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Harmonic Mean Labeling of Subdivision and Related Graphs

Mathematics

KEYWORDS : graph labeling,mean labeling,harmonic mean labeling.

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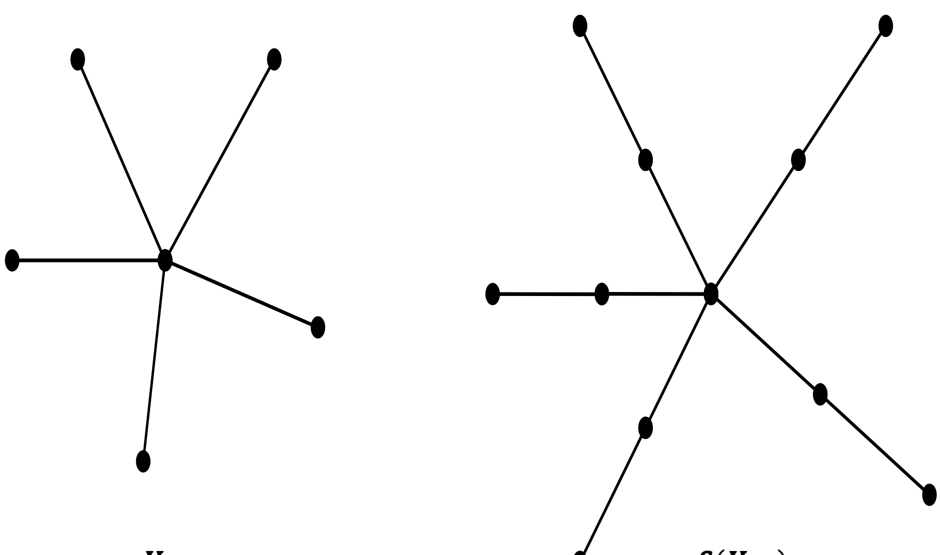
ABSTRACT

Let G(V,E) be a graph with p vertices and q edges.An assignment (or labeling) is a 1-1 function f:V(G)→Z.If f is an assignment, then for any edge e=uv, we define an induced edge label f^* (uv)= (2f(u)f(v))/(f(u)+f(v)) or (2f(u) f(v))/(f(u)+f(v)) .An assignment f is called a harmonic mean labeling if f:V(G)→{1,2,…,q+1} such that f^* (uv) are all distinct.If G admits a harmonic mean labeling,we say that G is a harmonic mean graph.In this paper,we establish harmonic mean labels of some well known sub-division graphs and some disconnected graphs.

1 Introduction

In this paper,we consider only finite,simple and undirected graphs.Let G(V,E) be a graph with p vertices and q edges.For notations and terminology we follow [1].In a graph G,the subdivision of an edge uv is the process of deleting the edge uv and introducing a new vertex w and the new edges uw and vw.If every edge of G is subdivided exactly once, then the resultant graph is denoted by S(G) and is called the *subdivision graph* of G. For example,a star $K_{1,5}$ and its subdivision graph S($K_{1,5}$) are given in Figure 1.



$K_{1,5}$ $S(K_{1,5})$

Figure 1

The *union* of two graphs G_1 and G_2 is a graph $G_1 \cup G_2$ with vertex set $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2).mG$ denotes the disjoint union of m copies of G.

Bistar $B_{n,m}$ is the graph obtained from $K_{1,n} \cup K_{1,m}$ by joining the central vertices of $K_{1,n}$ and $K_{1,m}$ by means of an edge. The newly added edge is called the central edge of $B_{n,m}$. For example the bistar $B_{4,5}$ is shown in Figure 2.

