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Bianchi type i Viscous Fluid Cosmological Models with Cosmological Term $\Lambda(t)$

Mathematics
KEYWORDS :

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ABSTRACT

In this paper, we have studied anisotropic Bianchi-type I space time with bulk and shear viscosity in the context of general relativity. Exact solution of Einstein field equations are presented by a suitable functional form for the Hubble parameter , which yields a model of the universe that describe an early deceleration and late time acceleration.

Introduction:-

Cosmology is the scientific study of the large scale properties of the universe as a whole. It endeavours to use the scientific method to understand the origin, evolution and ultimate fate of the entire universe. Cosmology is soundly based on observations, mostly astronomical and laws of physics. In recent cosmological observations of Type Ia Supernova (SNeIa)[1,2], cosmic microwave back ground (CMB) anisotropies [3].

In the investigation of relativistic cosmological models, distribution of matter can be satisfactorily described by a perfect fluid due to the large scale distribution of matter. However, observation of large entropy per baryon and the remarkable degree of isotropy of the cosmic background radiation suggest the analysis of dissipative effects in cosmology .Dissipative effects, including both bulk and shear viscosity, plays a very important role in the evolution of the universe. The viscosity theory of relativistic fluid was first suggested by Eckart [4]. The inclusion of dissipative term in the energy-momentum tensor of cosmic fluids seems to be the best motivated generalization of the matter term of the gravitational field equations. Misner [5,6] suggested that the strong dissipation due to the neutrino viscosity may considerably reduce the anisotropy of the black-body radiation. According to Weinberg [7] and Harrison [8], the viscosity mechanism in cosmology can explain the anomalously high entropy per baryon in the present universe. A uniform cosmological model filled with fluid which possesses pressure and bulk viscosity was developed by Murphy [9]. Padmanabhan and chitre [10] studied that presence of bulk viscosity leads inflationary like solutions in general relativity. Another characteristic of bulk viscosity is that it acts like a negative energy field in an expanding universe [11]. The nature of cosmological solutions for homogeneous Bianchi type I model was studied by Bilinski and Khalatnikov [12] by taking into account dissipative process due to viscosity. Bianchi type I solution in the case of siff matter with shear viscosity being the power function of energy density were obtained by Banerjee et al.[13] whereas models with bulk viscosity as a power function of energy density and stiff matter were investigated by Huang [14]. The effect of bulk viscosity on the cosmological evolution has been studied by number of workers namely, Peebles [15], Saha [16], Bali and Pradhan [17], Bali and Kumawat [18], Bali [19], Singh and Baghel [20], Singh et. al [21].

Models with dynamic cosmological term $\Lambda(t)$ are becoming popular as they solve the cosmological constant problem in a natural way. There is a significant observational evidence for the detection of cosmological constant Λ or a component of material content of the universe that varies slowly with the time and space and so acts like Λ. Some of the recent discussions on the cosmological constant problem and consequence on cosmology with a time varying cosmological constant have been studied by Dolgov [22], Sahni and Starobinsky [23], Padmnabhan [24], Vishwakarma [25], Pradhan et al.[26] and Singh et al [27].

Motivated from the studied outlined above, in this paper, we have investigated Bianchi type I models the simplest anisotropic generalization of FRW models with flat space slices. Matter content is taken to viscous fluid. We obtain exact solutions of Einstein field equations assuming a functional from of the Hubble parameter. Cosmological consequence of the resulting models have been discussed.

The metric and the field equations:-

We consider homogeneous and anisotropic Bianchi type-I space time is described by the line element

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \qquad (1)$$

Where A,B and C are the metric function of cosmic time t.

