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Zc-open sets and Zs-open sets in topological spaces.

Mathematics

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ABSTRACT

In this paper we introduce the concept of Zc-open sets and Zs-open sets and relates this concept to Z-open, θ-semiopen sets and extremally disconnected, separation spaces.

1.Introduction:

The topological spaces have been investigated from different aspects in the recent past years. In 2011,A.I.EL-Magharabi and A.M.Mubarki [9] introduced the concept of Z-open sets. Open sets ofcourse semi-open sets stand among the most important and most researched points in every part of mathematics especially in Topology.

Throughout this paper (X,τ) or simply X represent topological space with topology τ.No separation axioms are assumed unless otherwise stated explicitly. For a subset A of X, the closure of A and the interior of A will be denoted by $cl(A)$ and $int(A)$ respectively. The end or omission of the theorem or proposition is denoted by.

Stone defined a subset A of a topological space (X,τ) to be regular open(resp.regular closed) [15]if $A=int(cl(A))$ (resp. $A=cl(int(A))$).The delta interior[16] of a subset A of X is the union of all regular open sets of X contained in A and is denoted by δ-$int(A)$.A subset A of a space X is called δ-open if it is the union of regular open sets.A subset A of (X,τ) is called δ-closed[16] if $A= \delta$-$cl(A)$,where δ-$cl(A)=\{x\in X: A\cap int(cl(U))\neq \emptyset, U\in\tau \text{ and } x\in U\}$.A subset A of a space (X,τ) is called α-open[11](resp. δ-semiopen[12],semiopen[8], δ-preopen [13], preopen[10],b-open[2],e-open[6])if $A\subseteq int(cl(int(A)))$,(resp. $A\subseteq cl(\delta\text{-}int(A))$, $A\subseteq cl(int(A))$, $A\subseteq int(\delta\text{-}cl(A))$, $A\subseteq int(cl(A))$, $A\subseteq int(cl(A))\cup cl(int(A))$, $A\subseteq cl(\delta\text{-}int(A))\cup int(\delta\text{-}cl(A))$).

Joseph and Kwack [7] introduced that a subset A of a space X is called θ-semiopen if for each $x\in A$, there exists a semi-open set G such that $x\in G\subset cl(G)\subset A$.Velicko[16] defined a subset A of a space X is called θ-open if for each $x\in A$, there exists an open set G such that $x\in G\subset cl(G)\subset A$.Noiri [12]introduced semi θ-open set as, if for each $x\in A$, there exists a semiopen set G such that $x\in G\subset s\,cl(G)\subset A$. Recall that a set A of a space X is said to be semi-regular if it is both semiopen and semi closed.

Recall that a space X is called T_1 if for every two distinct points x, y in X ,there exists two open sets one containing x but not y and the other containing y not x.
Ahmed[1] defined a topological space (X,τ) to be s^{**}-normal if and only if for every semi-closed set F, and every semi-open set G containing F, there exists an open set H such that $F\subset H\subset cl(H)\subset G$.

The family of all open (resp.,semiopen, α-open,pre-open, θ-semiopen ,semi- θ-open , θ-open, δ-open,regular open,semi closed,regular closed) subsets of X are denoted by τ (resp., $SO(X)$, $\alpha O(X)$,$PO(X)$, $\theta SO(X)$, $S\theta O(X)$, $\theta O(X)$, $\delta O(X)$, $RO(X)$, $SC(X)$, $RC(X)$.

In preliminary section 2, we provide basic definitions and results which are carry out our work.

In section 3,we introduce the definition of *Zc*-open set and analyse the relation between *Zc*-open sets and θ-semiopen, *Z*-open sets.

In section 4,we introduce *Zs*-open sets and study some of their properties.

2.Preliminaries:

We recall that a space X is regular if for each pair consisting a point x and a closed set B disjoint from x, there exists disjoint open sets containing x and B respectively .Equivalently, A space X is called regular if for each $x \in X$ and each open set H containing x there exists an open set G such that $x \subset G \subset cl(G) \subset H$.

