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Research Paper

Certain Properties of Discrete Sushila Distribution and its Application

Statistics

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Prof. Munindra Borah — Department of Mathematical Sciences, Tezpur University, Napaam, Tezpur, assam, 784028

Krihna Ram Saikia — Research Scholar, Department of Mathematical Sciences, Tezpur University, Napaam, Tezpur, assam, 784028

ABSTRACT

An attempt has been made to introduce a new discrete distribution, discrete analogue of the continuous Sushila distribution, which we shall call a discrete Sushila distribution. The probability mass function and probability generating function of the distribution have been obtained. Size biased and Zero truncated forms of the distribution have been investigated. Certain recurrence relations for probabilities and moment have been derived. Estimation of its parameters of discrete Lindley distribution using method of moments has been discussed. The distribution has been fitted to some well known data set for testing its goodness of fit.

1. Introduction

Shanker et al (2013) introduced the following two parameter continuous Sushila distribution with parameter α and θ, of which the Lindley distribution is a particular case.

$$f(x;\,\theta,\alpha)=\frac{\theta^2}{\alpha(\theta+1)}\left(1+\frac{x}{\alpha}\right)e^{-\frac{\theta}{\alpha}x};\qquad x>0,\theta>0,\alpha>0 \qquad (1.1)$$

The superiority of the Sushila distribution have been illustrated by authors with an example.

It is sometimes impossible or inconvenient to measure the life length of a device, on a continuous scale. In practice, we come across situation, where lifetime of a device is considered to be a discrete random variable. For example, in the case of an on off switching device, the lifetime of the switch is a discrete random variable. If the lifetimes of individuals in some populations are grouped or when lifetime refers to a integral numbers of cycles of some sort, it may be desirable to treat it as a discrete random variable. When a discrete model is used with lifetime data, it is usually a multinomial distribution. This arises because effectively the continuous data have been grouped. Such situations may demand another discrete distribution, usually over the non negative integers. Such situations are best treated individually, but generally one tries to adopt one of the standard discrete distributions.

Some of those works are by Nakagawa and Osaki (1975), where the discrete Weibull distribution is obtained; Roy (2004) studied discrete Rayleigh distribution; Kemp (2008) derived discrete Half normal distribution. Krishna and Pundir (2009) investigated the discrete Burr and the discrete Pareto distribution. Gomez-Deniz (2010) derived a new generalization of the geometric distribution obtained from the generalized exponential distribution of Marshall and Olkin (1997). Chakraborty and Chakrvarty (2012) derived two parameter discrete gamma distributions. Fitting of the distribution and the performance of different estimation methods are also compared through simulation. Borah and Begum (2000) derived certain discrete distributions and studied their properties. Borah and Deka Nath (2001) derived certain inflated discrete distributions.

2. Discretization of Continuous Distribution

Discretization of continuous distribution can be done using different methodologies. In this paper we deal with the derivation of a new discrete distribution 'discrete Sushila distribution' which takes values in $\{0, 1, \ldots\ldots\ldots\ldots..\}$. This new distribution is generated by discretizing the continuous survival function of Sushila distribution, which is may be obtained as

$$s(x) = \int_x^\infty f(x;\ \theta, \alpha)dx$$
$$= \frac{e^{-\frac{\theta}{\alpha}x}\{\alpha(\theta+1)+\theta x\}}{\alpha(\theta+1)} \tag{2.1}$$

$$s(x+1) = \frac{e^{-\frac{\theta}{\alpha}(x+1)}\{\alpha(\theta+1)+\theta(x+1)\}}{\alpha(\theta+1)} \tag{2.2}$$

The probability mass function (pmf) of discrete Sushila (DS) distribution may be obtained as
$P(X = x) = S(x) - S(x+1)$

$$P(X = x) = \frac{e^{-\frac{\theta}{\alpha}x}\left[\{\alpha(\theta+1)+\theta x\}\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right]}{\alpha(\theta+1)} \qquad x = 0, 1, 2, \ldots \theta > 0, \alpha > 0. \tag{2.3}$$