We assume, the energy momentum tensor of a viscous fluid has the form

$$T_{ij} = (\rho+\bar{p})u_i u_j + \bar{p}g_{ij} - \eta u_{ij} \quad (2)$$

with

$$\bar{p} = p - \left(\xi - \frac{2}{3}\eta\right)u^i_{;j}$$
$$\bar{p} = p - (3\xi - 2\eta)H \qquad (3)$$

And

$$u_{ij} = u_{i;j} + u_{j;i} + u_i u^{\alpha} u_{j;\alpha} + u_j u^{\alpha} u_{i;\alpha} \qquad (4)$$

Here ξ and η stand for the bulk and shear viscosity coefficient respectively; ρ is the matter density; p is the isotropic pressure and u^i is the four velocity vector satisfying

$$u^i u_i = -1 \qquad (5)$$

We choose the coordinates to be commoving, so that

$$v^1 = 0 = v^2 = v^3, v^4 = 1 \qquad (6)$$

The Einstein's field equations (in gravitational units $8\pi G = c = 1$) with time-dependent cosmological term Λ are given by

$$R_{ij} - \frac{1}{2}R_k^k g_{ij} = -T_{ij} + \Lambda g_{ij} \tag{7}$$

The Einstein's field equation (7) for the line element (1) becomes

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - 2\eta\frac{\dot{A}}{A} = -p + \left(\xi - \frac{2\eta}{3}\right)\theta + \Lambda \tag{8}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} - 2\eta\frac{\dot{B}}{B} = -p + \left(\xi - \frac{2\eta}{3}\right)\theta + \Lambda \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - 2\eta\frac{\dot{C}}{C} = -p + \left(\xi - \frac{2\eta}{3}\right)\theta + \Lambda \tag{10}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} = \rho + \Lambda \tag{11}$$

Where the dot (.) means ordinary differentiation with respect to t and θ is the scalar of expansion given by

$$\theta = u^i_{;i} \tag{12}$$

The average scale factor R of Bianchi type-I model (1) is defined as

$$S = (ABC)^{1/3} \tag{13}$$

A volume scale factor V is given by

$$V = S^3 = ABC \tag{14}$$

In analogy with FRW universe, we define a generalized Hubble parameter H and generelized deceleration parameter q as

$$H = \frac{\dot{S}}{S} = \frac{(H_1 + H_2 + H_3)}{3} \tag{15}$$

$$q = \frac{-\dot{H}}{H^2} - 1 = 2 - \frac{3V\ddot{V}}{\dot{V}^2} \tag{16}$$

Where

$$H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}, H_3 = \frac{\dot{C}}{C},$$

are directional Hubble's factor along x,y and z directions respectively. The physical quantities of observational interest in cosmology i.e. expansion scalar θ and shear tensor σ_i^j for the metric (1) leads to

$$\theta = 3H \tag{17}$$

$$\sigma_1^1 = H_1 - H, \sigma_2^2 = H_2 - H, \sigma_3^3 = H_3 - H, \sigma_4^4 = 0 \tag{18}$$

Shear scalar σ is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{6}\left\{\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)^2 + \left(\frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right)^2\right\} \tag{19}$$

where

$$\sigma_{ij} = u_{i;j} + \frac{1}{2}\left(u_{i;k}u^k u_j + u_{j;k}u^k u_i\right) + \frac{1}{3}\theta\left(g_{ij} + u_i u_j\right) \tag{20}$$

For the physical relevant model, the Hubble parameter ***H*** and deceleration parameter ***q*** are the most important observational quantities. The first quantity sets the present time scale of expansion while the second one reveals that the present state of evolution of universe is speeding up instead of slowing down as expected before the type Ia supernova observations [1,2]. The law of variation for Hubble parameter was initially proposed by Berman [28] for FRW models that yields a constant value of deceleration parameter. Singh [29] studied a cosmological models by assuming a functional relation between Hubble parameter ***H*** and average scale factor ***S***. We assume a functional form for the Hubble parameter ***H*** which yields a model of the universe that describes an early deceleration and late time acceleration. We consider

$$H = m\tanh(t) + n\coth(t) \tag{21}$$

where*m* and *n* are constants.

For this assumption, we obtain scale factor ***S***, spatial volume ***V***, expansion scalar $\boldsymbol{\theta}$ and deceleration parameter ***q*** as

$$V = S^3 = [\cosh^m(t)\sinh^n(t)]^3 \tag{22}$$

$$\theta = 3[m\tanh(t) + n\coth(t)] \tag{23}$$

$$q = \frac{-[m\,\mathrm{sech}^2(t) - n\,\mathrm{csch}^2(t)]}{(m+n)^2 + n^2\mathrm{csch}^2(t) - m^2\mathrm{sech}^2(t)} - 1 \tag{24}$$

We observe that spatial volume is zero and expansion scalar is infinite at *t=0*, which shows that the universe starts evolving with zero volume at initial epoch with an infinite rate of expansion. At $t = 0, q = -1 - \frac{1}{n} > 0$ provided $-1 < n < 0$ and for $t = \infty$, $q = -1$. Thus the model universe represents initial deceleration and late time acceleratimg expansion. The derived model can be utilized to describe the dynamics of the universe consistent with recent cosmological observations.

