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Research Paper

Some Results on n -Edge Magic Labeling –Part 1

Mathematics

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S.Vimala

Assistant Professor in Mathematics, Mother Teresa Women's University, Kodaikanal

ABSTRACT

Let G (V, E) be a graph with order p and size q and G be simple and finite graph. n-edge magic introduced by [8] and extended from 1-edge magic labelling[9] and 0 –edge magic labelling[6]. In this paper extend n – edge magic labeling to Bistar, $K_{1,t}$ º P_{t-1}, Möbius Ladder and wheel Graph.

1. Introduction

All graphs in this paper are finite, simple, without loop, planer and undirected. Labeling of a graph G is a mapping that carries graph elements to integers. The origin of this labeling is introduced by Kotzig and Rosa[1,2]. Dealing with labeling have domain either the set of all vertices, or the set of all edges, or the set of all vertices and edges, respectively. This named as vertex labeling, or an edge labeling, or a total labeling, depending on the graph elements that are being labeled. Magic graph defined many others it is helping unsolved applications. Edge –Anti magic labelling is the motivation [4,5] of 0-edge magic and 1 – edge magic labelling. Later on n –edge magic introduced by Neelam Kumari [8] and solved for some class like P_n, C_n (n being odd +ve integer).This paper extend to some other class like above mentioned.

2. Preliminaries and Main Results

An **edge-magic labeling** of a (p; q)-graph G is a bijective function f : V(G) ∪ E(G)→{1; 2; : : : ; p + q} such that f(u) + f(v) + f(uv)=k is a constant for any edge uv of G. In such a case, G is said to be edge-magic and k is called the valence of f.

0-Edge Magic Labeling: Let G = (V, E) be a graph where V = {v_i, $1 \le i \le n$}, and E = {v_i v_{i+1},$1 \le i \le n$}. Let f : V→ {-1, 1}, and f * : E → {0}, such that all uv є E, f *(uv) = f (u) + f (v) = 0 then the labeling is said to be 0- Edge Magic labeling.

A (p, q) graph G is said to be **(1,0) edge-magic** with the common edge count k if there exists a bijection f : V(G) → {1, …, p} such that for all e = uv є E(G), f(u) + f(v) = k. It is said to be (1, 0) edge anti-magic if for all e = (u,v) є E(G), f(u) + f(v) are distinct.

A (p,q) graph G is said to be **(0,1) vertex-magic** with the common vertex count k if there exists a bijection f: E(G) → {1, …, q} such that for each u є V(G), e є Σ f(e) =k for all e = uv є E(G) with v є V(G). It is said to be (0, 1) vertex-antimagic if for each u є V (G), e є Σ f(e) are distinct for all e = uv є E(G) with v є V(G).

Let G= (V, E) be a graph where V= {vi, $1 \le i \le t$} and E = {v_i v_{i+1},$1 \le i \le t-1$}. Let f: V → {-1, 2} and f*: E → {1} such that for all uv Є E, f*(u v) = f (u) + f (v) =1 then the labelling is said to be **1-Edge Magic Labeling**.

A (p,q) graph G is said to be **(1,1) edge-magic** with the common edge count k if there exists a bijection f : V(G) ∪ E(G) → {1, …, p+q} such that f(u) + f(v) +f(e) = k for all e = uv ϵ E(G). It is said to be (1,1) edge-antimagic if f(u) + f(v) + f(e) are distinct for all e = uv ϵ E(G).

G_+ = GʘK$_1$ is a graph obtained by joining exactly one pendant edge to every vertex of a graph G.

A **sun S_t** is a cycle on t vertices with an edge terminating in a vertex of degree 1 attached to each vertex on the cycle.

A **complete binary tree** T is a tree with a central vertex of degree 2 , all other vertices that are not leaves of degree 3, and all leaves at the same distance from the central vertex.

Let G=(V, E) be a graph where V= { vi, 1≤i≤ t } and E = {vi v i+1,1≤i≤t-1}.
Let f: V → {-1, n+1} and f*: E → {n} such that for all uv E, f*(u v) = f (u) + f (v) = n then the labeling is said to be **n-Edge Magic Labeling**.

Theorem 1 P_t admits n-Edge Magic Labeling for all t[8]
Theorem 2 C_t admits n-Edge Magic Labeling when t is even[8]

Theorem 3 A sun graph S_t is n-edge magic only when t is even [8]

Theorem 4 If G admits n-Edge Magic Labeling then GʘK$_1$ admits n-Edge Magic Labeling[8]

Theorem 5 Let G =$S_{m,t}$ be a double star graph then G admits n-Edge Magic Labeling[8]

The results extends to Bistar, $K_{1,t} \circ P_{t-1}$, wheel graph and Möbius Ladder graphs.

Theorem 6 Let G = Bn,n be a Bistar graph then G admits n-Edge Magic Labeling.

Proof: Let G = (V,E) be a Bistar graph denoted by Bn,n and v_1 and v_2 are two vertices in Bn,n which are not pendent.