3. Recurrence Relations for Discrete Sushila Distribution

Probability generating function may DS distribution be obtained as
$G(t) = \sum_{x=0}^{\infty} t^x P_x$

$$= \frac{\left[\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right]\left(1-te^{-\frac{\theta}{\alpha}}\right)+\theta te^{-\frac{\theta}{\alpha}}\left(1-e^{-\frac{\theta}{\alpha}}\right)}{\alpha(\theta+1)\left(1-te^{-\frac{\theta}{\alpha}}\right)^2} \tag{3.1}$$

(A) Probability Recurrence Relation:

Probability recurrence relation of DS distribution may be obtained as

$$P_r = e^{-\frac{\theta}{\alpha}}\left[2P_{r-1} - e^{-\frac{\theta}{\alpha}}P_{r-2}\right] \qquad \text{for } r > 1 \tag{3.2}$$

where $P_0 = \frac{\left[\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right]}{\alpha(\theta+1)}$ (3.3)

$$P_1 = \frac{e^{-\frac{\theta}{\alpha}}\left[\{\alpha(\theta+1)+\theta\}\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right]}{\alpha(\theta+1)} \quad (3.4)$$

(B) Factorial Moment Recurrence Relation

Factorial moment generating function of DS distribution may be written as
$M(t) = G(1+t)$

$$M(t) = \frac{\left[\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right]\left(1-e^{-\frac{\theta}{\alpha}}-te^{-\frac{\theta}{\alpha}}\right)+\theta(1+t)e^{-\frac{\theta}{\alpha}}\left(1-e^{-\frac{\theta}{\alpha}}\right)}{\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}-te^{-\frac{\theta}{\alpha}}\right)^2} \quad (3.5)$$

Factorial moment generating function may be obtained as

$$\mu'_{[r]} = \frac{e^{-\frac{\theta}{\alpha}}}{\left(1-e^{-\frac{\theta}{\alpha}}\right)^2}\left[2\left(1-e^{-\frac{\theta}{\alpha}}\right)e^{-\frac{\theta}{\alpha}}r\mu'_{[r-1]} - e^{-\frac{2\theta}{\alpha}}r(r-1)\mu'_{[r-2]}\right], \text{ where } r>1 \quad (3.6)$$

$$\mu'_{[1]} = \frac{e^{-\frac{\theta}{\alpha}}\left[\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)+\theta\right]}{\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)^2} \quad (3.7)$$

$$\mu'_{[2]} = \frac{2e^{-\frac{2\theta}{\alpha}}\left[\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)+2\theta\right]}{\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)^3} \quad (3.8)$$

The more general form of factorial moment may also be obtained as

$$\mu'_{[r]} = \frac{r!e^{-\frac{r\theta}{\alpha}}\left[\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)+r\theta\right]}{\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)^{r+1}} \quad (3.9)$$

The first four moments of the distribution have been obtained as

$$\mu_1 = \frac{e^{-\frac{\theta}{\alpha}}\left[\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)+\theta\right]}{\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)^2} \quad (3.10)$$

$$\mu_2 = \mu'_{[2]} + \mu'_{[1]} - \mu'^2_{[1]} \quad (3.11)$$

$$\mu_3 = \mu'_{[3]} + 3\mu'_{[2]} + \mu'_{[1]} - 3\mu'_{[2]}\mu'_{[1]} - 3\mu'^2_{[1]} + 2\mu'^3_{[1]} \quad (3.12)$$

$$\mu_4 = \mu'_{[4]} + 6\mu'_{[3]} + 7\mu'_{[2]} + \mu'_{[1]} - 4\mu'_{[3]}\mu'_{[1]} - 12\mu'_{[2]}\mu'_{[1]} - 4\mu'^2_{[1]} + 6\mu'_{[2]}\mu'^2_{[1]} + 6\mu'^3_{[1]} - 3\mu'^4_{[1]} \quad (3.13)$$

4. Size- biased Discrete Sushila (SBDS) Distribution

Size biased distributions arise in practice when observations from a sample are recorded with unequal probabilities, having probability proportional to some measure of unit size. Fisher (1934) first introduced Size biased distributions to model ascertainment bias which were later formalized by Rao (1965) in a unifying theory. Van Deusen (1986) discussed size- biased distribution theory and applied it to fitting distribution of diameter at breast height (DBH) data arising from horizontal point sampling (HPS). Later, Lappi and Baily(1987) used size-biased distributions to analyze HPS diameter increment data. Most of the statistical applications of these distributions, especially to analyse of data relating to human population and ecology can be found in Patil and Rao (1977,1978). Gove (2003) reviewed some of the recent results on size biased distribution pertaining to parameter estimation in forestry, with special emphasis on Weibull family. Adhikari and Srivastava (2013, 2014) and Dutta and Borah (2014a) studied some properties of certain size biased distributions.