Definition 2.1 [9] : A subset A of a space X is said to be

i) *Z*-open set if $A \subseteq cl(\delta\text{-}int(A)) \cup int(cl(A))$,

ii) *Z*-closed set if $int(\delta\text{-}cl(A)) \cap cl(int(A)) \subseteq A$. The family of all *Z*-open (resp. *Z*-closed sets) subsets of a space (X,τ) will be denoted by $ZO(X)$ (resp., $ZC(X)$).

We recall that a topological space X is said to be extremally disconnected [14] if $cl(G)$ is open for every open set G of X.

Theorem 2.2[4] : A space X is extremally disconnected if and only if $RO(X) = RC(X)$.
Type equation here.

Theorem 2.3[16**]:** A space X is extremally disconnected if and only if $\delta O(X) = \theta SO(X)$.

Theorem 2.4[17]: If X is s^{**}-normal, then $S\theta O(X) = \theta O(X) = \theta SO(X)$. Type equation here.

Definition 2.5[4]: A space X is called locally indiscrete if every open subset of X is closed.

Results 2.6[9]: We get the following

i)Every δ-semiopen set is *Z*-open ,

ii)Every pre-open set is *Z*-open,

iii)Every *Z*-open set is b-open,

iv)Every *Z*-open set is e-open.

But the converse of the above are not necessarily true in general as given by the following examples 2.7 and 2.8.

Example 2.7: Let $X=\{a,b,c,d\}$ with $\tau=\{\emptyset,\{a\},\{c\},\{a,b\},\{a,c\},\{a,b,c\},\{a,c,d\},X\}$ then

i) A subset $\{a\}$ of X is *Z*-open but not δ-semiopen.

ii) A subset $\{a,d\}$ of X is b-open but not *Z*-open.

iii) A subset $\{b,c\}$ of X is e-open set but not *Z*-open.

Example 2.8: Let $X=\{a,b,c\}$ with $\tau=\{\emptyset,\{a\},\{b\},\{a,b\},X\}$ then $\{b,c\}$ is Z-open set but not preopen.

Theorem 2.9[9]: Let X be a topological space with topology . Then
i)If $A\in \delta O(X)$ and $B\in ZO(X)$, then $A\cap B$ is Z-open.
ii) If $A\in \tau$ and $B\in ZO(X)$,then $A\cap B$ is Z-open.
iii) If $A\in \alpha O(X,\tau)$ and $B\in ZO(X)$ then $A\cap B$ is $ZO(X,\tau_A)$.

Lemma 2.10 [9]: Let (X,τ) be a topological space. Then
i)The union of arbitrary Z-open sets is Z-open.
ii)The intersection of arbitrary Z-closed sets is Z-closed.

Remark 2.11 [9]: Intersection of any two Z-open sets is need not be Z-open.

Lemma 2.12 : A space X is locally indiscrete if and only if every semi open set in X is closed.

Definition 2.13: A topological space (X,τ) is called
i) semi-T_1[1] if for every two distinct points x, y in X **,**there exist two semi open sets one containing x but not y and the other containing y not x.
ii)semi-regular [5] if for each $x\in X$ and each $H\in SO(X)$ containing x there exists $G\in SO(X)$ such that $x\in G\subseteq scl(G)\subseteq H$.

Proposition 2.14[9]: Let (X,τ) be a topological space.Then the closure of a Z-open subset of X is semiopen.
Proof: Let $A\in ZO(X)$.Then $cl(A)\subseteq cl(cl(\delta\text{-}int(A))\cup int(cl(A)))\subseteq cl(\delta\text{-}int(A))\cup cl(\text{int}(cl(A)))=cl(int(cl(A)))$.Therefore $cl(A)$ is semi open.

3.*Zc*-open sets:

In this section we introduce a new class of Zc-open sets using Z-open and closed sets.

Definition 3.1: A subset A of a space X is Zc-open`if for each $x\in A\in ZO(X)$,there exists a closed set F such that $x\in F\subset A$. A subset A of a space X is Zc-closed if $X-A$ is Zc-open. The family of all Zc-open(resp. Zc-closed) subsets of a topological space (X,τ) is denoted by $ZcO(X,\tau)$ or $ZcO(X)$ (resp. $ZcC(X,\tau)$ or $ZcC(X)$).