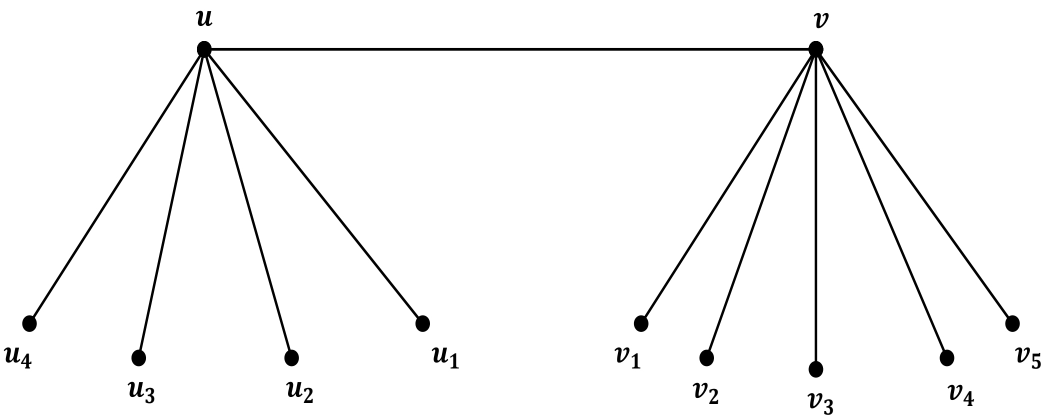


Figure 2

Somasundaram and Ponraj have introduced the concept of mean labeling of graphs in [3]. An assignment $f: V(G) \rightarrow \{0,1,2,\dots,q\}$ is called a *mean labeling* if whenever each edge $e = uv$ is labeled with $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ if $f(u) + f(v)$ is even and $\left\lceil \frac{f(u)+f(v)+1}{2} \right\rceil$ if $f(u) + f(v)$ is odd, then the resulting edge labels are all distinct.Any graph that admits a mean labeling is called a *mean graph*.For example,a graph G with a mean labeling is illustrated in Figure 3.

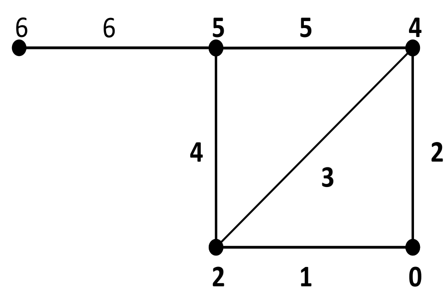


Figure 3

Many results on mean labeling have been proved in [4] and [5].In a similar way, Somasundaram, Ponraj and Sandhya [6] have introduced the concept of harmonic mean labeling of a graph. An assignment $f: V(G) \rightarrow \{1,2,\dots,q+1\}$ is called a *harmonic mean labeling* if whenever each edge $e = uv$ is labeled with $\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ then the edge labels are distinct. Any graph that admits a harmonic mean labeling is called a *harmonic mean graph*.For example,a graph G with a harmonic mean labeling is illustrated in Figure 4.

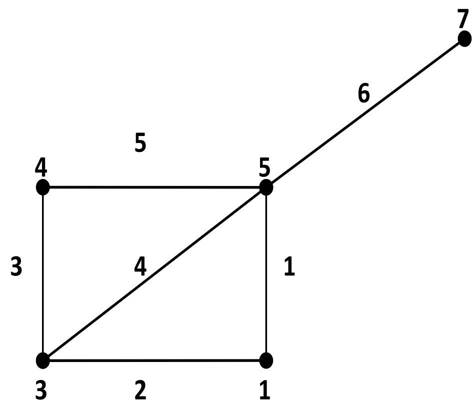


Figure 4

More results on harmonic mean labeling have been proved in [7].A well collection of results on graph labeling has been done in the survey [2].

In this paper,we establish the harmonic mean labeling of some standard graphs like subdivision of star $S(K_{1,n})$,subdivision of bistar $S(B_{n,m})$,the disconnected graph $S(K_{1,n}) \cup kC_m$ etc.

2 Main Results

Theorem 2.1

The disconnected graph $S\left(K_{1,n}\right) \cup kC_m$ is a harmonic mean graph for $1 \leq n \leq 5$, $k \geq 0$ and $m \geq 3$.

Proof

Let $V\left(S\left(K_{1,n}\right) \cup kC_m\right) =$

$$\{v; u_1, u_2, \dots, u_n; v_1, v_2, \dots, v_n; w_{11}, w_{12}, \dots, w_{1m};\ w_{21}, w_{22}, \dots, w_{2m}; \dots w_{k1}, w_{k2}, \dots, w_{km}\}$$

and $E\left(S\left(K_{1,n}\right) \cup kC_m\right) = \{vu_i, u_iv_i \mid 1 \leq i \leq n\} \cup \left[\bigcup_{i=1}^{k} \left(\left(\bigcup_{j=1}^{m-1}\{w_{ij}w_{ij+1}\}\right) \cup \{w_{im}w_{i1}\}\right)\right]$.

Here $p = 2n + km + 1$ and $q = 2n + km$.