From equations (8)- (12), we obtain

$$\dot{\rho} + (\overline{p} + \rho)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \dot{\Lambda} = 4\eta\sigma^2 \tag{25}$$

From equations (8)- (11), we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\left(\frac{\dot{C}}{C} + 2\eta\right) = 0 \tag{26}$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)\left(\frac{\dot{A}}{A} + 2\eta\right) = 0 \tag{27}$$

From equation (26) and (27), we have

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{S^3} e^{-2\int \eta dt} \tag{28}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{S^3} e^{-2\int \eta dt} \tag{29}$$

where k_1 and k_2 are constant.

From (15), (28) and (29), we have

$$\frac{\dot{A}}{A} = \frac{\dot{S}}{S} + \frac{2k_1 + k_2}{3S^3} e^{-2\int \eta dt} \tag{30}$$

$$\frac{\dot{B}}{B} = \frac{\dot{S}}{S} + \frac{-k_1 + k_2}{3S^3} e^{-2\int \eta dt} \tag{31}$$

$$\frac{\dot{C}}{C} = \frac{\dot{S}}{S} - \frac{k_1 + 2k_2}{3S^3} e^{-2\int \eta dt} \tag{32}$$

Equation (8)-(11) and (25) can be written in terms of H, σ and q as

$$p - \Lambda = (2q - 1)H^2 - \sigma^2 + \zeta\theta \tag{33}$$

$$\rho + \Lambda = 3H^2 - \sigma^2 \tag{34}$$

$$\dot{\rho} + 3(\overline{p} + \rho)H + \dot{\Lambda} = 4\eta\sigma^2 \tag{35}$$

From equations (33) and (34), we get

$$\dot{H} = -3H^2 + \frac{(\rho - p)}{2} + \frac{3\zeta H}{2} + \Lambda \tag{36}$$

which is the Raychaudhuri equation for present distribution. Thus equation shows that the shear viscosity does not affect the mean expansion rate but bulk viscosity and positive cosmological term Λ increase the rate of expansion.

We assume that the matter content obeys equation of state (EoS)

$$p = \omega\rho, \qquad (0 \le \omega \le 1) \tag{37}$$

In order to obtain analytical models, we assume specific form of coefficients of shear viscosity η and bulk viscosity ζ in the following models:

Model I:-

We assume that and $\eta = \eta_0$ are $\zeta = \zeta_0$ constants. For this assumption, equation (30)-(32) become

$$\frac{\dot{A}}{A} = \frac{\dot{S}}{S} + \frac{2k_1 + k_2}{3S^3} e^{-2\eta_0 t} \tag{38}$$

$$\frac{\dot{B}}{B} = \frac{\dot{S}}{S} + \frac{-k_1 + k_2}{3S^3} e^{-2\eta_0 t} \tag{39}$$

$$\frac{\dot{C}}{C} = \frac{\dot{S}}{S} - \frac{k_1 + 2k_2}{3S^3} e^{-2\eta_0 t} \tag{40}$$

In this case, shear scalar σ turns out to be

$$\sigma = \frac{ke^{-2\eta_0 t}}{S^3} \tag{41}$$

where

$$k^2 = \left(\frac{k_1^2 + k_2^2 + k_1 k_2}{3}\right)$$

For the model, we obtain matter density ρ, vacuum energy density Λ and shear scalar σ have the following expressions:

$$(1+\omega)\rho = 2[ncsch^2(t) - msech^2(t)] - \frac{2k^2}{e^{4\eta_0 t}[cosh^m(t) sinh^n(t)]^6}$$
$$+3\zeta_0[m\tanh(t) + n\coth(t)] \tag{42}$$