Example 3.2: Consider X={a,b,c,d}with $\tau=\{\emptyset,X,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\},\{a,b,c\}\}$.
Then the family of closed sets are $\{\emptyset,X,\{b,c,d\},\{a,b,d\},\{a,c,d\},\{c,d\},\{a,d\},\{b,d\},\{d\}\}$.
The family of Z-open sets are :
$ZO(X)=\{\emptyset,X,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\},\{a,d\},\{b,d\},\{c,d\},\{a,b,c\},\{a,c,d\},\{a,b,d\},\{b,c,d\}\}$
The family of Zc-open sets are : $ZcO(X)=\{\emptyset,X,\{a,d\},\{b,d\},\{c,d\},\{a,b,d\},\{b,c,d\},\{a,c,d\}\}$.
The family of Zc-closed sets are: $ZcC(X)=\{\emptyset,X,\{b,c\},\{a,c\},\{a,b\},\{c\},\{a\},\{b\}\}$.

Proposition 3.3:(i) A subset A of a space X is Zc-open if and only if A is Z-open and it is the union of closed sets. That is where A is Z-open and F_α is closed sets for each α.
(ii)A subset A of a space X is Zc-closed if and only if A is Z-closed and it is an intersection of open sets. ■

Remark 3.4: It is clear from the definition of *Zc*-open(resp. *Zc*-closed) sets, that every *Zc*-open(resp. *Zc*-closed) subset of a space *X* is *Z*-open, but the converse is not true in general as shown in example 3.2 where $\{a\},\{b\},\{c\}$ belongs to *ZO*(*X*) whereas $\{a\},\{b\},\{c\}$ does not belongs to *ZcO*(*X*) and $\{a,d\},\{b,d\}$ belongs to *ZO*(*X*) whereas $\{a\},\{b\},\{c\}$ does not belongs to *ZcC*(*X*).

Proposition 3.5:(i) Let $\{A_\alpha : \alpha \in \Lambda\}$ be a collection of *Zc*-open sets in a topological space *X*. Then $\cup\{ A_\alpha : \alpha \in \Lambda\}$ is *Zc*-open.
(ii) Let $\{A_\alpha : \alpha \in \Lambda\}$ be a collection of *Zc*-closed sets in a topological space *X*. Then $\cap\{ A_\alpha : \alpha \in \Lambda\}$ is *Zc*-closed.
Proof:(i) Let *A* be *Zc*-open set for each α. Then A_α is *Z*-open. By lemma 2.10 (i), $\cup\{ A_\alpha : \alpha \in \Lambda\}$ is *Z*-open. Let $x \in \cup\{ A_\alpha : \alpha \in \Lambda\}$, there exists $\alpha \in \Lambda$ such that $x \in A_\alpha$ for some α. Since A_α is *Z*-open for each α, there exists a closed set *F* such that $x \in F \subset A_\alpha \subset \cup\{ A_\alpha : \alpha \in \Lambda\}$. So $x \in F \subset \cup\{ A_\alpha : \alpha \in \Lambda\}$. Hence $\cup\{ A_\alpha : \alpha \in \Lambda\}$ is *Zc*-open set.
(**ii**) Obvious

Result 3.6:(i) Intersection of two *Zc*-open sets need not be *Zc*-open.
(ii) Union of two *Zc*-closed sets need not be *Zc*-closed.

Example 3.7: Let $X=\{a,b,c\}$, $\tau=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}$. Then closed sets are $\{\emptyset,X,\{b,c\},\{c,a\},\{c\}\}$. We get $ZO(X)= \{\emptyset,X,\{a\},\{b\},\{a,b\},\{b,c\},\{a,c\}\}$. Thus $ZcO(X)=\{ \emptyset, X,\{a,c\},\{b,c\}\}$ and $ZcC(X)=\{ \emptyset,X,\{b\},\{a\}\}$. Here $\{a,c\} \in ZcO(X)$ and $\{b,c\} \in ZcO(X)$ but $\{a,c\}\cap\{b,c\}=\{c\}\notin ZcO(X)$. Also $\{a\},\{b\} \in ZcC(X)$ but $\{a,b\}\notin ZcC(X)$.