Define a function $f: V\left(S\left(K_{1,n}\right) \cup kC_m\right) \rightarrow \{1,2, \dots, q+1\}$ by

$f(v) = 2n + 1;$

$f(u_i) = n + i\ , 1 \leq i \leq n;$

$f(v_i) = n + 1 - i\ ,1 \leq i \leq n$ and

$f\left(w_{ij}\right) = (2n + 1) + (i - 1)m + j, 1 \leq i \leq k, 1 \leq j \leq m.$

Then the induced edge labels of $S\left(K_{1,n}\right)$ are given below:

$f^*(u_n v_n) = 1;$
$f^*(vu_i) = n + 1 + i, 1 \le i \le n;$
$f^*(u_i v_i) = n + 2 - i, 1 \le i \le n - 1;$
and the set of all edge labels of kC_m is $\{2(n+1), 2n+3, \dots, 2n+km+1\}$.
Therefore the set of all edge labels of $S(K_{1,n}) \cup kC_m$ is $\{1,3,4, \dots .2n+km+1\}$.
Hence $S(K_{1,n}) \cup kC_m$ is a harmonic mean graph for $1 \le n \le 5, k \ge 0$ and $m \ge 3$.
Hence the theorem. ∎
For example,the case when $n = 3, k = 3$ and $m = 4$ is illustrated in Figure 5.

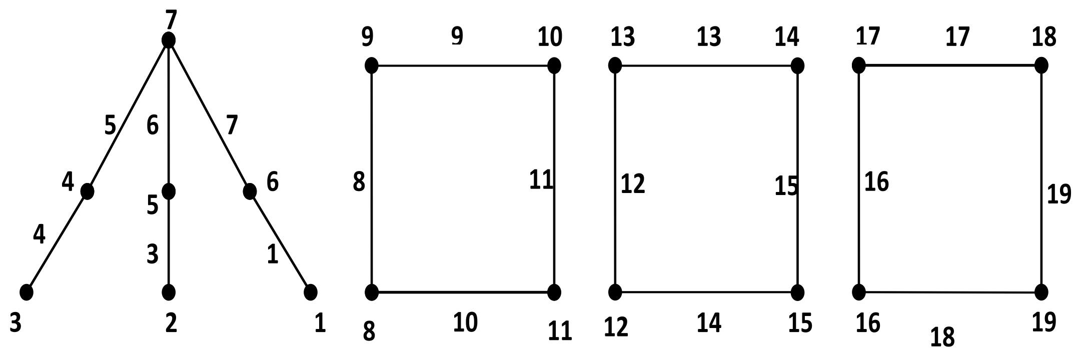


Figure 5

Theorem 2.2

The disconnected graph $S(B_{3,4}) \cup kC_m$ is a harmonic mean graph for $k \ge 0$ and $m \ge 3$.

Proof:

Let $V(S(B_{3,4}) \cup kC_m) = \{u; u_1, u_2, \dots, u_3; v_1, v_2, \dots, v_3; y; x; x_1, x_2, \dots, x_4; y_1, y_2, \dots y_4; w_{11}, w_{12}, \dots, w_{1m}; w_{21}, w_{22}, \dots, w_{2m}; \dots w_{k1}, w_{k2}, \dots, w_{km}\}$.

and $E(S(B_{n_1,n_2}) \cup kC_m) = \{u_i v_i, uv_i, uy, yx, xx_j, x_j y_j \mid 1 \le i \le 3, 1 \le j \le 4\} \cup \left[\bigcup_{i=1}^{k}\left(\left(\bigcup_{j=1}^{m-1}\{w_{ij}w_{ij+1}\}\right) \cup \{w_{im}w_{i1}\}\right)\right]$.

Here $p = 17 + km$ and $q = 16 + km$.
Define a function $f: V(S(B_{3,4}) \cup kC_m) \to \{1,2, \dots, q+1\}$ by
$f(u) = 7;$
$f(u_i) = 3 + i, 1 \le i \le 3;$
$f(v_i) = 4 - i, 1 \le i \le 3;$
$f(y) = 8;$
$f(x) = 17;$
$f(x_j) = 12 + j, 1 \le j \le 4;$
$f(y_j) = 8 + j, 1 \le j \le 4$ and
$f(w_{ij}) = 17 + (i-1)m + j, 1 \le i \le k, 1 \le j \le m.$
Then the induced edge labels of $S(B_{3,4})$ are given below:
$f^*(u_n v_n) = 1;$

