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Research Paper

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Homo Cordial Graphs

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ABSTRACT

Let G=(V,E) be a graph with p vertices and q edges. Let f be a mapping from V to {0,1}. The induced edge label of an edge uv is 1 if f(u) = f(v) or 0 if f(u)≠f(v). Then f is called a homo cordial labeling if the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. The graph that admits a homo cordial labeling is called a homo cordial graph. In this paper, we prove that some standard trees, corona related graphs are homo cordial.

1 Introduction:

Throughout this paper, by a graph G (V, E) we mean a finite, undirected graph with vertex set V and edge set E. For notations and terminology we follow [1]. Cycle on n vertices is denoted by C_n. Path on n vertices is denoted by P_n. Complete graph on n vertices is denoted by K_n.

The disjoint union of two graphs G_1 and G_2 is denoted by $G_1 \cup G_2$. The *join* of two graphs G_1 and G_2 is denoted by $G_1 + G_2$, and is obtained from $G_1 \cup G_2$ by adding an edge between each vertex in G_1 and each vertex in G_2. The graph $P_n + K_1$ is called the *fan graph* and $P_n + 2K_1$ is called the *double fan*.

The *corona* $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 and joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2. The graph $P_n \odot K_1$ is called the *Comb graph*.

For example, the comb graph $P_4 \odot K_1$ is shown in Figure 1.

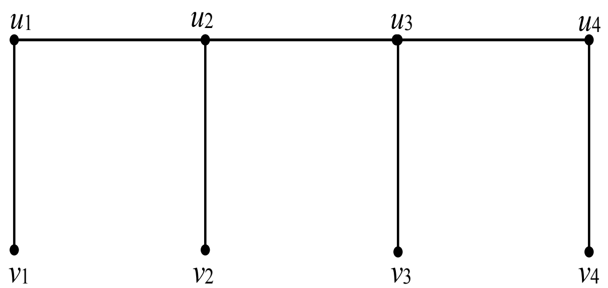
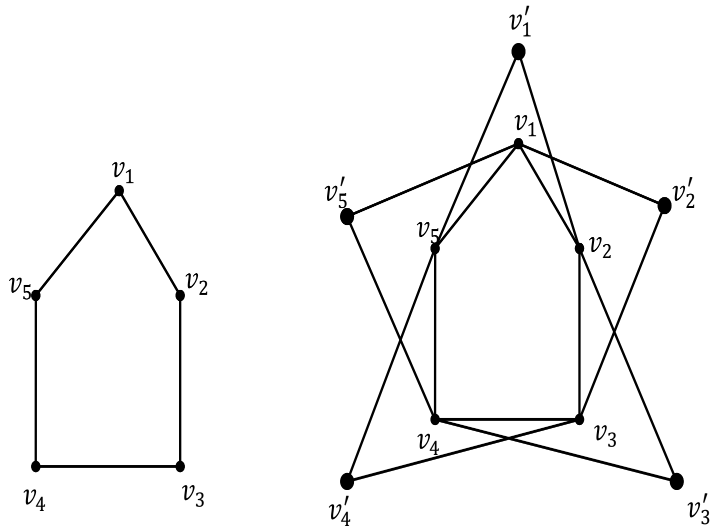


Figure 1

For each vertex v of a graph G, take a new vertex v'. Join v' to all the vertices of G adjacent to *v*. The graph S(G) thus obtained is called *splitting graph* of G.

For example, the graph $S(C_5)$ is shown in Figure 2.



C_5 $S(C_5)$

Figure 2

Recently P.Lawerence Rozario Raj and S.Koilraj have given cordial labeling for the Splitting graph of some standard Graphs in [4].

A.Nellai Murugan and V.Baby Suganya have provided Cordial labeling of Path Related Splitting Graphs in [5].

The assignment of number or symbols subject to certain conditions to the vertices of a graph is known as *graph labeling*. A mapping f: V (G)→{0,1} is called a *binary vertex labeling* of G and f(v) is called the *label of the vertex v* of G under f.

For an edge e=uv, the induced edge labeling f^*:E(G)→{0,1} is given by $f^*(e) = |f(u) - f(v)|$.

Let $v_f(0)$ and $v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively, under f. Let $e_f(0)$ and $e_f(1)$ be the number of edges having labels 0 and 1 respectively, under f*.

