Octahedral Sum Labeling Graph

Mathematics

KEYWORDS : Octahedral sum label, Graceful and Harmonious Label, Tetrahedral sum label, and Pentatopic sum label.

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ABSTRACT

Let G:(V,E) be a graph. A tetrahedral sum labeling of a graph G is a one to one function f:V→N_0 that induces a bijection f: E→{ B_(1 ,) B_(2,............) B_(|E|)} wheref*(uv)= f(u)+ f(v) ;∀uv∈ E. The graph which admits such labeling is a tetrahedral sum labeling. In this paper we define octahedral sum labeling of a graph G and a graph which admits octahedral sum labeling as octahedral sum labeling graph. Also we discuss some special graphs which admits octahedral sum labeling such as Paths, Stars, Bistar.*

1. INTRODUCTION

Let G :(V, E) be a finite, connected, undirected and simple graph with neither loop nor multiple edges. The order and size of the graph G are denoted by p and q respectively.For graph theoretic terminology we refer to Harary [1] and for graph labeling Gallian [2].

Graph labeling is a Strong Communication between Number theory [3] and structure of graphs. In the next section, we give the brief summary of definitions which are useful to the present investigations. Assigning some values to the vertices of the graph subject to certain conditions are known as Graph Labeling. Graph labeling trace their origins to labeling presented by Alex Rosa in 1967.

A vertex labeling is a function from a vertex V to a set of labels. A graph G with vertex label is called vertex labeled graph. An Edge labeling is a function from an edge set E to a set of labels and the graph with this label is called Edge labeled graph. Later on many types of labelings were defined and discussed by many researchers. The two best labeling methods are Graceful and Harmonious labeling, which plays a vital role in labeling.

2. Preliminaries

In this paper, we give some basic definitions of graceful, harmonious, tetrahedron and pentatopiclabelings, which are the base for the new labeling defined in this paper**.**

Definition2.1 Graceful Labeling

A Vertex labeling**f:V→**$\{\ \mathbf{0,1,2\ldots\ldots\ldots.p-1}\}$of a graph G is called Graceful if f is an injective mapping from the set of vertices V to the set of integers$\{\mathbf{0,1,2\ldots\ldots\ldots.p-1}\}$ such that the induced map f^***:E→** $\boldsymbol{N_0}$from the edge set E to the set of

integers,which is defined as f*(uv) = |f(u)-f(v)| **;**∀ **uv**∈ **E**which assigns different labels to each edge of G. A graph G is called graceful, if G admits a graceful labeling. Graceful labeling was introduced by Rosa in 1967 and is also named as β labeling. Several years later Golomb further extended the concept of graceful labeling.Rosa proved that all Eulerian graphs with order equivalent to 1 or 2(mod4) are not graceful.

Definition 2.2 Harmonious labeling

It was introduced by Graham and Sloane in 1980.A graph G is called Harmonious, if there is an injective mapping f^* from the vertex set V to the additive group of integer modulo q such that each edge uv ∈E assigned the label f*(uv) = |f(u)+f(v)|(modq),which are distinct.

Definition 2.3 Tetrahedral number

A Tetrahedral number (or) Triangular number is a figurative number that represents a pyramid with a triangular base and three sides, called a tetrahedron. The n^{th} tetrahedral number is defined by $\frac{n(n+1)(n+2)}{6}$and denoted by B_n. The tetrahedral numbers are 1,4,10,20,35,56,84,120………..

Definition 2.4 Tetrahedral Sum labeling

A tetrahedral sum labeling of a graph G is one-to-one function f :V→ N_0that induces a bijection f*:E→ $\{B_1, B_{2,\ldots\ldots\ldots} B_q\}$ defined by f*(uv)= f(u)+ f(v) ; ∀uv ∈ E. The graph which admits such labeling is called tetrahedral sum labeling graph.

Definition 2.5 Pentatope number

A Pentatope number is a figurative number given by$P_n=\frac{B_n(n+3)}{4}$where B_n is the n^{th}tetrahedral number. The n^{th}pentatope number is the sum of first n tetrahedral numbers.The n^{th}pentatope number is defined by$\frac{n(n+1)(n+2)(n+3)}{24}$and denoted by C_n. The pentatope numbers are 1, 5,15,35,70,126,210,330…

Definition 2.6Pentatopic Sum labeling

A pentatopicsum labeling of a graph G is a One-to-One function f:V→ N_0 that induces a bijection f*:E→ $\{C_1, C_{2,\ldots\ldots\ldots} C_q\}$ defined byf* (uv)= f(u)+ f(v) ; ∀uv ∈

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E.The graph which admits such labeling is called pentatopic sum labeling graph.