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$f^*(uu_i) = 4 + i, 1 \leq i \leq 3;$
$f^*(u_i v_i) = 5 - i, 1 \leq i \leq 2;$
$f^*(uy) = 8;$
$f^*(yx) = 9;$
$f^*(xx_j) = 13 + j, 1 \leq j \leq 4;$
$f^*(x_j y_j) = 9 + j, 1 \leq j \leq 4;$
and the set of all edge labels of kC_m is $\{18,19, \dots ,17 + km\}$.
Therefore the set of all edge labels of $S(B_{3,4}) \cup kC_m$ is $\{ 1,3,4, \dots .17 + km \}$.
Hence $S(B_{3,4}) \cup kC_m$ is a harmonic mean graph for $k \geq 0$ and $m \geq 3$.
Hence the theorem. ■
As an example harmonic mean labeling of $S(B_{3,4}) \cup 3C_4$ is shown Figure 6.

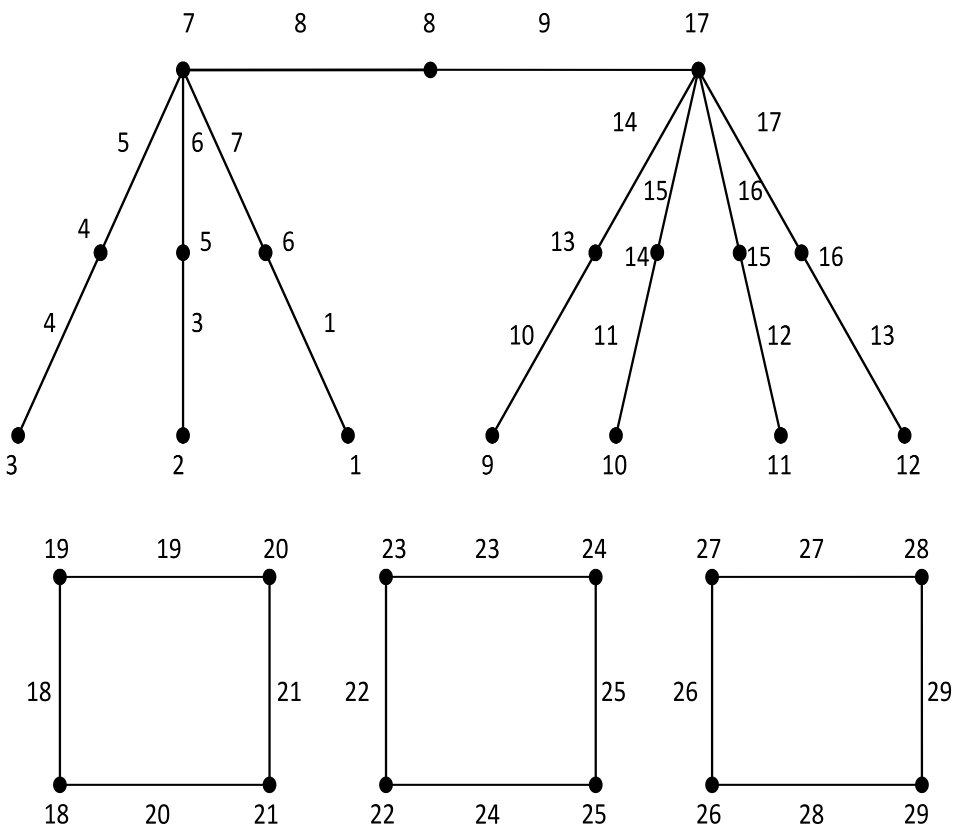

Figure 6

Definition

Let G be a graph with vertex set $v_1, v_2, \dots , v_p; u_1, u_2, \dots , u_q$ and edge $e_1, e_2, \dots , e_q$.Then G^* is a graph with vertex set $V(G^*) = \{ v_1, v_2, \dots , v_p;\ u_1, u_2, \dots , u_q \}$ in which two vertices v_i and v_j adjacent iff they are adjacent in G; u_i and u_j are adjacent iff e_i and e_j are adjacent in G; v_i and u_j are adjacent iff e_j is incident with v_i.For example the graph P_5^* is shown in Figure 7.