Proposition 3.8: If the family of all *Z*-open sets form a topology on *X*, then the family of *Zc*-open sets also form a topology on *X*.
Proof: Clearly $\emptyset, X \in ZcO(X)$ and by proposition 3.5, the union of any family of *Zc*-open set is *Zc*-open. To complete the proof it is enough to show that the finite intersection of *Zc*-open sets is *Zc*-open. Let *A* and *B* be two *Zc*-open sets, then by Remark 3.4 , *A* and *B* are *Z*-open sets. Since *ZO*(*X*) is a topology on *X* ,so $A\cap B$ is *Z*-open by lemma 2.10. Let $x \in A\cap B$, then $x \in A$ and $x \in B$, so there exists *F* and *E* such that $x \in F \subset A$ and $x \in E \subset B$ this implies that $x \in F\cap E \subset A\cap B$. Since any intersection of closed sets is closed, $F\cap E$ is closed. Thus $A\cap B$ is *Zc*-open.

Proposition 3.9: The set *A* is *Zc*-open in *X* if and only if for each $x \in A$, there exists a *Zc*-open set *B* such that $x \in B \subset A$.
Proof: Assume, *A* is *Zc*-open in (X,τ). Then for each $x \in A$,say $A=B$ is *Zc*-open containing *x* such that $x \in B \subseteq A$. Conversely, suppose that for each $x \in A$, there exists a *Zc*-open set *B* such that $x \in B \subseteq A$. Thus $A= \cup B_x$ where $B_x \in ZcO(X)$ for each *x* and hence *A* is *Zc*-open set.

Proposition 3.10: If a topological space (X,τ) is locally indiscrete then $SO(X) \subset ZcO(X)$.
Proof: Let *A* be any subset of a space *X* and $A\in SO(X)$. If $A=\emptyset$ then $A \in ZcO(X)$. If $A\neq \emptyset$ then $A \subset cl(int(A))$. Since *X* is locally indiscrete ,then $int(A)$ is closed. In general $int(A) \subset A$, this implies that for each $x \in A$, $x \in int(A) \subset A$. Therefore *A* is *Zc*-open and hence $SO(X) \subset ZcO(X)$.

Proposition 3.11: Let (X,τ) be a topological space. If X is regular, then $\tau \in ZcO(X)$.
Proof: Let A be any subset of X and A is open. If $A=\emptyset$ then $A \in ZcO(X)$. If $A\neq \emptyset$, since X is regular so for each $x \in A \subset X$ there exists an open set G such that $x \in G \subset cl(G) \subset A$. Thus we have $x \in cl(G) \subset A$. Since $A \in \tau$ and hence $A \in ZO(X)$.Therefore $\tau \in ZcO(X)$.

Proposition 3.12: If a space X is T_1 ,then (i) $ZO(X)= ZcO(X)$
(ii)$ZC(X)= ZcC(X)$).
Proof:(i) Let A be any subset of a space X and let $A \in ZO(X)$.If $A= \emptyset$, then $A \in ZcO(X)$. If $A\neq \emptyset$,then there is $x \in A$. Since the space X is T_1, then every singleton set is closed and hence $x \in \{x\} \subset A$. Hence $A \subseteq ZcO(X)$.Thus $ZO(X) \subset ZcO(X)$.But we know that $ZcO(X) \subset ZO(X)$. Hence $ZO(X)= ZcO(X)$.
(ii)Obvious

Proposition 3.13: Every θ-semiopen set of a space X is a Zc-open set.
Proof: Let A be a θ-semiopen set in X, then for each $x \in A$, there exists a semi-open set G such that , $x \in G \subset cl(G) \subset A$. So $\cup\{x\} \in \cup G \subset \cup cl(G) \subset \cup A$ for each $x \in A$ which implies that, $A=\cup G$ which is a semi-open set and $A= \cup cl(G)$ is a union of closed sets. Therefore by proposition 3.3, A is a Zc-open set. Converse of the above proposition need not be true in general as shown in the following example.

Example 3.14: Since a space X with cofinite topology is T_1 and then the family of Z-open and Zc-open sets are identical. Hence any open set G is Zc-open but not θ-semiopen.

Result 3.15: (i) Every θ-open set is Zc-open.
(ii)Every Regular-closed set is Zc-open.
(iii)Every θ-closed set is closed.
(iv)Every Regular open set is Zc-closed.

