Research Paper

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Derivation of the Schrodinger Equation from Classical Physics

Physics

KEYWORDS : 4-d space-time discrete, Schrödinger equation, Planck length, Quantification of space–time, minimal length, imaginary time

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ABSTRACT

In this work, the Schrödinger equation is deduced in a very simple manner. The starting point is the assumption that the Universe and particles are formed by four-dimensional Planck atoms. The wave function is the ratio between the kinetic energy that the electron has when it is unobserved and the energy that it acquires due the observation.

Introduction

In 1923, De Broglie suggested that all matter should behave as waves with a wavelength given by , where λ is the wavelength of the particle, h is Planck's constant, m is the mass of the particle and v its velocity.

Motivated by the hypothesis of De Broglie, in 1926 Erwin Schrödinger conceived an equation as a way to describe the wave behaviour of particles of matter. The equation was later called the Schrödinger equation.

On the one hand, despite much debate, it is accepted that the square of the wave function at a point represents the probability density at that point. Max Born gave the wave function a different probabilistic interpretation than that given by De Broglie and Schrödinger, an interpretation that Einstein never shared.

On the other hand, Schrödinger published two attempts to derive the equation that takes his name [1, 2]. There have also been attempts by other authors to obtain Schrödinger's equation from different principles [3–9].

Discrete space–time

General relativity implies that space–time is continuous. However, there is no experimental evidence for it. Are space and time continuous? Or are we only convinced of that continuity as a result of education? In recent years, both physicists and mathematicians have asked if it is possible that space and time are discrete.

If we could probe to size scales that were small enough, would we see "atoms" of space, irreducible pieces of volume that cannot be broken into anything smaller? [10].

Minimum volume, length or area are measured in units of Planck [10]. Planck's constant, h, which represents the elementary quantum of action, has an important role in quantum mechanics. There are several theories that predict the existence of a minimum length [11–12]. Besides black hole physics, these theories are related to quantum gravity, such as string theory and double special relativity [13–15].

Quantification of space–time maintains the relativistic invariance [16] and causation and allows us to distinguish elementary particles from themselves in a simple and natural way [17].

Discrete space–time is used as a model by other authors to present the solution of the Schrödinger equation for a free particle [18] or for electromagnetic waves and the Helmholtz equation [19].

Heisenberg said that physics must have a fundamental length scale, and with Planck's constant h and the speed of light, allow the derivation of the masses of the particles [20–21].

Planck's length has been considered as the shortest distance having any physical meaning.

It is shown that Planck's length is a lower bound to a proper physical length in any space–time. It is impossible to construct an apparatus which will measure length scales smaller than the Planck length [22].

Electron wave particle duality

Suppose the universe is made of atoms of space–time and discrete particles have four spatial dimensions, with radius equal to the Planck diameter r_p. To simplify the drawing, only three dimensions are considered r(x,y) y u.

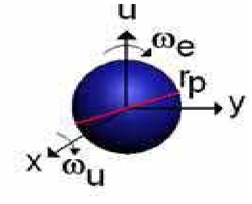

Figure 1. Rotations of the particle

The particle may rotate both in three-dimensional space and the fourth dimension (u, Fig. 1), which gives rise to the following combinations:

- 0 rotations.
- 1 spatial rotation: w_e.
- 1 rotation in the fourth dimension: w_u.
- 2 rotations, one space: w_e, and the other in the fourth dimension: w_u.

If we suppose that we have a particle of mass m, which rotates at velocity ω_e, the potential of the gravitational field at the distance *r*, will be :

$$\frac{Gm}{r} = v^2 \quad (1)$$

where G is the gravitational constant, and v the velocity. Let us assume that this is the linear speed of rotation of the particle.

The atoms of space and time are united by Planck's force, so that turning one of them, will drag it to adjacent atoms, so that the linear velocity of rotation (Fig. 2) will increase as we move away from the rotating atom, to the speed of light *c*, in the distance r, then : y (2)

$$v = \omega_e r_p \quad y \quad c = \omega_e r \qquad (2)$$

Substituting in Equation (1) and taking into account the Planck radius ($r_p = (G\hbar/c^3)^{1/2}$,: (3)

$$E = m\,c^2 = \hbar\,\omega_e = {}^{\hbar c}/_{r} \quad (3)$$

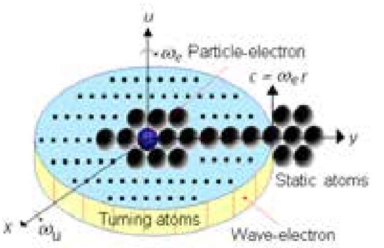

Figure 2. Two-dimensional representation of the electron
$\hbar$ is the reduced Planck constant and r is the distance at which the adjacent space and time atoms rotate at the speed of light, and coincides with the wavelength of the particle (r = λ).