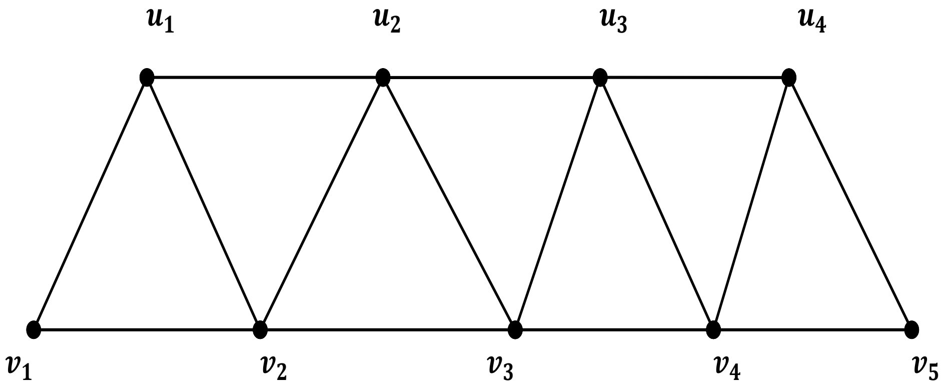

Figure 7

Theorem 2.3

The graph $P_n^* \cup kC_m$ is a harmonic mean graph for $k \geq 0, m \geq 3$ and $n \geq 2$.

Proof Let $V(P_n^* \cup kC_m) =$
$\{ v_1, v_2, \dots, v_n \; ; \; u_1, u_2, \dots, u_{n-1}; w_{11}, w_{12}, \dots, w_{1m}; w_{21}, w_{22}, \dots, w_{2m}; \dots w_{k1}, w_{k2}, \dots, w_{km}\}$
and $E(P_n^*) = \{ v_i v_{i+1} \; ; \; v_i u_i \; ; \; u_i v_{i+1} \mid 1 \leq i \leq n-1 \; ; \; u_i u_{i+1} \mid 1 \leq i \leq n-2\} \cup$

$$\left[\cup_{i=1}^{k}\left(\left(\cup_{j=1}^{m-1}\{w_{ij}w_{ij+1}\}\right) \cup \{w_{im}w_{i1}\}\right)\right].$$

Here $p = 2n + km - 1$ and $q = 4n - 5 + km$.
Define a function $f : V(P_n^* \cup kC_m) \to \{ 1, 2, \dots, q+1\}$ by
$f(v_n) = 4(n-1)$;
$f(v_i) = 4i - 3, 1 \leq i \leq n-1$;
$f(u_i) = 4i - 1, 1 \leq i \leq n-1$ and
$f(w_{ij}) = 4(n-1) + (i-1)m + j, 1 \leq i \leq k, 1 \leq j \leq m$.
Then the induced edge labels of P_n^* are given below:
$f^*(v_i v_{i+1}) = 4i - 2, 1 \leq i \leq n-1$;
$f^*(u_i u_{i+1}) = 4i, 1 \leq i \leq n-2$;
$f^*(v_i u_i) = 4i - 3, 1 \leq i \leq n-1$;
$f^*(u_i v_{i+1}) = 4i - 1, 1 \leq i \leq n-1$;
and the set of all edge labels of kC_m is $\{ 4n-3, 4n-4, \dots, 4(n-1) + km \}$.
Therefore the set of all edge labels of $P_n^* \cup kC_m$ is $\{ 1, 2, 3, \dots, 4(n-1) + km \}$.
Hence $P_n^* \cup kC_m$ is a harmonic mean graph for $k \geq 0, m \geq 3$ and $n \geq 2$.
Hence the theorem. ■
For example,the case when $n = 5, k = 3$ and $m = 4$ is illustrated in Figure 8.

