{0.5}{b}$, fuzzy πgr-closed sets = {1, 0, μ},   fuzzy w-closed sets = {1, 0}. Hence the μ in fuzzy πgr-closed set need not be fuzzy w-closed set.

Theorem 3.10. Every fuzzy g-closed set in X is fuzzy πgr-closed in X.

Proof. Let λ be a fuzzy g-closed set in X. Let μ be a fuzzy π-open such that $\lambda \leq \mu$. Since λ is fuzzy g-closed, we have gcl(λ) = $\lambda \leq \mu$, such that gcl(λ) $\leq$ rcl(λ) $\leq \mu$. Hence λ is fuzzy πgr-closed set in X.

Remark 3.11. The converse of the theorem 3.10 is not true in general as shown in the following example.

Example 3.12. Consider X = {a, b} and τ = {1, 0, λ, μ, γ} is a fuzzy topology on X. The fuzzy sets are define as
$\lambda = \frac{0.2}{a} + \frac{0.4}{b}$, $\mu = \frac{0.8}{a} + \frac{0.9}{b}$, $\gamma = \frac{0.6}{a} + \frac{0.5}{b}$, fuzzy πgr-closed sets = {1, 0, μ}, fuzzy g-closed sets = {1, 0} . Hence the μ in fuzzy πgr-closed set need not be fuzzy g-closed set.

Theorem 3.13. Every fuzzy semi-closed set in X is fuzzy πgr-closed in X.

Proof. Let λ be a fuzzy semi-closed set in X. Let μ be a fuzzy π-open such that $\lambda \leq \mu$. Since λ is fuzzy semi-closed, we have scl(λ) = $\lambda \leq \mu$, such that scl(λ) $\leq$ rcl(λ) $\leq \mu$. Hence λ is fuzzy πgr-closed set in X.

Remark 3.14. The converse of the theorem 3.13 is not true in general as shown in the following example.

Example 3.15. Consider X = {a,b} and τ = {1 ,0 ,λ, μ ,γ} is a fuzzy topology on X. The fuzzy sets are define as
$\lambda = \frac{0.4}{a} + \frac{0.3}{b}$, $\mu = \frac{0.6}{a} + \frac{0.9}{b}$, $\gamma = \frac{0.6}{a} + \frac{0.5}{b}$, fuzzy πgr-closed sets = {1, 0, μ}, fuzzy semi-closed sets = {1, 0, λ, γ}. Hence the μ in fuzzy πgr-closed set need not be fuzzy semi-closed set.

Theorem 3.16. Every fuzzy pre-closed set in X is fuzzy πgr-closed in X.

Proof. Let λ be a fuzzy pre-closed set in X. Let μ be a fuzzy π-open such that $\lambda \leq \mu$. Since λ is fuzzy pre-closed, we have pcl(λ) = $\lambda \leq \mu$, such that pcl(λ) $\leq$ rcl(λ) $\leq \mu$. Hence λ is fuzzy πgr-closed set in X.

Remark 3.17. The converse of the theorem 3.16 is not true in general as shown in the following example.

Example 3.18. Consider X = {a, b} and τ = {1, 0, λ, μ, γ} is a fuzzy topology on X. The fuzzy sets are define as
$\lambda = \frac{0.4}{a} + \frac{0.4}{b}$, $\mu = \frac{0.6}{a} + \frac{0.7}{b}$, $\gamma = \frac{0.6}{a} + \frac{0.5}{b}$, fuzzy πgr-closed sets = {1, 0, μ}, fuzzy pre-closed sets = {1, 0} . Hence the μ in fuzzy πgr-closed set need not be fuzzy pre-closed set.

Theorem 3.19. Every fuzzy α-closed set in X is fuzzy πgr-closed in X.

Proof. Let λ be a fuzzy α-closed set in X. Let μ be a fuzzy π-open such that $\lambda \leq \mu$. Since λ is fuzzy α-closed, we have αcl(λ) = $\lambda \leq \mu$, such that αcl(λ) $\leq$ rcl(λ) $\leq \mu$. Hence λ is fuzzy πgr-closed set in X.