The above said labeling was introduced by S.Murugesan, D.Jayaraman and J.Shiama [4], which is motivated by the concept of tetrahedral and pentatopic sum labeling. In this paper we define a new type of labeling called octahedral sum labeling.

3. Main Results

Definition 3.1 Octahedral number

In number theory, an Octahedral number is a figurate number that represents the number of spheres in an Octahedron formed from closed packet spheres.The n^{th} Octahedral number O_n can be obtained by the

formula$\frac{n(2n^2+1)}{3}$.

The first few Octahedral numbersare 1,6,19,44,85,146,231,344,489,670,891………

Definition 3.2 Octahedral Sum labeling

An Octahedral Sum Labeling of a graph G is a One-One function f :V→ N_0that induces a injection f*:E→ $\{O_1, O_2, \ldots, O_q\}$ defined byf* (uv)= f(u)+ f(v) ; ∀uv ∈ E.The graph which admits such labeling is called Octahedral sum labeling graph.

Example 3.3

An Octahedral sum labeling graph with 7 vertices

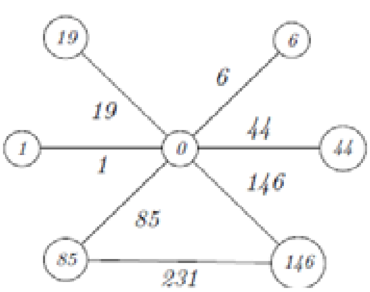

Figure 1: Octahedral sum labeling graph

Theorem 3.4

Path P_n admits octahedral sum labeling
Proof:
Let P_n: $\{v_0,v_1,v_2,.....v_n\}$ be the path.
Let $u_iv_i = v_iv_{i+1}$ be the edges for i={0,1,2….n-1}

Define f:V(G)→N_0 such that
$f(v_i)=\frac{i(i+1)(2i+1)}{6}$;i=0,1,2,…….
Hence, we see that the induced edge labels are first n octahedral numbers.

Example 3.5

This example shows that P_6 admits Octahedral sum labeling.

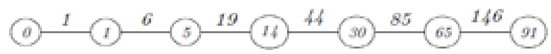

Figure 2: PathP_6

Theorem 3.6

Star $K_{1,n}$ admits Octahedral sum labeling.
Proof:
Let v_0 be the apex vertex, and
let $\{v_1,v_2,……v_n\}$ be the pendent vertices of the star $K_{1,n}$
Define $f(v_0)=0$and $f(v_i)=\frac{i(2i^2+1)}{3}$,$1\leq i \leq n$
Hence, we see that the induced edge labelsare first noctahedral numbers.

Example 3.7

Star graph $K_{1,7}$ admits Octahedral sum labeling

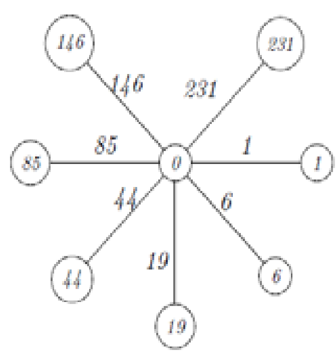

Figure 3: Star $K_{1,7}$

Theorem 3.8

The Subdivision of the star graph S($K_{1,n}$})admits Octahedral sum labeling.

Proof:

Let $V(S(K_{1,n})) = \{v_0, v_i, u_i ; 1 \leq i \leq n\}$

$E(S(K(1,n)) = \{v_0v_i , v_iu_i ; 1 \leq i \leq n \}$, where u_i are the pendent vertices in the star.

Define f by $f(v_0) = 0$

$f(v_i) = \frac{i(2i^2+1)}{3}$, $1 \leq i \leq n$

$f(u_i) = \frac{n(2n^2+1)}{3} + 2ni(n+i)$; $1 \leq i \leq n$

The induced edge labels are the first 2n octahedral labels. Hence $S(K_{1,n})$ admits Octahedral sum labeling.