and

$$\Lambda(t) = 3[m\tanh(t) + n\coth(t)]^2 - \frac{2[ncsch^2(t) - msech^2(t)]}{1+\omega} \quad (43)$$
$$+ \frac{(1-\omega)}{(1+\omega)} \frac{k^2}{e^{4\eta_0 t}[cosh^m(t)sinh^n(t)]^6}$$
$$- \frac{3\zeta_0[m\tanh(t)+n\coth(t)]}{1+\omega}$$

also

$$\sigma = \frac{k}{e^{2\eta_0 t}[cosh^m(t)sinh^n(t)]^3} \quad (44)$$

We observed that matter density ρ , cosmological term Λ and shear scalar σ all diverge at $t = 0$. Also from (42) the energy density ρ approaches to $3(m+n)\zeta_0/_{1+\omega}$ and from (43) cosmological term Λ approaches to $3(m+n)^2 - \left\{3(m+n)\zeta_0/_{1+\omega}\right\}$ and $\sigma \to 0$as $t \to \infty$.

We observe that the effect of bulk viscosity is to increase the value of matter density and to decrease the value of vacuum energy density. At late times cosmological term Λtends to a genuine cosmological constant.

For this model

$$\frac{\sigma}{\theta} = \frac{k}{3e^{2\eta_0 t}[ncosh^2(t)+msinh^2(t)][cosh^{3m-1}(t)sinh^{3n-1}(t)]} \quad (45)$$

From (45), we observes that the anisotropy σ/θ is decreasing function of *t* and tends to *0*as $t \to \infty$. Hence, the model approaches isotropy for large value of t. The shear viscosity is found to be responsible for the faster removal of initial anisotropies in the universe. Thus, the shear viscosity plays an important role in the process of isotropization of large scale structure of the universe.

Model-II:-

We assume that shear viscosityη calling as

$$\eta = \frac{\eta_0 \alpha}{1 - e^{-\alpha t}} \quad (46)$$

and bulk viscosity given by

$$\zeta = \frac{\zeta_0}{1 - e^{-\alpha t}} \quad (47)$$

where α, η_0, ζ_0 are constants.

So from equation (30)-(32), we get

$$\frac{\dot{A}}{A} = \frac{\dot{S}}{S} + \frac{2k_1 + k_2}{3S^3(e^{\alpha t} - 1)^{2\eta_0}} \tag{48}$$

$$\frac{\dot{B}}{B} = \frac{\dot{S}}{S} + \frac{-k_1 + k_2}{3S^3(e^{\alpha t} - 1)^{2\eta_0}} \tag{49}$$

$$\frac{\dot{C}}{C} = \frac{\dot{S}}{S} - \frac{k_1 + 2k_2}{3S^3(e^{\alpha t} - 1)^{2\eta_0}} \tag{50}$$

In this case, shear scalar σ turns out to be

$$\sigma = \frac{k}{S^3(e^{\alpha t} - 1)^{2\eta_0}} \tag{51}$$

where

$$k^2 = \left(\frac{k_1^2 + k_2^2 + k_1 k_2}{3}\right)$$

For the model, we obtain matter density ρ, vacuum energy density Λ and shear scalar σ have the following expressions:

$$(1 + \omega)\rho = 2[ncsch^2(t) - msech^2(t)] - \frac{2k^2}{(e^{\alpha t} - 1)^{4\eta_0}[cosh^m(t) sinh^n(t)]^6} + 3\frac{\zeta_0}{1 - e^{-\alpha t}}[m\tanh(t) + n\coth(t)] \tag{52}$$

And

$$\Lambda(t) = 3[m\tanh(t) + n\coth(t)]^2 - \frac{2[ncsch^2(t) - msech^2(t)]}{1 + \omega} + \frac{(1 - \omega)}{(1 + \omega)}\frac{k^2}{(e^{\alpha t} - 1)^{4\eta_0}[cosh^m(t) sinh^n(t)]^6} - \frac{3\zeta_0}{1 - e^{-\alpha t}}\frac{[m\tanh(t) + n\coth(t)]}{1 + \omega} \tag{53}$$

also

$$\sigma = \frac{k}{(e^{\alpha t} - 1)^{2\eta_0}[cosh^m(t) sinh^n(t)]^3} \tag{54}$$

We observed that matter densityρ , cosmological term Λ and shear scalar σ all diverge at $t = 0$ whereas $\eta \to \eta_0 \alpha$and $\zeta \to \zeta_0$. Also from (52) the energy density ρ approaches to $3(m+n)\zeta_0/_{1+\omega}$ and from (53) cosmological term Λ approaches to

$3(m+n)^2 - \left\{3(m+n)\zeta_0/_{1+\omega}\right\}$ and $\sigma \to 0$as $t \to \infty$.