Proposition 3.16: Let (X,τ) be an extremally disconnected space. If $A \in \delta O(X)$, then $A \in ZcO(X)$.
Proof: Let $A \in \delta O(X)$. If $A= \emptyset$ then $A \in ZcO(X)$. If since given X is extremally disconnected then by theorem 2.3 , $\delta O(X)= \theta\ SO(X)$. Hence $A \in \theta$SO(X).But $\theta SO(X) \subset ZcO(X)$ in general. Therefore we get $A\in ZcO(X)$.

Proposition 3.17: Let (X,τ) be an extremally disconnected space.If $A\in RO(X)$ then $A\in ZcO(X)$.
Proof: It follows from proposition 3.16 and the fact that $RO(X) \subset \delta O(X)$.

Proposition 3.18: Let (X,τ) be an s^{**}-normal space. If $A \in S\theta O(X)$, then $A \in ZcO(X)$.
Proof: Let $A \in S\theta O(X)$. If $A=\emptyset$ then $A\in ZcO(X)$. If $A\neq \emptyset$, since X is s^{**}-normal by Theorem 2.4, $S\theta O(X) = \theta SO(X)$.Hence $A \in \theta SO(X)$.But $\theta SO(X) \in ZcO(X)$ in general. Hence $A \in ZcO(X)$.

Proposition 3.19: If (X,τ) is finite, then $\theta SO(X)= ZcO(X)$.
Proof: By proposition 3.13 we have $\theta SO(X) \subset ZcO(X)$.Remaining to show, $ZcO(X) \subset \theta SO(X)$. Let $A \in ZcO(X)$. Then A is Z-open set and by proposition 3.3, $A=\cup\ F_\alpha$

where F_α is closed set for each α. Since X is finite, there exists a closed set F in X such that $F=\cup F_\alpha$ and hence $A=F$. This implies that A is both Z-open and closed. Therefore $A \in \theta ZO(X)$. Hence $\theta SO(X)= ZcO(X)$.

Proposition 3.20: Let (X,τ) be a topological space and A, $B \subseteq X$. If $A \in ZcO(X)$ and B is clopen ,then $A \cap B \in ZcO(X)$.
Proof: Let $A \in ZcO(X)$ and B is clopen .Then A is Z-open set. This implies by theorem 2.9 (ii) that $A \cap B \in ZO(X)$.Now let $x \in A \cap B$. Then $x \in A$ and $x \in B$. Therefore there exist a closed set F such that $x \in F \subseteq A$.Since B is clopen ,so B is closed set which implies that $F \cap B$ is closed.Therefore $x \in F \cap B \subseteq A \cap B$. Thus $A \cap B$ is Zc-open set in X.

Corollary 3.21: Let (X,τ) be a topological space and A, $B \subseteq X$. If $A \in ZcO(X)$ and B is open then $cl(A \cap B)$is semiopen.
Proof: $A \cap B \in ZcO(X)$, by Theorem 3.20.Then by Remark 3.4 $A \cap B$ is Z-open. Then $cl(A \cap B)$ is semiopen by proposition 2.14.

4. *Zs*-open sets

In this section we introduce *Zs*-open sets using Z-open and semiclosed sets and investigate some of their characterizations.

Definition 4.1: A subset A of a space X is *Zs*-open if for each $x \in A \in ZO(X)$,there exists a semiclosed set F such that $x \in F \subset A$. The family of all *Zs*-open subsets of a topological space (X,τ) is denoted by $ZsO(X,\tau)$ or $ZsO(X)$.

Example 4.2: Let X={a,b,c,d}with $\tau=\{\emptyset, X, \{a\},\{c\},\{a,b\},\{a,c\},\{a,b,c\}\{a,c,d\}\}$.The family of semi closed sets are $SC(X)=\{\emptyset, X, \{b\},\{c\},\{d\},\{a,b\},\{b,c\},\{b,d\},\{c,d\},\{a,b,d\}\{b,c,d\}\}$. The family of Z-open sets are $ZO(X)=\{\emptyset, X, \{a\},\{c\},\{a,b\},\{a,c\},\{c,d\},\{a,b,c\}\{a,c,d\}\}$. The family of *Zs*-open sets are $ZsO(X)=\{\emptyset, X, \{a,b\},\{c,d\},\{a,b,c\}\}$.