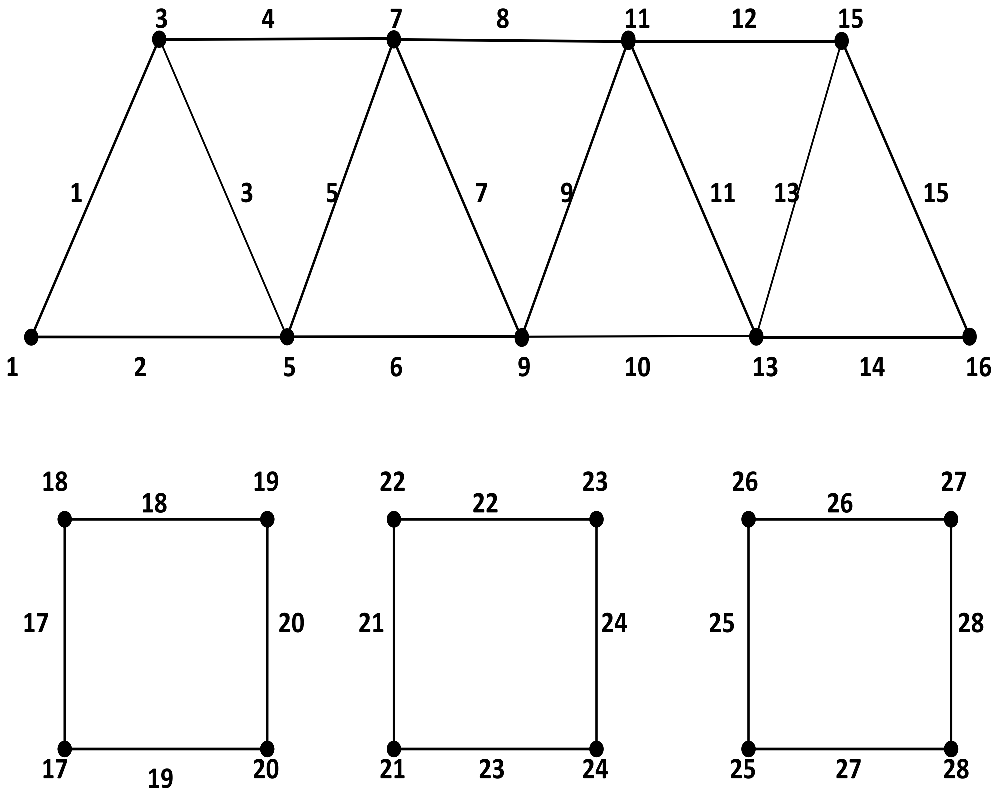

Figure 8

3 Super harmonic mean labeling

Definition

An assignment $f: V(G) \rightarrow \{1,2,\ldots,p+q\}$ is called a *super harmonic mean labeling* if whenever each edge $e = uv$ is labeled with $\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ then the edge labels are distinct.Any graph that admits a super harmonic mean labeling is called a *super harmonic mean graph*.For example,a graph G with a super harmonic mean labeling of $K_{1,3}$ is illustrated in Figure 9.

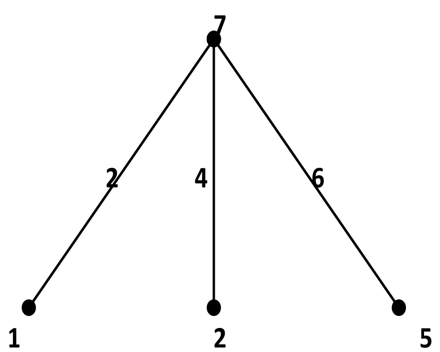

Figure 9

Theorem 3.1

The disconnected graph $S(K_{1,n}) \cup kC_m$ is a super harmonic mean graph for $1 \leq n \leq 5$, $m \geq 3$ and $k \geq 0$.

Proof

Let $V(S(K_{1,n}) \cup kC_m) =$
$\{v; u_1, u_2, \dots, u_n; v_1, v_2, \dots, v_n; w_{11}, w_{12}, \dots, w_{1m}; w_{21}, w_{22}, \dots, w_{2m}; \dots w_{k1}, w_{k2}, \dots, w_{km}\}$
and $E(S(K_{1,n}) \cup kC_m) = \{vu_i, u_iv_i \mid 1 \leq i \leq n\} \cup \left[\cup_{i=1}^{k}\left(\left(\cup_{j=1}^{m-1}\{w_{ij}w_{ij+1}\}\right) \cup \{w_{im}w_{i1}\}\right)\right]$.
Here $p = 2n + km + 1$ and $q = 2n + km$.
Define a function $f: V(S(K_{1,n}) \cup kC_m) \rightarrow \{1,2, \dots, p+q\}$ by
$f(v) = 4n + 1$;
$f(u_i) = 4i - 3, 1 \leq i \leq n$;
$f(v_i) = 4i - 1, 1 \leq i \leq n$ and
$f(w_{ij}) = 2[(2n+1) + (i-1)m] + 2j - 1, 1 \leq i \leq k, 1 \leq j \leq m$.
Then the induced edge labels of $S(K_{1,n})$ are given below:
$f^*(vu_i) = 4i + 2, 1 \leq i \leq n - 1$;
$f^*(vu_n) = 4n$;
$f^*(u_iv_i) = 4i - 3, 1 \leq i \leq n$;
and the set of edge labels of kC_m is $\{ 4n + 3, 4n + 5, \dots, 2(2n + km) + 1 \}$.
Therefore the set of all edge labels of $S(K_{1,n}) \cup kC_m$ is $\{ 1,5,9, \dots .4n + 2km + 1 \}$.
Hence $S(K_{1,n}) \cup kC_m$ is a super harmonic mean graph for $1 \leq n \leq 5, m \geq 3$ and $k \geq 0$. ■
For example,the case when $n = 5, k = 3$ and $m = 4$ is illustrated in Figure 10.