A binary vertex labeling of G is called a *cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is *cordial* if it admits a cordial labeling.

Cordial labeling was introduced by *Cahit (1987*) as a weakend version of graceful and harmonious..

Let f be a mapping from V to {0,1}. The induced edge label of an edge uv is the label 1 if f(u) = f(v) or 0 if f(u)≠f(v). Then f is called a *homo cordial labeling*, if the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. The graph that admits a homo cordial labeling is called a *homo cordial graph*.

For example, the homo cordial labeling of $C_5 \odot K_1$ is shown in Figure 3.

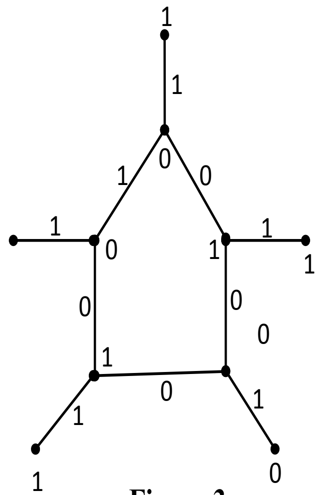


Figure 3

In [6] and [7], homo cordial labeling for path related graphs and cycle related graphs have been estabtished.

In this paper, we prove that some well known trees, corona related graphs are homo cordial graphs.

Theorem 1 The graph $K_{1,n} \cup K_{1,m}$ (n-odd, m-even) is a homo cordial graph, $if\ m+n$ is odd for $m, n\ \geq 1$.

Proof

We have m+n≡1 (mod 2). Then m and n are of different parity.

Without loss of generality, assume that n is odd and m is even.

Let $V(K_{1,n} \cup K_{1,m}) = \{u, u_i, v, v_i\ ; 1 \leq i < n, 1 \leq i \leq m\}$ and

$E(K_{1,n} \cup K_{1,m}) = \{uu_i, vv_i\ ; 1 \leq i < n, 1 \leq i \leq m\}$

Define $f: V(K_{1,n} \cup K_{1,m}) \rightarrow \{0,1\}$ by

$f(u) = 1,$

$f(v) = 0,$

$$f(u_i) = \begin{cases} 1 & \text{if i is odd} \quad 1 \leq i \leq n \\ 0 & \text{if i is even} \end{cases}$$

$$f(v_i) = \begin{cases} 1 & \text{if i is odd} \\ 0 & \text{if i is even} \end{cases} \qquad 1 \le i \le n$$

The induced edge labelings are,

$$f^*(uu_i) = \begin{cases} 1 & \text{if i is odd} \\ 0 & \text{if i is even} \end{cases} \qquad 1 \le i \le n$$

$$f^*(vv_i) = \begin{cases} 0 & \text{if i is odd} \\ 1 & \text{if i is even} \end{cases} \qquad 1 \le i < n$$

Here,

$$v_f(1) = v_f(0) + 1 \text{ and}$$

$$e_f(1) = e_f(0) + 1.$$

Therefore the graph $K_{1,n} \cup K_{1,m}$ satisfies the condition $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence $K_{1,n} \cup K_{1,m}$ (n-odd, m-even) is a homo cordial graph, $n, m \ge 1$. ■

Example 2 The homo cordial labelings of $K_{1,5} \cup K_{1,4}$ are shown in Figure 4.

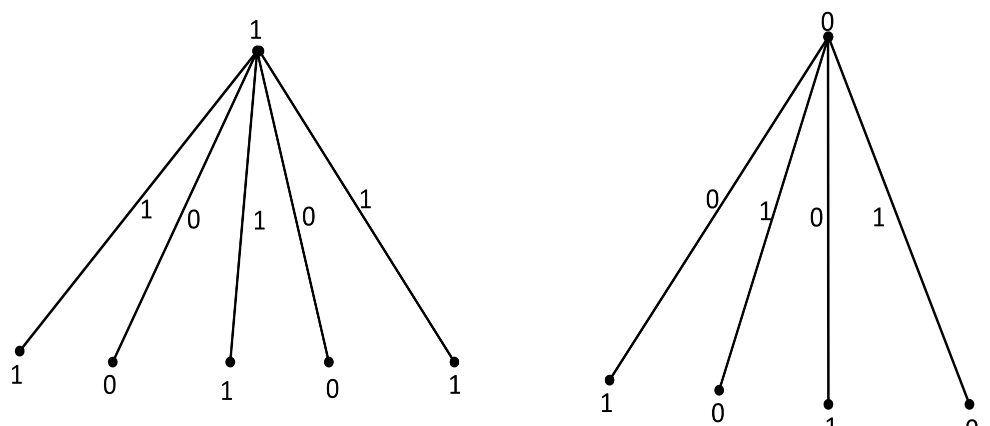


Figure 4

Lemma 3 The graph $K_{1,n} \cup P_n$ (n-odd) is a homo cordial graph, $n \ge 3$.