If a random variable X have DS distribution with parameter α and θ then a simple size-biased distribution is given by its probability function

$f_s(x;\ \theta,\alpha) = \frac{xP_x}{\mu}$, P_x and μ denote respectively pmf and mean of DS distribution (4.1)

The pmf of discrete Sushila distribution with parameter α and θ given in (2.3) where mean of DS distribution may be obtained as $\mu = \frac{e^{-\frac{\theta}{\alpha}}\left[\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)+\theta\right]}{\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)^2}$

Therefore pmf of Size-biased discrete Sushila (SBDS) distribution with parameter α and θ has may be derived from (4.1) as

$$f_s(x;\ \theta,\alpha) = \frac{xe^{-\frac{\theta}{\alpha}(x-1)}\left(1-e^{-\frac{\theta}{\alpha}}\right)^2\left[\{\alpha(\theta+1)+\theta x\}\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right]}{\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)+\theta} \qquad where\ \ x = 1,2,3,\ldots \qquad (4.2)$$

5. Recurrence Relation of Size- biased Discrete Sushila (SBDS) Distribution

Probability generating function for SBDS may be obtained as

$$G_s(t) = \sum_{x=0}^{\infty} t^x P_x$$

$$= \frac{t\left(1-e^{-\frac{\theta}{\alpha}}\right)^2\left[\left\{\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right\}\left(1-te^{-\frac{\theta}{\alpha}}\right)+\theta\left(1-e^{-\frac{\theta}{\alpha}}\right)\left(1+te^{-\frac{\theta}{\alpha}}\right)\right]}{\left\{\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)+\theta\right\}\left(1-te^{-\frac{\theta}{\alpha}}\right)^3} \qquad (5.1)$$

(A) Probability Recurrence relation:

Probability recurrence relation of SBDJ distribution may be obtained as

$$P_r = e^{-\frac{\theta}{\alpha}}\left[3P_{r-1} - 3e^{-\frac{\theta}{\alpha}}P_{r-2} + e^{-\frac{2\theta}{\alpha}}P_{r-3}\right] \qquad \text{where } r > 2 \qquad (5.2)$$

where $P_1 = \frac{\left(1-e^{-\frac{\theta}{\alpha}}\right)^2\left[(\alpha(\theta+1)+\theta)\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right]}{\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)+\theta}$

(5.3)

$$P_2 = \frac{2e^{-\frac{\theta}{\alpha}}\left(1-e^{-\frac{\theta}{\alpha}}\right)^2\left[(\alpha(\theta+1)+2\theta)\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right]}{\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)+\theta}$$

(5.4)

(B) Factorial Moment Recurrence Relation

Factorial moment generating function of SBDS may be written as
$M_s(t) = G_s(1+t)$

$$M_s(t) = \frac{(1+t)\left(1-e^{-\frac{\theta}{\alpha}}\right)^2\left[\left\{\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right\}\left(1-e^{-\frac{\theta}{\alpha}}-te^{-\frac{\theta}{\alpha}}\right)+\theta\left(1-e^{-\frac{\theta}{\alpha}}\right)\left(1+e^{-\frac{\theta}{\alpha}}+te^{-\frac{\theta}{\alpha}}\right)\right]}{\left\{\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)+\theta\right\}\left(1-e^{-\frac{\theta}{\alpha}}-te^{-\frac{\theta}{\alpha}}\right)^3}$$

(5.6)