Proposition 4.3: A subset A of a space X is *Zs*-open if and only if A is Z-open and it is the union of semi closed sets. That is $A=\cup F_\gamma$ where A is Z-open and F_γ is semiclosed for each γ.
Proof: Obvious.

Remark 4.4: It is clear from the definition that every *Zs*-open subset of a space X is Z-open, but the converse is not true in general as shown in the example 4.2 ,where $\{a\} \in ZO(X)$ but $\{a\} \notin ZsO(X)$.

Proposition 4.5: If a space X is semi-T_1 ,then $ZO(X)= ZsO(X)$.

Proposition 4.6: i)Arbitrary union of *Zs*-open sets is *Zs*-open .

The following example shows that the intersection of two *Zc*-open sets need not be *Zc*-open in general.

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Example 4.7: Let X={a,b,c} with $\tau=\{\emptyset, X, \{a\},\{b\},\{a,b\}\}$.The semi closed sets are :
$SC(X)=\{\emptyset, X, \{a\},\{b\},\{c\},\{b,c\},\{c,a\}\}$.Then
$ZO(X)=\{\emptyset, X, \{a\},\{b\},\{a,b\},\{b,c\},\{c,a\}\}$.Hence,
$ZsO(X)=\{\emptyset, X, \{a\},\{b\},\{a,b\},\{b,c\},\{c,a\}\}$.Here $\{b,c\},\{c,a\}\in ZsO(X)$ but
$\{b,c\}\cap\{c,a\}=\{c\}\notin ZsO(X)$.

Proposition 4.8: Every semi- θ-open subset of a space X is a Zs-open.
Proof: Suppose that the subset A of X is semi- θ-open. Then by definition ,if for each $x \in A$, there exists a semi-open set G such that $x \in G \subset scl(G) \subset A$. Hence $scl(G)$ is the semi-closed set containing x and contained in A .Hence A is Zs-open.

Proposition 4.9: i) Every θ-semiopen subset of a space X is a Zs-open.
ii) Every Zc-open set is Zs-open.
iii) Every semi-regular subset of X is Zs-open.
The Examples 4.12 (i)and (ii) shows that the converse of above proposition is not true.

Proposition 4.10: Let (X,τ) be a semi regular space, then $\tau \subseteq ZsO(X)$.
Proof: Let A be any non-empty open subset of X. Then for each $x \in A$, there exists a semi-open set G such that $x \in G \subset scl(G) \subset A$. This implies that $x \in scl(G) \subset A$. Hence A is Zs-open.

Proposition 4.11: i) If A is a semiopen subset of a space X, then $scl(A)$ is Zs-open.
ii)If A is a semiclosed subset of a space X, then $sint(A)$ is Zs-open.

Example 4.12:(i) Consider $X=\{a,b,c,d\}$ with same topology as in example 4.2 we have,

$SC(X)=\{\emptyset, X, \{b\},\{c\},\{d\},\{a,b\},\{b,c\},\{b,d\},\{c,d\},\{a,b,d\}\{b,c,d\}\}$
$SO(X)=\{\emptyset, X, \{a,b\},\{a,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{a\},\{c\},\{a,c\}\}$
$ZsO(X)=\{\emptyset, X, \{a,b\},\{c,d\},\{a,b,c\}\}$.Hence the set $\{a,b,c\}$ is Zs-open set but it is not θ-semiopen and also not semi-regular set.

(ii)Consider $X=\{a,b,c,d\}$ with same topology as in example 3.2 we have, Zc-open sets,
$ZcO(X)=\{\emptyset, X,\{a,d\},\{b,d\},\{c,d\},\{a,c,d\},\{a,b,d\},\{b,c,d\}\}$ and Zs-open sets,
$ZsO(X)=\{\emptyset, X,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\},\{a,d\},\{b,d\},\{c,d\},\{a,b,c\},\{a,c,d\},\{a,b,d\},\{b,c,d\}\}$.Here the sets $\{a\},\{b\},\{c\},\{a,b\}$are all Zs-open sets but not Zc-open.

We get the following diagram of implications:

open → semi- θ-open ← θ-semiopen

↓ ↓ ↓

Z-open ← Zs-open ← Zc-open

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