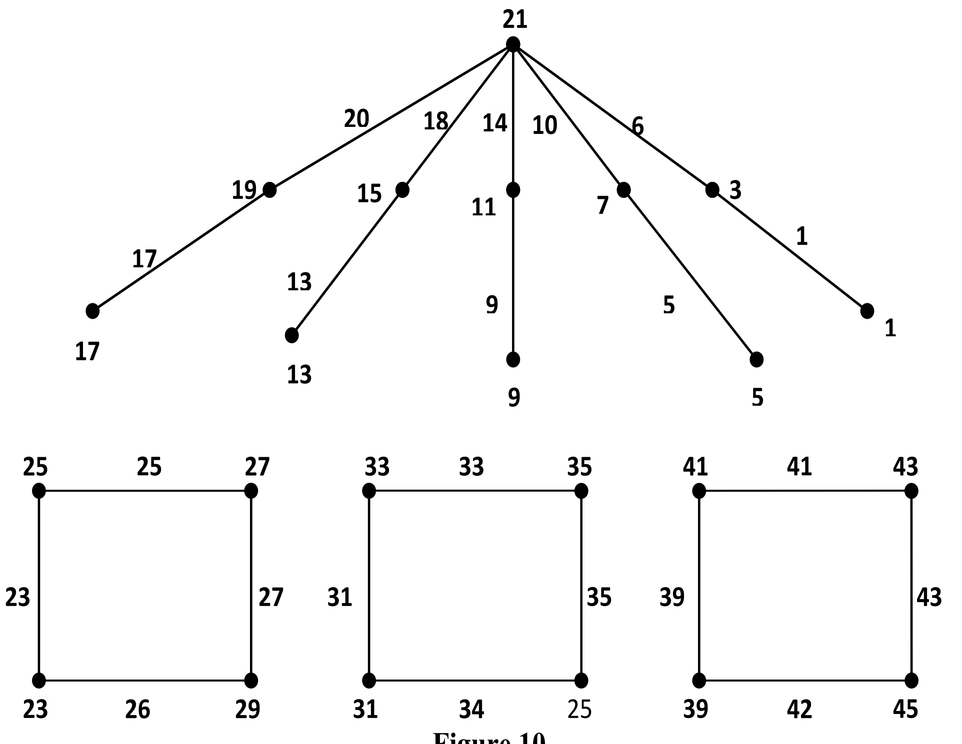

Figure 10

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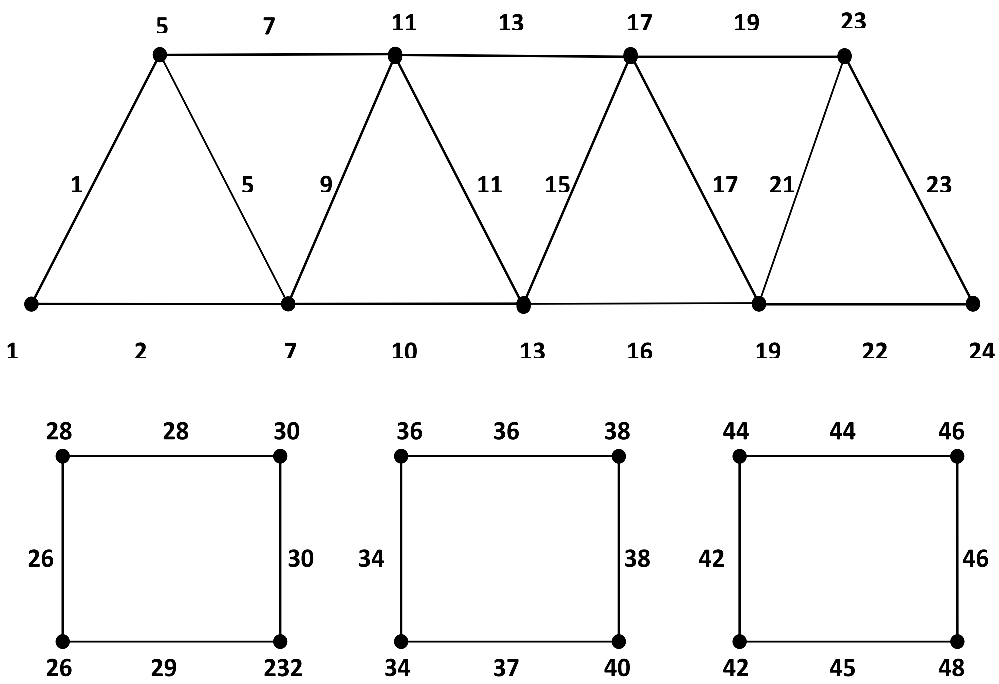
Theorem 3.2