Remark 3.20. The converse of the theorem 3.19 is not true in general as shown in the following example.

Example 3.21. Consider X = {a,b} and τ = { 1, 0, λ, μ, γ} is a fuzzy topology

on X. The fuzzy sets are define as
$\lambda = \frac{0.3}{a} + \frac{0.4}{b}$, $\mu = \frac{0.7}{a} + \frac{0.9}{b}$, $\gamma = \frac{0.6}{a} + \frac{0.5}{b}$, fuzzy πgr-closed sets = {1, 0, μ}, fuzzy α-closed sets = {1, 0} . Hence the

μ in fuzzy πgr-closed set need not be fuzzy α -closed set.

Theorem 3.22. Every fuzzy b-closed set in X is fuzzy πgr-closed in X.

Proof. Let λ be a fuzzy b-closed set in X. Let μ be a fuzzy π-open such that $\lambda \leq \mu$. Since λ is fuzzy b-closed, we have bcl(λ) = $\lambda \leq \mu$, such that bcl(λ) $\leq$ rcl(λ) $\leq \mu$. Hence λ is fuzzy πgr-closed set in X.

Remark 3.23. The converse of the theorem 3.22 is not true in general as shown in the following example.

Example 3.24. Consider X = {a,b} and τ = {1, 0, λ, μ, γ} is a fuzzy topology on X. The fuzzy sets are define as
$\lambda = \frac{0.3}{a} + \frac{0.4}{b}$, $\mu = \frac{0.7}{a} + \frac{0.9}{b}$, $\gamma = \frac{0.6}{a} + \frac{0.5}{b}$, fuzzy πgr-closed sets = {1, 0, μ}, fuzzy b-closed sets = {1, 0, λ, γ}. Hence the μ in fuzzy πgr-closed set need not be fuzzy b-closed set.

Theorem 3.25. Every fuzzy semi pre-closed set in X is fuzzy πgr-closed in X.

Proof. Let λ be a fuzzy semi pre -closed set in X. Let μ be a fuzzy π-open such that $\lambda \leq \mu$. Since λ is fuzzy semi pre-closed, we have spcl(λ) = $\lambda \leq \mu$, such that spcl(λ) $\leq$ rcl(λ) $\leq \mu$. Hence λ is fuzzy πgr-closed set in X.

Remark 3.26. The converse of the theorem 3.25 is not true in general as shown in the following example.

Example 3.27. Consider X = {a, b} and τ = {1, 0, λ, μ, γ} is a fuzzy topology on X. The fuzzy sets are define as
$\lambda = \frac{0.4}{a} + \frac{0.4}{b}$, $\mu = \frac{0.6}{a} + \frac{0.7}{b}$, $\gamma = \frac{0.6}{a} + \frac{0.5}{b}$, fuzzy πgr-closed sets = {1, 0, μ}, fuzzy semi pre-closed sets = {1, 0, λ, γ} . Hence the μ in fuzzy πgr-closed set need not be fuzzy semi pre -closed set.

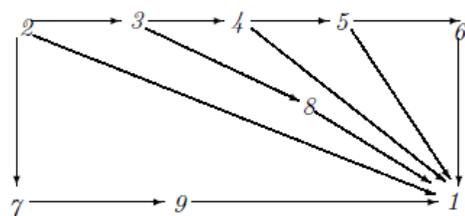**Remark 3.28.** The above discussion are shown in the following diagram .

Figure 3.1

1. $f\pi gr$-closed   6. fsp-closed..

2. fr-closed,   7. fw-closed.

3. $f\alpha$-closed,   8. fs-closed.

4. $fpre$-closed,   9. fg-closed.

5. fb-closed,

Theorem 3.29. The finite union of two fuzzy πgr-closed sets is fuzzy πgr- closed.