Proof

Let V($K_{1,n} \cup P_n$) = $\{u, u_i, v_i \; ; 1 \leq i < n\}$ and

E($K_{1,n} \cup P_n$) = $\{(uu_i, 1 \leq i \leq n); (v_i v_{i+1}, 1 \leq i \leq n-1)\}$

Define $f: V(K_{1,n} \cup P_n) \rightarrow \{0,1\}$ by

$$f(u) = 1,$$

$$f(u_i) = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 0 & \text{if } i \text{ is even} \end{cases} \qquad 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} 0 & i \equiv 1,2 \pmod 4 \\ 1 & i \equiv 0,3 \pmod 4 \end{cases} \qquad 1 \leq i \leq n$$

The induced edge labelings are,

$$f^*(uu_i) = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 0 & \text{if } i \text{ is even} \end{cases} \qquad 1 \leq i \leq n$$

$$f^*(vv_{i+1}) = \begin{cases} 1 & i \equiv 1,3 \pmod 4 \\ 0 & i \equiv 0,2 \pmod 4 \end{cases} \qquad 1 \leq i \leq n-1$$

Here,

$$v_f(1) = v_f(0) + 1.$$

$$e_f(1) = e_f(0) + 1.$$

Therefore the graph $K_{1,n} \cup P_n$ (n-odd) satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence $K_{1,n} \cup P_n$ (n-odd) is a homo cordial graph $n \geq 3$. ■

Example 4 The homo cordial labeling of $K_{1,5} \cup P_5$ is shown in Figure 5.

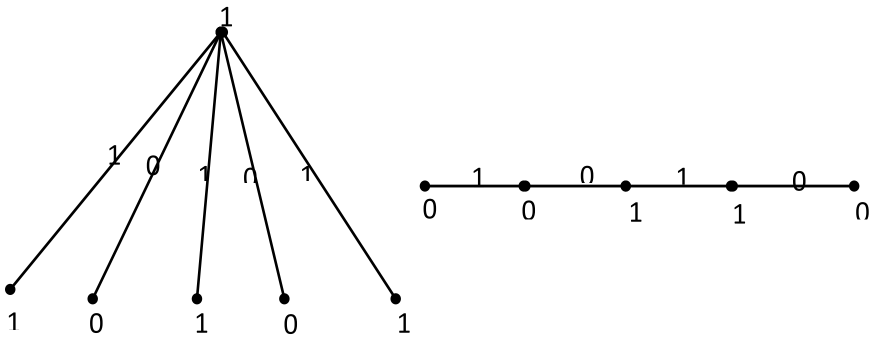


Figure 5

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Lemma 5 The graph $K_{1,n} \cup P_n$ (n-even) is a homo cordial graph, $n \geq 2$.

Proof

Let $V(K_{1,n} \cup P_n) = \{u, u_i, v_i \,;\, 1 \leq i \leq n\}$ and

$E(K_{1,n} \cup P_n) = \{(uu_i, 1 \leq i \leq n); (v_i v_{i+1}, 1 \leq i \leq n-1)\}$

Define $f: V(K_{1,n} \cup P_n) \rightarrow \{0,1\}$ by

$$f(u) = 1,$$

$$f(u_i) = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 0 & \text{if } i \text{ is even} \end{cases} \quad 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} 0 & i \equiv 0,1 \ (\text{mod } 4) \\ 1 & i \equiv 2,3 \ (\text{mod } 4) \end{cases} \quad 1 \leq i \leq n$$

The induced edge labelings are,

$$f^*(uu_i) = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 0 & \text{if } i \text{ is even} \end{cases} \quad 1 \leq i \leq n$$

$$f^*(vv_{i+1}) = \begin{cases} 1 & i \equiv 0 (\text{mod} 4) \\ 0 & i \equiv 1 (\text{mod } 4) \end{cases} \quad 1 \leq i \leq n-1$$

Here,

$v_f(0) = v_f(1) + 1$ and

$e_f(1) = e_f(0) + 1$.