The graph $P_n^* \cup kC_m$ is a super harmonic mean graph for $k \geq 0, m \geq 3$ and $n \geq 2$.

Proof Let $V(P_n^* \cup kC_m) =$
$\{ v_1, v_2, \dots, v_n ; u_1, u_2, \dots, u_{n-1}; w_{11}, w_{12}, \dots, w_{1m}; w_{21}, w_{22}, \dots, w_{2m}; \dots w_{k1}, w_{k2}, \dots, w_{km}\}$
and $E(P_n^*) = \{ v_i v_{i+1} ; v_i u_i ; u_i v_{i+1} \mid 1 \leq i \leq n-1 ; u_i u_{i+1} \mid 1 \leq i \leq n-2\} \cup$

$$\left[\cup_{i=1}^{k}\left(\left(\cup_{j=1}^{m-1}\{w_{ij}w_{ij+1}\}\right) \cup \{w_{im}w_{i1}\}\right)\right].$$

Here $p = 2n + km - 1$ and $q = 4n - 5 + km$.
Define a function $f: V(P_n^* \cup kC_m) \to \{ 1, 2, \dots, p+q \}$ by
$f(v_n) = 6(n-1)$;
$f(v_i) = 6i - 5, 1 \leq i \leq n-1$;
$f(u_i) = 6i - 1, 1 \leq i \leq n-1$ and
$f(w_{ij}) = 2[\,3(n-1) + (i-1)m + j\,], 1 \leq i \leq k, 1 \leq j \leq m$.
Then the induced edge labels of P_n^* are given below:
$f^*(v_1 v_2) = 2$;
$f^*(v_i v_{i+1}) = 6i - 2, 2 \leq i \leq n-1$;
$f^*(u_i u_{i+1}) = 6i + 1, 1 \leq i \leq n-2$;
$f^*(v_1 u_1) = 1$;
$f^*(v_i u_i) = 6i - 3, 2 \leq i \leq n-1$;
$f^*(u_i v_{i+1}) = 6i - 1, 1 \leq i \leq n-1$;
and the set of all edge labels of kC_m is $\{2(3n-2), 2(3n-1), \dots, 2(3(n-1) + km)\}$.
Therefore the set of all edge labels of $P_n^* \cup kC_m$ is $\{ 1, 2, 3, \dots, 6(n-1) + 2km \}$.
Hence $P_n^* \cup kC_m$ is a super harmonic mean graph for $k \geq 0, m \geq 3$ and $n \geq 2$.
Hence the theorem.
For example, the case when $n = 5, k = 3$ and $m = 4$ is illustrated in Figure 11. ■



References

[1] R.Balakrishnan and K.Ranganathan, A Text Book of graph Theory, Springer – Verlag, New York, Inc (1999).

[2] J.A.Gallian, 2010, A dynamic survey of graph labelling. The Electronic Journal of Combinatorics 17#DS6.

[3] S.Somasundaram and R.Ponraj, Mean Labeling of graphs,National Academy Science Letters vol: 26,(2003),210-213.

[4] S.Somasundaram and R.Ponraj, Some Results on Mean Labeling of graphs, Pure and applied Mathematika sciences vol: 58, (2003), 29-35.

[5] S.Somasundaram and R.Ponraj, On Mean graphs of order≤5,Journal of Decision and mathematical sciences vol:9,(2004),1-3.

[6] S.Somasundaram and R.Ponraj and S.S.Sandhya, Some Results on Harmonic Mean Graphs, International Journal contemporary Mathematical sciences,vol:7, 2012, no: 4,197-208.

[7] S.Somasundaram and R.Ponraj and S.S.Sandhya, Harmonic Mean Labeling of Graphs,communicated.