Proof. Assume that λ and γ are fuzzy πgr-closed sets in X. Let μ be a fuzzy π-open set in X such that $\lambda \cup \gamma \leq \mu$. Then $\lambda \leq$

μ and γ ≤ μ. Since λ and γ are fuzzy πgr-closed sets, rcl(λ) ≤ μ and rcl(γ) ≤ μ. Hence rcl(λ∪γ) = rcl(λ)∪rcl(γ) ≤ μ. That is , rcl(λ∪γ) ≤ μ. Hence λ∪γ is an πgr- closed set in X.

Example 3.30. Consider X = {a,b} and τ = {1, 0, λ, μ} is a fuzzy topology on X. The fuzzy sets are define as

$\lambda = \frac{0.5}{a} + \frac{0.5}{b}$, $\mu = \frac{0.3}{a} + \frac{0.5}{b}$, fuzzy πgr-closed sets = {1, 0, λ, μ}, λ is fuzzy πgr-closed set and μ is fuzzy πgr-closed union of two fuzzy πgr-closed sets is fuzzy πgr-closed sets.

Theorem 3.31. The finite intersection of two fuzzy πgr-closed sets fuzzy πgr-closed.

Proof. The proof is obvious.

Example 3.32. Consider X = {a,b} and τ = {1, 0, λ, μ} is a fuzzy topology on X. The fuzzy sets are define as

$\lambda = \frac{0.5}{a} + \frac{0.5}{b}$, $\mu = \frac{0.3}{a} + \frac{0.5}{b}$, fuzzy πgr- closed sets = {1, 0, λ, μ}, λ is fuzzy πgr-closed set and μ is fuzzy πgr-closed set intersection of two fuzzy πgr-closed sets is fuzzy πgr-closed set.

Theorem 3.33. If a subset λ of X is both are fuzzy π-open and are fuzzy πgr-closed, then it is fuzzy-closed .

Proof. Let λ be a subset of X which is both are fuzzy π-open and are fuzzy πgr-closed . Then rcl(λ) ≤ μ whenever λ ≤ μ and μ is fuzzy π-open. Also, λ ≤ rcl(λ), which implies rcl(λ) = λ. That is λis fuzzy regular closed . Hence λ is fuzzy closed.

Theorem 3.34. If λ is fuzzy πgr-closed sets and λ ≤ γ ≤ rcl(λ), then γ is also fuzzy πgr-closed subset of X.

Proof. Let λ be fuzzy πgr-closed set in X and γ ≤ μ, where μ is fuzzy π-open. Since λ ≤ γ, λ ≤ μ. Since λ is fuzzy πgr-closed, rcl(λ) ≤ μ. Given γ ≤ rcl(λ). Then rcl(γ) ≤ rcl(λ) ≤ μ. Which implies rcl(γ) ≤ μ and hence γ is fuzzy πgr-closed .

Theorem 3.35. If λ is fuzzy πgr-closed , then rcl(λ)−λ does not contain any non-empty fuzzy π-closed set .

Proof. Let F be a non-empty fuzzy π-closed set such that F ≤ rcl(λ) − λ. Which implies F < X − λ. The above implies λ ≤ X − F. Since λis fuzzy πgr-closed, X − F is fuzzy π-open. Since rcl(λ) ≤ X − F, F ≤ X − rcl(λ). Thus, F ≤ rcl(λ)∩(X −rcl(λ)) which implies F ≤ φ, which is a contradiction. Thus F = φ, whence rcl(λ)−λ does not contain any non-empty fuzzy π- closed set .

Corollary 3.36. Let λ be fuzzy πgr-closed set in X. Then λ is fuzzy regular closed iff rcl(λ) − λ is fuzzy π-closed .

Proof. Necessity: Let λ be fuzzy regular-closed. Then rcl(λ) = λ and so rcl(λ) − λ = φ, which is fuzzy π-closed .

Sufficiency: Suppose rcl(λ) − λ is fuzzy π-closed. Then rcl(λ) − λ = φ, since λ is fuzzy πgr-closed, we have rcl(λ) = λ which implies λ is fuzzy regular closed.

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