Also

$$\frac{\sigma}{\theta} = \frac{k}{3(e^{\alpha t}-1)^{2\eta_0}[ncosh^2(t)+msinh^2(t)][cosh^{3m-1}(t)sinh^{3n-1}(t)]} \quad (55)$$

From (55), we find that the anisotropy σ/θ is decreasing function of *t* and tends to zero as $t \to \infty$. Hence the model approaches isotropy for large value of *t*.

Model-III:-

We assume that shear viscosityη calling as

$$\eta = \frac{1}{t+\eta_0} \quad (56)$$

and bulk viscosity given by

$$\zeta = \frac{1}{t+\zeta_0} \quad (57)$$

whereη_0 are ζ_0 are constants.

So from equation (30)-(32), we get

$$\frac{\dot{A}}{A} = \frac{\dot{S}}{S} + \frac{2k_1+k_2}{3S^3(t+\eta_0)^2} \quad (58)$$

$$\frac{\dot{B}}{B} = \frac{\dot{S}}{S} + \frac{-k_1+k_2}{3S^3(t+\eta_0)^2} \quad (59)$$

$$\frac{\dot{C}}{C} = \frac{\dot{S}}{S} - \frac{k_1+2k_2}{3S^3(t+\eta_0)^2} \quad (60)$$

In this case, shear scalar σ turns out to be

$$\sigma = \frac{k}{S^3(t+\eta_0)^2} \tag{61}$$

So for this model, we obtain matter density ρ, vacuum energy density Λ and shear scalar σ have the following expressions:

$$(1+\omega)\rho = 2[ncsch^2(t) - msech^2(t)] - \frac{2k^2}{(t+\eta_0)^4[cosh^m(t)sinh^n(t)]^6} + \frac{3}{t+\zeta_0}[m\tanh(t) + n\coth(t)] \tag{62}$$

and

$$\Lambda(t) = 3[m\tanh(t) + n\coth(t)]^2 - \frac{2[ncsch^2(t) - msech^2(t)]}{1+\omega} + \frac{(1-\omega)}{(1+\omega)}\frac{k^2}{(t+\eta_0)^4[cosh^m(t)sinh^n(t)]^6} - \frac{3}{t+\zeta_0}\frac{[m\tanh(t)+n\coth(t)]}{1+\omega} \tag{63}$$

also

$$\sigma = \frac{k}{(t+\eta_0)^2[cosh^m(t)sinh^n(t)]^3} \tag{64}$$

We observed that the model has singularity at $t = 0$. All physical quantities matter density ρ , cosmological term Λ and shear scalar σ all infinite at $t = 0$ whereas $\eta \to 1/\eta_0$ and $\zeta \to 1/\zeta_0$. Also from (63) cosmological term Λ approaches to $3(m+n)^2$ and ρ, η, ζ and σ becomes zero as $t \to \infty$.

We find that cosmological term Λ is decaying function of time and it approaches a small value at late times. Thus, solution tends to de-sitter universe with $H = m + n = \sqrt{\Lambda/3}$ for the large value of t.

Also for this model, anisotropy

$$\frac{\sigma}{\theta} = \frac{k(t+\eta_0)^{-2}}{3[ncosh^2(t)+msinh^2(t)][cosh^{3m-1}(t)sinh^{3n-1}(t)]} \tag{65}$$

From (65), we find that the anisotropy σ/θ is decreasing function of t and tends to zero as $t \to \infty$. Hence the model approaches isotropy for large value of t. The shear viscosity is found to be responsible for faster removal of initial anisotropies in the universe.