Let u_i's are n pendent vertices to v_1 and v_j's are n pendent vertices to v_2

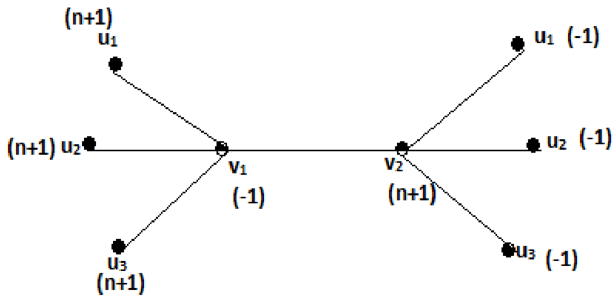
Define as in

Let f: v → { -1, n+1} such that

	f(v_1)	=	-1	
	f(v_2)	=	n+1	
	f(u_i)	=	n+1	for $1 \le i \le n$
	f(u_j)	=	-1	for $1 \le j \le n$
f*(v_1, u_i) =	-1+(n+1)	=	n	if $1 \le i \le n$
f*(v_2, u_j) =	(n+1)-1	=	n	if $1 \le j \le n$

Hence the Proof.

Example : $B_{3,3}$



Theorem 7 Let $G = K_{1,t} \circ P_{t-1}$ be a graph then G admits n-Edge Magic Labeling.

Proof: Let G = (V,E) be $K_{1,t} \circ P_{t-1}$ graph and v_i pendent vertices in $K_{1,t} \circ P_{t-1}$

Let u_i's vertices and v_i's are n pendent vertices to u_1

Let f: v → { -1, n+1} such that

$$f(u_i) = \begin{cases} -1, & if\ i\ is\ odd \\ n+1, & if\ i\ is\ even \end{cases}$$

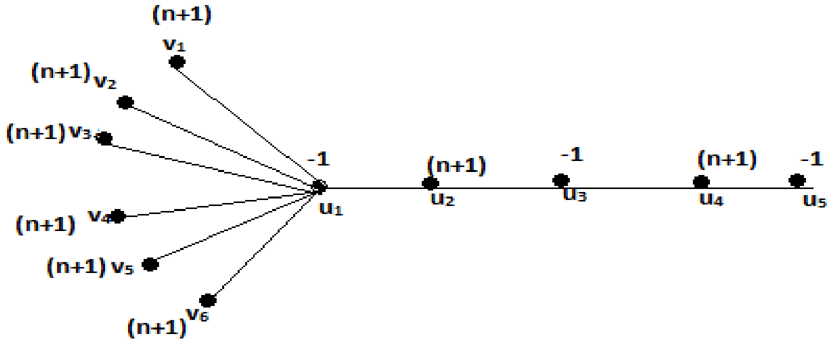
$$f(v_i) = n+1 \qquad 1 \le i \le t$$

$$f^*(u_1 v_i) = -1+(n+1) = n \qquad 1 \le i \le t$$

$$f^*(u_i u_{i+2}) = -1+(n+1) = n$$

Hence the proof.

Example : $K_{1,6} \circ P_5$



Theorem 8 Let G = Wn be a graph then G admits n-Edge Magic Labeling n ≡ 0 (mod 2)

Proof: Let G = (V,E) be a Wheel graph denoted by Wn and u_1 is the Hub in Wn

Let v_i is the vertices of Wn

Let f: V → {-1, n+1}

$$f(v_i) = \begin{cases} -1, & if\ i\ is\ odd \\ n+1, & if\ i\ is\ even \end{cases}$$

$f(u_1) = -1$

$f^*(u_1\, v_i) = (n+1)-1 = n$ i is even

$f^*(v_i\, v_{i+1}) = -1+(n+1) = n$ i is even and odd

Hence the proof.

Example : W_6

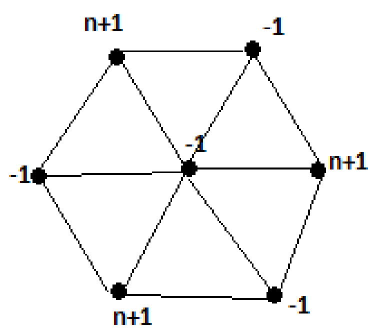


FIG 4.10

Theorem 9 Let G = M_n (Mobius Ladder) be a graph then G admits n-Edge Magic Labeling n ≡ 0 (mod 2)

Proof: Let G = (V, E) be a Mobius Ladder graph denoted by M_n and v_i and u_i are vertices in M_n each vertices are adjacent.

Let f: V → {-1, n+1} such that

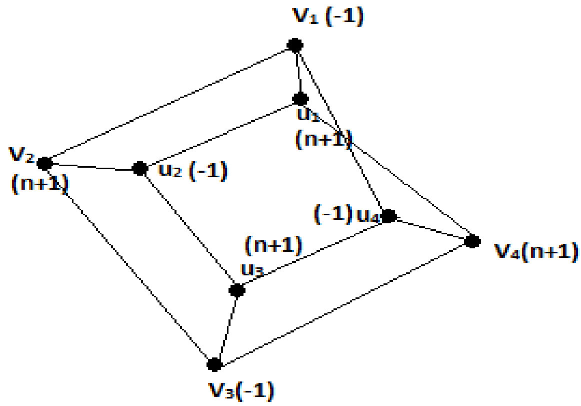
$$f(v_i) = \begin{cases} -1, & if\ i\ is\ odd \\ n+1, & if\ i\ is\ even \end{cases}$$

$$f(u_i) = \begin{cases} -1, & if\ i\ is\ even \\ n+1, & if\ i\ is\ odd \end{cases}$$

$f^*(v_i\, v_{i+1})$	=	-1+(n+1)	=	n	$1 \leq i \leq n-1$
$f^*(u_i\, u_{i+1})$	=	(n+1)+(-1)	=	n	$1 \leq i \leq n-1$
$f^*(v_iu_i)$	=	-1+(n+1)	=	n	$1 \leq i \leq n$

Hence the Proof

Example :M_4



3. Conclusion

The author extending the work to many graph with applications in the future articles.

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