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Research Paper

Analysis of Various Performance Measures of a Cold Standby System with Arrival Time of Server

Mathematics

KEYWORDS : Cold Standby System, Waiting Time of Server, Preventive Maintenance, Priority, Performance Measures.

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ABSTRACT

The present study deals with the performance measures of a cold standby system with arrival time of server. For this purpose, two stochastic models are proposed here in which one unit is operative initially and other is kept in cold standby. There is a single server who always remains available in model-I whereas it takes some time to arrive in the system to perform repair activities in model-II. In both models, preventive maintenance is performed after a pre fixed maximum operation time and priority is given to operation of system over preventive maintenance. The failure time, maximum operation time follows exponential distribution whereas waiting time, preventive maintenance and repair are arbitrary distributed. Recurrence relations of various measures of system performance are obtained by using semi-Markov processes and RPT. To compare the performance of stochastic models difference graphs of several reliability measures are depicted.

INTRODUCTION

Many researcher including Gopalan and Nagarwall (1985), Goel and Sharma (1989), Cao and Wu (1989) suggested reliability models for repairable systems under a set of assumptions. Malik and Nandal [2010] carried out the economic analysis of repairable systems by using the concept of random appearance and disappearance of the server. Kumar et al. [2012] obtained various reliability measures of a computer system using concept of preventive maintenance after maximum operation time and maximum repair time.Barak and Malik [2014] and Kumar et al. [2015] suggested a stochastic model for a cold standby system with arrival time of server subject to MOT and MRT. Kumar and Baweja [2015] carried out the cost benefit analysis of a cold standby system with arrival time of server and preventive maintenance. In all the existing literature, researcher tried to develop reliability models under the assumption of preventive maintenance after maximum operation time and immediate arrival of the server.

But no research work is found under the combine set of assumption such as priority to operation over preventive maintenance, arrival time of server and repair. For this purpose, two stochastic models are proposed here in which one unit is operative initially and other is kept in cold standby. There is a single server who always remains available in model-I whereas it takes some time to arrive in the system to perform repair activities in model-II. In both models, preventive maintenance is performed after a pre fixed maximum operation time and priority is given to operation of system over preventive maintenance. The failure time, maximum operation time follows exponential distribution whereas waiting time, preventive maintenance and repair are arbitrary distributed. Recurrence relations of various measures of system performance are obtained by using semi-Markov processes and RPT. To compare the performance of stochastic models difference graphs of several reliability measures are depicted.

NOTATIONS

O	:	The unit is operative and in normal mode.
Cs	:	The unit is in cold standby.
λ	:	Constant failure rate of the unit.
α_0	:	Constant rate of Maximum Operation Time.
Pm/PM	:	The unit is under preventive Maintenance/ under preventive maintenance continuously from previous state.
FUr/FUR	:	The unit is failed and is under repair / under repair continuously from previous state
FWr / FWR	:	The unit is failed and is waiting for repair/ waiting for repair from previous state
Furp/FURP	:	The unit is failed and is under replacement / under replacement continuously from previous state

TRANSITION STATES AND PROBABILITIES

In view of the above notations and assumptions the system may be in one of the following states:

Model-I

$S_0 = (o, Cs)$, $S_1 = (o, Fur)$, $S_2 = (o, Pm)$, $S_3 = (PM, Fwr)$ and $S_4 = (FUR, Fwr)$

Model-II

$S_0 = (o, Cs)$, $S_1 = (o, Fwr)$, $S_2 = (o, Fur)$, $S_3 = (o, WPm)$, $S_4 = (o, Pm)$,

$S_5 = (Fwr, PM)$, $S_6 = (FUR, Fwr)$, $S_7 = (WPM, Fwr)$, $S_8 = (Pm, FWR)$,

$S_9 = (Fur, FWR)$, $S_{10} = (Fwr, FWR)$

By considering simple probabilistic arguments and using below mentioned formula, we easily obtained transition probabilities for all possible states of both models.

$$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)dt \tag{1}$$

RELIABILITY MEASURES

Mean time to system faliure (mtsf)

By simple probablistics arguments, we obtain the following recurrence relations for cdf (denoted by $V_i(t)$) of first passage time from the regenerative state S_i to a failed state treated as absorbing state:

$$V_i(t) = \sum_j Q_{i,j}(t) \Theta V_j(t) + \sum_k Q_{i,k}(t), \quad \begin{cases} i = 0,1,2 \;\; for \;\; Model - I \\ i = 0,1,2,3,4 \;\; for \;\; Model - II \end{cases} \tag{2}$$

The mean time to system failure (MTSF) is obtained by taking LST of above relations and using the formula appended below

$$\lim_{s \to 0} \frac{1 - V_0^{**}(s)}{s}.$$

AVAILABILITY ANALYSIS

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at t = 0. The recursive relations for $A_i(t)$ are given as

$$A_i(t) = \sum_j q_{i,j}(t) \Theta A_j(t) + M_i(t), \quad \begin{cases} i = 0,1,2 \;\; for \;\; Model - I \\ i = 0,1,2,3,4 \;\; for \;\; Model - II \end{cases} \tag{3}$$

where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ up at time t without visiting to any other regenerative state.