Equation (3) can be expressed in terms of Planck, as:

$$E_P = m_P\,c^2 = \hbar\,\omega_P = {}^{\hbar c}/_{\lambda_P} \qquad (4)$$

where: E_p is the Planck energy, m_p is the Planck mass, ω_p is the Planck rotation and λ_p is the Planck wavelength on $\hbar$.

Eliminating the Planck constant in Equations (3) and (4) results in:

m_P λ_P= m λ=cte. (5)

m_P ω_e= m ω_P=cte. (6)

ω_P λ_P= ω_e λ=cte. (7)

where λ is the wavelength and ω_e is the rotation of the Planck atom which results in the elementary particle.

In any case, we have an equation and two unknowns: rotation and mass, mass and wavelength or wavelength and rotation. Therefore, there is a relationship between the mass and the rotation of the Planck atom. As the rotation of the Planck atom decreases, the mass of the particle decreases to a minimum value.

We can consider **the electron as a Planck particle that is in the state of minimum energy**. The Planck particle turns into angular velocity (w_e), dragging adjacent space atoms to a distance equal to its wavelength.

Therefore, any perturbation will increase the electron rotation, resulting in an increase in its mass, but always verifying Equations (5), (6) and (7). This increase in energy produced by the disturbance will be removed as radiation, so that the electron quickly returns to its lowest energy state.

Wavelength
The hypothesis that the mass is an intrinsic property of matter, along with the assumption that particles are points, leads to the conclusion that mass is independent of the energy used in the measurement. However, in a space–time continuum, Equations (5), (6) and (7) have infinite solutions. If we consider discrete space–time, the number of solutions is finite and the wavelength must be an integer number of Planck wavelengths. Therefore, these equations indicate that the mass of any particle or electron can be any value between the minimum value (mass of the particle or electron at rest) and the maximum value or Planck mass.

When the mass of the electron (m_e) is measured using little energy or, as is the same, a large wavelength (Fig. 3), it is equivalent to observing the particle at great distances (λ_1). When we want to accurately measure the position, we use high energy photons or small wavelengths (λ_2), which is equivalent to closely observing the particle. Under these conditions, the photon energy causes an increase in the rotation of the particle, which means an increase in mass (m'_e). Namely, the observation modifies the energy of the particle. In any case, relations (5), (6) and (7) must be fulfilled at all times.

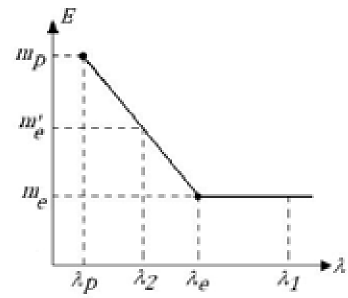

Figure 3. Mass particle depending on the wavelength of the measurement
Moreover, the wavelength is the distance at which two Planck masses exert the same force as two masses *m*, supposedly points and separated by a distance equal to the Planck radius r_p.

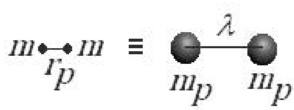

Figure 4. Force and wavelength

Therefore, we can consider the wave length λ as the minimum distance at which we can measure and consider that mass is *m*.

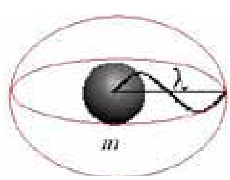

Figure 5. Minimum distance at which m is constant.

Real and imaginary time
In special relativity, length contraction takes place in the direction of movement. In Newton's law of gravity and Coulomb's law, the forces depend on the distance between the two bodies.

Similarly, we can consider a tetra dimensional space, the circular area, formed by the direction of photon motion of wavelength λ, used to carry out the observation, and the fourth dimension u.

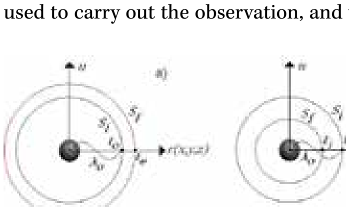

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Figure 6. Observation area formed by the direction of observation and the fourth dimension

If we define time squared as the surface variation for the square of the speed of light, we obtain: The time will be real and positive for long wavelengths (Fig 6.):

and the time will be imaginary for short wavelengths (Fig. 6b),

$$t_e^2 = \frac{\Delta S}{c^2} = \frac{S_f - S_i}{c^2} \geq 0 \quad (9)$$

wavelengths below the wavelength of the particle.

$$t_i^2 = \frac{\Delta S}{c^2} = \frac{S_f - S_i}{c^2} \leq 0 \quad (10)$$

Schrödinger equation.

a) Particle at rest. Let us first consider the case of a particle at rest. In these conditions, the potential energy is zero.