Model-IV:-

We assume that shear viscosityη calling as

$$\eta = 3\eta_0 \frac{\dot{S}}{S} \quad , \qquad \eta_0 = constant \tag{66}$$

and bulk viscosity given by

$$\zeta = \zeta_0 \frac{\dot{S}}{S} \quad , \qquad \zeta_0 = constant \tag{67}$$

So from equation (30)-(32), we get

$$\frac{\dot{A}}{A} = \frac{\dot{S}}{S} + \frac{2k_1 + k_2}{3S^{3+6\eta_0}} \tag{68}$$

$$\frac{\dot{B}}{B} = \frac{\dot{S}}{S} + \frac{-k_1 + k_2}{3S^{3+6\eta_0}} \tag{69}$$

$$\frac{\dot{C}}{C} = \frac{\dot{S}}{S} - \frac{k_1 + 2k_2}{3S^{3+6\eta_0}} \tag{70}$$

In this case, shear scalar σ turns out to be

$$\sigma = \frac{k}{S^{3+6\eta_0}} \tag{71}$$

Thus, coefficient of shear viscosity η and bulk viscosity ζ for the model come out to be

$$\eta = 3\eta_0[m\tanh(t) + n\coth(t)] \tag{72}$$

$$\zeta = \zeta_0[m\tanh(t) + n\coth(t)] \tag{73}$$

So for this model, we obtain matter densityρ, vacuum energy density $\Lambda$and shear scalar σhave the following expressions:

$$(1+\omega)\rho = 2[ncsch^2(t) - msech^2(t)] - \frac{2k^2}{[cosh^m(t)sinh^n(t)]^{6+12\eta_0}} + 3\zeta_0[m\tanh(t) + n\coth(t)] \tag{74}$$

and

$$\Lambda(t) = 3[m\tanh(t) + n\coth(t)]^2 - \frac{2[ncsch^2(t) - msech^2(t)]}{1+\omega} + \frac{(1-\omega)}{(1+\omega)}\frac{k^2}{[cosh^m(t)sinh^n(t)]^{6+12\eta_0}} - 3\zeta_0\frac{[m\tanh(t)+n\coth(t)]^2}{1+\omega} \quad (75)$$

$$\sigma = \frac{k}{[cosh^m(t)sinh^n(t)]^{3+6\eta_0}} \quad (76)$$

Also for this model, anisotropy

$$\frac{\sigma}{\theta} = \frac{k}{3[m\tanh(t)+n\coth(t)][cosh^m(t)sinh^n(t)]^{3+6\eta_0}} \quad (77)$$

We observed that the model has singularity at $t = 0$. We observed that model starts with big-bang from its singular state at $t = 0$ with all physical quantities matter density ρ, cosmological term Λ and shear scalar σ bulk viscosity ζ, shear viscosity η and anisotropy σ/θ all infinite. At large value of time σ and σ/θ becomes zero also $\eta \to 3\eta_0(m+n)$, $\zeta \to \zeta_0(m+n)$, $\rho \to 3\zeta_0(m+n)^2/(1+\omega)$ and cosmological term Λ approaches to $3(m+n)^2 - [3\zeta_0(m+n)^2/(1+\omega)]$. In the absence of bulk viscosity, at large value of time ρ tends to zero and $\Lambda \to 3(m+n)^2$= constant. Thus, solution tends to de-sitter universe with $H = m + n = \sqrt{\Lambda/3}$ for the large value of t.

Conclusion:-

In this paper, we have studied anisotropic Bianchi-type I space time with bulk and shear viscosity in the context of general relativity. Exact solution of Einstein field equations are presented by a suitable functional form for the Hubble parameter , which yields a model of the universe that describe an early deceleration and late time acceleration. We have found that cosmological term being very large at initial times relaxes to genuine cosmological constant at late times. Also observed that shear viscosity plays more important role in the isotropization process of the universe. Therefore it may be possible that the isotropy observed in the present universe, is a consequence of the viscous effects in the cosmic fluid right from the beginning of the evolution of the universe.

In the absence of viscosity, we obtain the solution of Einstein's field equations (5) for perfect fluid distribution in Bianchi type I space time.

Finally, the exact solutions presented in this paper may be useful for better understanding of the evolution of the universe in Bianchi type I space time with viscous effects.

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