Factorial moment recurrence relation of SBDS may be obtained from (5.6) as

$$\mu'_{[r]} = \frac{e^{-\frac{\theta}{\alpha}}}{\left(1-e^{-\frac{\theta}{\alpha}}\right)^3}\left[3\left(1-e^{-\frac{\theta}{\alpha}}\right)^2 r\mu'_{[r-1]} - 3e^{-\frac{\theta}{\alpha}}\left(1-e^{-\frac{\theta}{\alpha}}\right)r(r-1)\mu'_{[r-2]} + e^{-2\frac{\theta}{\alpha}}\left(1-e^{-\frac{\theta}{\alpha}}\right)r(r-1)(r-2)\mu'_{[r-3]}\right]$$

(5.7)

6. Zero truncated discrete Sushila (ZTDS) distribution:

When the data to be modelled originate from a generating mechanism that structurally excludes zero counts, the discrete Sushila (DS) distribution must be adjusted to count for the missing zeros. Hospital length of stay data is good example. In this paper we consider the Zero-truncated DS distribution with pmf. Dutta and Borah (2014b) studied some properties of certain Zero- truncated and Zero- modified distributions.

$$P(X=x) = \frac{e^{-\frac{\theta}{\alpha}x}\left[\{\alpha(\theta+1)+\theta x\}\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right]}{\alpha(\theta+1)}, \qquad x = 0, 1, 2, \ldots\ldots\ldots$$

(6.1)

where $P_0 = P(X=0) = \frac{\left[\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right]}{\alpha(\theta+1)}$

The pmf $f_z(x;\, \theta, \alpha)$ of Zero-truncated discrete Sushila (ZTDS) distribution is

$$f_z(x;\, \theta, \alpha) = \frac{e^{-\frac{\theta}{\alpha}(x-1)}\left[\{\alpha(\theta+1)+\theta x\}\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right]}{\alpha(\theta+1)+\theta}, \quad x = 1, 2, \ldots\ldots\ldots$$

(6.2)

(A) Probability Recurrence Relation for ZTDS Distribution

Probability generating function $G_z(t)$ of ZTDS distribution may be obtained as

$G_z(t) = \sum_{x=1}^{\infty} t^x P_x$

$$G_z(t) = \frac{t\left[\left\{\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right\}\left(1-te^{-\frac{\theta}{\alpha}}\right)+\theta\left(1-e^{-\frac{\theta}{\alpha}}\right)\right]}{\{\alpha(\theta+1)+\theta\}\left(1-te^{-\frac{\theta}{\alpha}}\right)^2} \qquad (6.3)$$

Probability recurrence relation may obtained from (6.3) as

$$P_r = e^{-\frac{\theta}{\alpha}}\left[2P_{r-1} - e^{-\frac{\theta}{\alpha}}P_{r-2}\right] \quad \text{for} \quad r > 2 \qquad (6.4)$$

where $P_1 = \frac{\left[\{\alpha(\theta+1)+\theta\}\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right]}{\alpha(\theta+1)+\theta}$ (6.5)

$$P_2 = \frac{e^{-\frac{\theta}{\alpha}}\left[\{\alpha(\theta+1)+2\theta\}\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right]}{\alpha(\theta+1)+\theta} \qquad (6.6)$$

(B) Factorial Moment Recurrence Relation

Factorial moment generating function of ZTDS distribution may be obtained as
$\mathrm{M(t)} = G_z(1+t)$

$$M(t) = \frac{(1+t)\left[\left\{\alpha(\theta+1)\left(1-e^{-\frac{\theta}{\alpha}}\right)-\theta e^{-\frac{\theta}{\alpha}}\right\}\left(1-e^{-\frac{\theta}{\alpha}}-te^{-\frac{\theta}{\alpha}}\right)+\theta\left(1-e^{-\frac{\theta}{\alpha}}\right)\right]}{\{\alpha(\theta+1)+\theta\}\left(1-e^{-\frac{\theta}{\alpha}}-te^{-\frac{\theta}{\alpha}}\right)^2} \qquad (6.7)$$

$$\mu'_{[r]} = \frac{e^{-\frac{\theta}{\alpha}}}{\left(1-e^{-\frac{\theta}{\alpha}}\right)^2}\left[2\left(1-e^{-\frac{\theta}{\alpha}}\right)e^{-\frac{\theta}{\alpha}}r\mu'_{[r-1]} - e^{-\frac{2\theta}{\alpha}}r(r-1)\mu'_{[r-2]}\right] \qquad (6.8)$$