The wave function is the ratio of the energy when the electron is not observed and when it is observed. Consider the function:

$$\Psi(r,t) = \frac{E_c}{E_H} = \frac{m\frac{v^2}{2}}{\frac{\hbar}{2t_i}} \quad (11)$$

E_c is the energy of the particle from the above distances to its wavelength distances and E_H is the energy of the particle from the lower distances to its wavelength, respectively. E_c is the energy of the particle when it is not observed and E_H is the energy of the particle when it is observed.

Multiplying by t_e (time outside the surface), that corresponds to measures with long wavelengths (low energy), gives:

$$\Psi(r,t) = \frac{E_c}{E_H} = \frac{m\,r^2}{\frac{\hbar\, t_e^2}{t_i}} \quad (12)$$

Consider the circular area formed by the temporal dimension of the direction u and the observation direction. From Equations (9) and (10) it follows that:

t_e =⟦it⟧_i (13)

where t_i is the time inside the surface of λ_o radio, and t_e is the time outside that area.

Substituting the above equation into Equation (12) results in:

Ψ(r,t)= -i (m⟦ r⟧^2)/(ℏ t_e) (14)

deriving from the outside time:

(δΨ(r,t))/(δ t_e)= -i (m⟦ r⟧^2)/(ℏ ⟦ t⟧_e^2) (15)

and deriving twice with respect to space:

∇^2 Ψ(r,t)= -i (2 m)/(ℏ t_e) (16)

The above two equations can be expressed as:

ℏ (δΨ(r,t))/(δ t_e)= -i (m⟦ r⟧^2)/(⟦ t⟧_e^2) (17)

⟦ℏ^2/(2 m) ∇⟧^2 Ψ(r,t)= -i ℏ/(t_e) (18)

Dividing one equation by the other and undoing the changes, we get:

(m⟦ r⟧^2)/(⟦ t⟧_e^2) t_e/ℏ= (m⟦ v⟧^2 ⟦ t⟧_e^2)/(ℏ⟦ t⟧_e)= (m⟦ v⟧^2)/(ℏ/⟦ t⟧_e)=(m⟦ v⟧^2)/((i ℏ)/⟦ t⟧_i) (19)

where is m⟦ v⟧^2/2 the energy outside the surface and ℏ/2⟦ t⟧_i is the energy inside the surface.

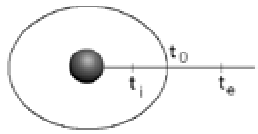

Figure 7. Real and imaginary time.

If we consider that, when t_e=t_0=⟦it⟧_i both energies are equal we get:

⟦ℏ^2/(2 m) ∇⟧^2 Ψ(r,t)= -iℏ (δΨ(r,t))/(δ t_e) (20)

the Schrödinger equation for any particle at rest.

b) Particle with Coulomb potential. The Schrödinger equation for an electron in the presence of Coulomb potential is:

⟦ℏ^2/(2 m) ∇⟧^2 Ψ(r,t)+iℏ (δΨ(r,t))/(δ t_e)+ Ψ(r,t) V(r,t) (21)

Since we know the Schrödinger equation and the wave function, we will calculate the value of the potential V(r,t,).

Adding Equations (17) and (18) results in:

⟦ℏ^2/(2 m) ∇⟧^2 Ψ(r,t)+iℏ (δΨ(r,t))/(δ t_e) = -(m⟦ r⟧^2)/(⟦ t⟧_e^2) -i ℏ/(t_e) (22)

And compared with Equation (21):

Ψ(r,t) V(r,t)= -2(1/2 m⟦ v⟧^2+ℏ/(2 t_i)) (23)

And considering the value of the wave function (Eq. (11)), gives:

V(r,t)= -2(1/2 m⟦ v⟧^2+ℏ/(2 t_i)) (ℏ/t_i)/(m⟦ v⟧^2) (24)

If we consider , which means that both energies are equal:

V(r,t)= -2 m⟦ v⟧^2 (25)

And, considering that in a circular orbit potential energy E_p is twice the kinetic results:

V(r,t)= -2 E_p (25)

What is the total potential of the orbital S, where there are two electrons? The answer is the expected potential, because in any orbit there are the same quantum numbers of positive and negative spin.

Conclusion

The definition of the wave function as the ratio of the kinetic energy that has the electron and the energy that acquire when it's been disturbed as a result of the observation, give the wave function a physical meaning. This energy should check the Heisenberg uncertainty principle at all times.

When we try to measure the precise position of the electron, the electron changes its energy, and, therefore, we vary its position. This position is recovered quickly, by issuing the energy absorbed.

The wave function has nothing to do with the probability of finding an electron in a given region of space. This function is the ratio of the unobserved electron energy and is acquired due to observation.

Einstein was right when he said:

"*I cannot but confess that I attach only a transitory importance to this interpretation. I still believe in the possibility of a model of reality – that is to say, of a theory which represents things themselves and not merely the probability of their occurrence.*"

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