Therefore the graph $K_{1,n} \cup P_n$ (n-even) satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence $K_{1,n} \cup P_n$ (n-even) is a homo cordial graph,where n ≥ 2 is even. ∎

Example 6 The homo cordial labelings of $K_{1,6} \cup P_6$ are shown in Figure 6.

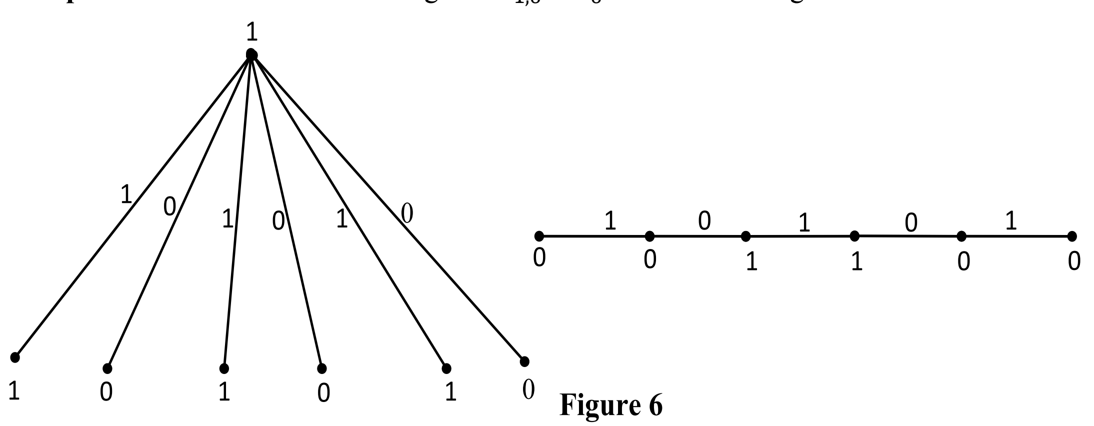

Figure 6

Combining Lemmas 3 and 5, we have the following theorem:

Theorem 7 The graph $K_{1,n} \cup P_n$ is a homo cordial graph.

Theorem 8 The graph $K_{1,n} \cup (P_m \odot K_1)$ (n-even,m-odd) is a homo cordial graph, $n \geq 2$, $m \geq 3$.

Proof

Let V($K_{1,n} \cup (P_n \odot K_1)$) = $\{u, u_i, v_j, w_j\ ; 1 \leq i \leq n, 1 \leq j \leq m\}$ an

E($K_{1,n} \cup (P_n \odot K_1)$)=$\{uu_i, v_jw_j\ ;\ 1 \leq i \leq n, 1 \leq j \leq m, v_j\ v_{j+1};\ 1 \leq j \leq m-1\}$

Define $f: V(K_{1,n} \cup (P_n \odot K_1) \rightarrow \{0,1\}$ by

$$f(u) = 1,$$

$$f(u_i) = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 0 & \text{if } i \text{ is even} \end{cases} \quad 1 \leq i \leq n$$

$$f(v_j) = 0,$$

$$f(w_j) = 1,$$

The induced edge labelings are,

$$f^*(uu_i) = \begin{cases} 1 & \text{if } i \text{ is even} \\ 0 & \text{if } i \text{ is odd} \end{cases} \quad 1 \leq i \leq n$$

$$f^*(v_j\ v_{j+1}) = 1, \quad 1 \leq i \leq n-1$$

$$f^*(v_jw_j) = 0, \quad 1 \leq i \leq n$$

Here,

$$v_f(1) = v_f(0) + 1 \text{ and}$$

$$e_f(0) = e_f(1) + 1.$$

Therefore the graph $(K_{1,n} \cup (P_n \odot K_1)$ (n-even, m-odd) satisfies the condition

$|v_f(0) - v_f(1)| \leq 1$and $|e_f(0) - e_f(1)| \leq 1$.

Hence $(K_{1,n} \cup (P_n \odot K_1\ )$ (n-even, m-odd) is homo cordial graph $n \geq 2, m \geq 3$.