Taking LT from Eqs. (3) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty) = \lim_{s \to 0} s A_0^*(s) \tag{4}$$

SOME OTHER RELIABILITY MEASURES

By using probabilistic arguments, recurrence relations for other reliability measures such as busy period of the server due to repair, preventive maintenance, expected number of preventive maintenance, repairs, and expected number of vistis by the server respectively denoted by $B_i^R(t)$, $B_i^P(t)$, $R_i^p(t)$, $R_i^r(t)$, & $N_i(t)$ are obtained from one regenerative state to other regenerative state for both models as follows:

$$B^r{}_i(t) = \sum_j q_{i,j}(t)\Theta B^r{}_j(t) + W_i(t), \quad \begin{cases} i = 0,1,2 \ for\ Model - I \\ i = 0,1,2,3,4 \ \ for\ Model - II \end{cases}$$

$$B^p{}_i(t) = \sum_j q_{i,j}(t)\Theta B^p{}_j(t) + W_i(t), \quad \begin{cases} i = 0,1,2 \ for\ Model - I \\ i = 0,1,2,3,4 \ \ for\ Model - II \end{cases}$$

$$R_i^r(t) = \sum_j Q_{i,j}^{(n)}(t) \circledR \left[\delta_j + R_j^r(t)\right], \quad \begin{cases} i = 0,1,2 \ for\ Model - I \\ i = 0,1,2,3,4 \ \ for\ Model - II \end{cases}$$

$$R_i^{pm}(t) = \sum_j Q_{i,j}^{(n)}(t) \circledR \left[\delta_j + R_j^{pm}(t)\right], \quad \begin{cases} i = 0,1,2 \ for\ Model - I \\ i = 0,1,2,3,4 \ \ for\ Model - II \end{cases}$$

$$N_i(t) = \sum_j Q_{i,j}^{(n)}(t) \circledR \left[\delta_j + N_i(t)\right], \quad \begin{cases} i = 0,1,2 \ for\ Model - I \\ i = 0,1,2,3,4 \ \ for\ Model - II \end{cases} \tag{5}$$

Taking LT of above relations and solving for B_0^{*P} (s), B_0^{*R} (s), $R_i^{*r}(s)$, $R_i^{*pm}(s)$ & $N^{**}{}_i(t)$. The time for which server is busy due to preventive maintenance, repair, expected number of preventive maintenance, repirs, and expected number of visits is given by

$$B_0^P = \lim_{s\to 0} sB_0^{*P}(s),\ B_0^R = \lim_{s\to 0} sB_0^{*R}(s),\ R_0^r(\infty) = \lim_{s\to 0} s\tilde{R}_0^r(s), R_0^p(\infty) = \lim_{s\to 0} s\tilde{R}_0^p(s)$$

$$,N_0(\infty) = \lim_{s\to 0} s\tilde{N}_0(s)$$

PROFIT ANALISIS

The profit incurred to the system model by defining various costs as K_i in steady state can be obtained as follows:

$$P = K_0A_0 - K_1B_0^R - K_2B_0^P - K_3R_0^R - K_4R_0^P - K_5N_0 - K_6R_0^{RP}$$

Comparative Study:

The MTSF and availability of the model-II is less than that of the model-I for a particular case by considering all random variables as exponential distributed. However, model-II is more profitable over model-I under the condition of availability of server (i.e. whether the server is immediately available).Thus, it is concluded that a system is less available but more profitable to use where server is not immediately available as compared to a system in which servers is immediately available.

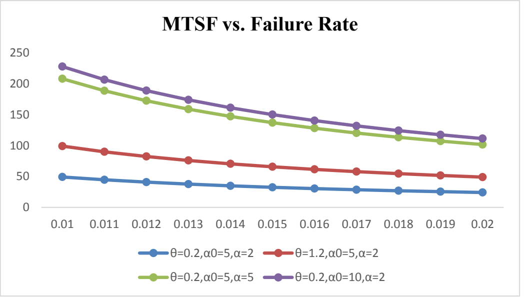

Fig 1: MTSF(Difference M1-M2) vs. Failure Rate

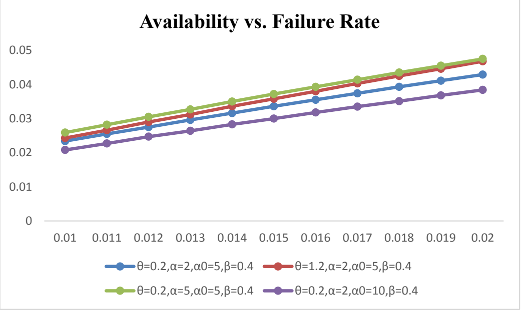

Fig 2: Availability(Difference M1-M2)vs. Failure Rate

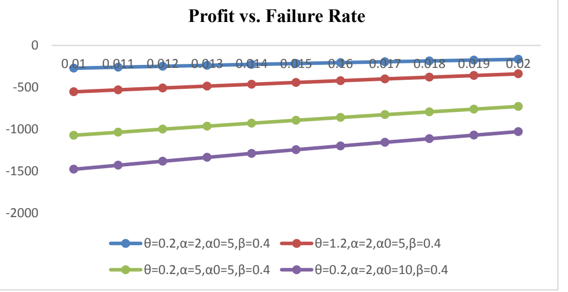

Fig 3: Profit (Difference M1-M2) vs. Failure Rate

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