$$\mu'_{[1]} = \frac{\left[(\theta+\alpha^2)\left(1-e^{-\frac{\theta}{\alpha}}\right)+\alpha\theta\right]}{(\theta+\alpha^2+\alpha\theta)\left(1-e^{-\frac{\theta}{\alpha}}\right)^2} \qquad (6.9)$$

$$\mu'_{[2]} = \frac{2e^{-\frac{\theta}{\alpha}}\left[(\theta+\alpha^2)\left(1-e^{-\frac{\theta}{\alpha}}\right)+2\alpha\theta\right]}{(\theta+\alpha^2+\alpha\theta)\left(1-e^{-\frac{\theta}{\alpha}}\right)^3} \qquad (6.10)$$

The more general form of factorial moment may also be written as

$$\mu'_{[r]} = \frac{r!e^{-\frac{(r-1)\theta}{\alpha}}\left[\{\alpha(\theta+1)-\theta\}\left(1-e^{-\frac{\theta}{\alpha}}\right)+r\theta\right]}{\{\alpha(\theta+1)+\theta\}\left(1-e^{-\frac{\theta}{\alpha}}\right)^{r+1}} \qquad (6.11)$$

7. Estimation of Parameters

Discrete Sushila distribution has two parameters to be estimated. The first two moments are used to get the initial guest values then Newton - Raphson iterative method has been used to get the suitable estimates.

8. Goodness of Fit

The discrete Sushila distribution has been fitted to three number of data- sets to which earlier the Poisson Lindley has been fitted. The fitting of the distribution have been presented in the following tables.

Table 1: Distribution of mistakes in copying groups of random digits. Data from Sankaran (1970)

No. of errors per group	Observed frequencies	Expected frequencies	
		Poisson-Lindley (θ)	DS distribution with parameter (α, θ)
0	35	33.1	32.91
1	11	15.3	14.98
2	8	6.8	6.91
3	4	2.9	3.53
4	2	1.2	1.67
Total	60	60.0	60.00
		$\hat{\theta} = 1.7434$	$\hat{\alpha} = 98.0$
			$\hat{\theta} = 74.0$
	χ^2	2.20	1.49
	d.f.	3	2

Table 2: Accidents to 647 women working on high explosive shells. Data from Sankaran (1970)

No. of accidents	Observed frequencies	Expected frequencies	
		Poisson-Lindley (θ)	DS distribution with parameter (α, θ)
0	447	439.5	437.35
1	132	142.8	141.93
2	42	45.0	46.02
3	21	13.9	14.97
4	3	4.2	4.97
$\geq$ 5	2	1.3	1.76
	647	647.00	647.00
		$\hat{\theta}$=2.729	$\hat{\alpha} = 75.0$
			$\hat{\theta} = 85.5$
	χ^2	4.82	4.13
	d.f.	3	2

Table 3: Distribution of Pyrausta nublilalis in 1937.

No. of accidents	Observed frequencies	Expected frequencies	
		Poisson-Lindley (θ)	DS distribution with parameter (α, θ)
0	33	31.5	30.89
1	12	14.2	13.95
2	6	6.1	6.31
3	3	2.5	2.88
4	1	1.0	1.35
5	1	0.7	0.62
Total	56	56.0	56.00
		$\hat{\theta} = 1.81081$	$\hat{\alpha} = 101$ $\hat{\theta} = 82$
	χ^2	0.53	0.44
	d.f.	3	2

Conclusions

In this article, a two parameter continuous Sushila distribution has been discretized. The proposed distribution may be called 'discrete Sushila' distribution. Several properties of the distribution such as recurrence relations for probabilities moments have been investigated. Size biased and Zero- truncated forms of the distribution have been discussed. Finally, an application of the proposed distribution has been tested by fitting the distribution to data sets. Three sets of real data have been considered. The first set of data represents the mistakes in copying groups of random digits and the second is regarding the distribution of Pyrausta nublilalis in 1937. The third set of data represents the number of accidents to 647 women working on high explosive shell in 5 weeks. This new proposed model seems to be suitable for modelling and thus provides a better alternative to discrete Poisson Lindley for modelling different types of count data.

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