Example 3.9

The Subdivision of the star graph S(K1,6) admits Octahedral sum labeling.

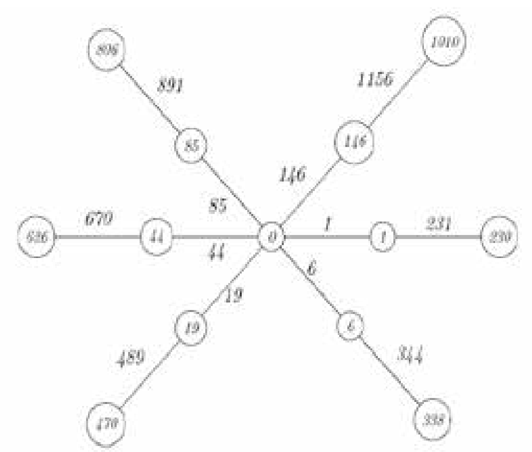

Figure 4: Subdivision of the star S(K1,6)

Theorem 3.10

The Bistar $B_{m,n}$ admits Octahedral sum labeling

Proof:

Let $V(B_{m,n}) = \{u, v, u_i, v_j / \quad 1 \leq i \leq n , 1 \leq j \leq m\}$

$E(B_{m,n}) = \{uv , uu_i, vv_j / 1 \leq i \leq n , 1 \leq j \leq m\}$

Define f by f(u)=0 and f(v)=1

$f(u_i) = \frac{(i+1)(2(i+1)^2+1)}{3}$, $1 \leq i \leq n$

$f(v_j) = \frac{(m+j+1)(2(m+j+1)^2+1)}{3} - 1$, $1 \leq j \leq m$

Hence,we see that the indued edge labels are the first m+n+1 octahedral numbers.

Example 3.11

The Bistar $B_{4,4}$ which admits Octahedral sum labeling.

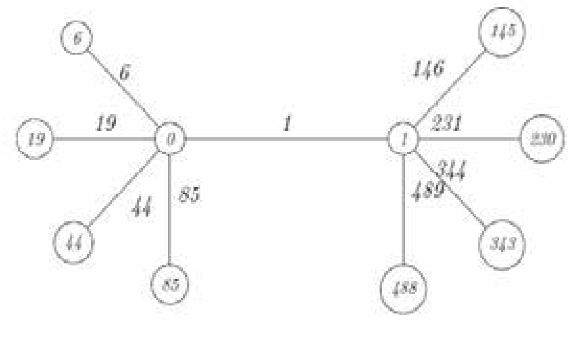

Figure 4: Bistar $B_{4,4}$

Theorem 3.12

The Complete graph K_n is not an Octahedral sum labeling graph $\forall$ $n \geq 3$

Proof:

Suppose assume, K_n admits an Octahedral sum labeling for $n \geq 3$

Let $\{v_0, v_1, v_2, \ldots v_{n-1}\}$ be the vertex set of K_n

Then every vertex set is adjacent to all other vertices of K_n.

Also the induced edge labels are $\{O_1, O_2, \ldots O_q\}$

Where $O_q = \frac{n(2n^2+1)}{3}$.

Now v_0 ,v_1 are labeled as 0 and O_1.Then the edge v_0v_1 will receive an octahedral number.

While all other remaining edges must receive an octahedral number by our assumption.

Let us disprove by induction, for n=3.

Consider K_3,

Let v_0 ,v_1,v_2 be the vertices with labels 0,1 and 5.The three edges v_1v_2, v_2v_0 must receive an Octahedral sum labeling.The edges v_0 ,v_1 ,v_2, may receives an octahedral sum label $0, O_1, O_2$, but v_0v_2 will not receive an octahedral sum label. The Complete graph with 3 vertices is not an octahedral sum labeling graph.

Similarly,if n=4,

K_3 is a subgraph of K_4.Therefore K_4 is also not an octahedral sum labeling graph.

Which is a Contradiction.

Hence, The Complete graph K_n is not an Octahedral sum labeling graph $\forall$ $n \geq 3$.

Conclusion:

We have discussed about octahedral sum labeling of graphs.These labelings can be extended by using some other arithmetic operations for different graphs.

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