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Example 9 The homo cordial labelings of $K_{1,4} \cup (P_5 \odot K_1)$ are shown in Figure 7.

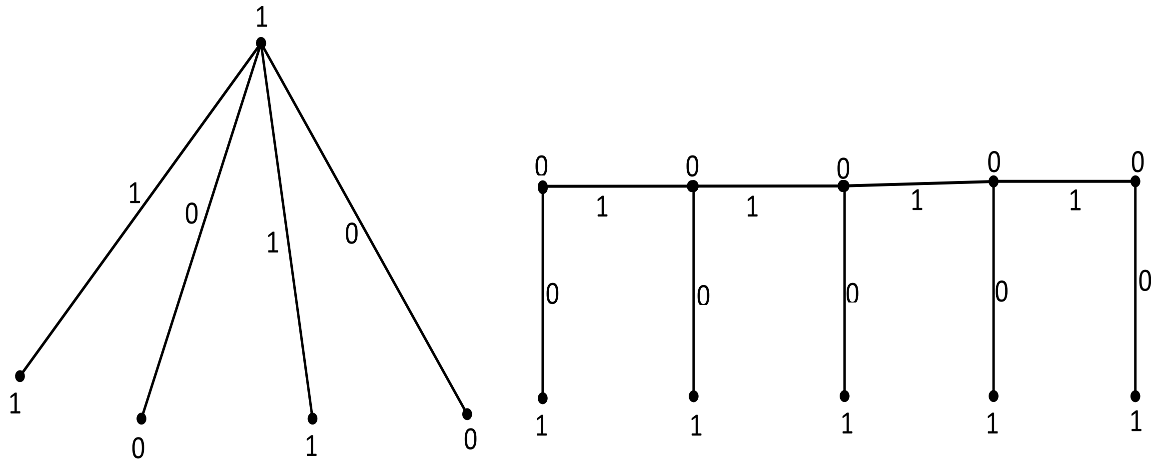


Figure 7

Theorem 10 The graph $(K_{1,n} \cup (P_n + K_1)$ (n-even) is homo cordial graph, $n > 2$.

Proof

Let V$(K_{1,n} \cup (P_n + K_1))$ = $\{u, u_i, v, v_i\ ; 1 < i \leq n\}$ and

E$(K_{1,n} \cup (P_n + K_1))$ = $\{uu_i, vv_i\ ;\ 1 < i \leq n, v_iv_{i+1}, 1 < i \leq n\}$

Define $f: V(K_{1,n} \cup (P_n + K_1)) \rightarrow \{0,1\}$ by

$$f(u) = 1,$$

$$f(u_i) = \begin{cases} 1 & \text{if i is odd} \\ 0 & \text{if i is even} \end{cases} \qquad 1 \leq i \leq n$$

$$f(v) = 0$$

$$f(v) = \begin{cases} 1 & i \equiv 0,1 (mod\ 4) \\ 0 & i \equiv 2,3 (mod\ 4) \end{cases} \qquad 1 \leq i \leq n$$

The induced edge labelings are,

$$f^*(uu_i) = \begin{cases} 1 & \text{if i is odd} \\ 0 & \text{if i is even} \end{cases} \qquad 1 \leq i \leq n$$

$$f^*(v_iv_{i+1}) = \begin{cases} 0 & i \equiv 1,3 (\text{mod } 4) \\ 1 & i \equiv 0,2 (\text{mod } 4) \end{cases} \qquad 1 \leq i \leq n-1$$

$$f^*(vv_i) = \begin{cases} 1 & i\equiv 2,3 \pmod 4 \\ 0 & i\equiv 0,1 \pmod 4 \end{cases} \qquad 1 \le i \le n$$

Here,

$v_f(0) = v_f(1)$ and

$e_f(0) = e_f(1) + 1.$

Therefore the graph $K_{1,n} \cup (P_n + K_1)$ (n-even) satisfies the condition $\left|v_f(0) - v_f(1)\right| \le 1$ and $\left|e_f(0) - e_f(1)\right| \le 1$

Hence $K_{1,n} \cup (P_n + K_1)$ (n-even) is homo cordial graph $n > 2$. ∎

Example 11 The homo cordial labelings of $K_{1,4} \cup (P_4+K_1)$ are shown in Figure 8.

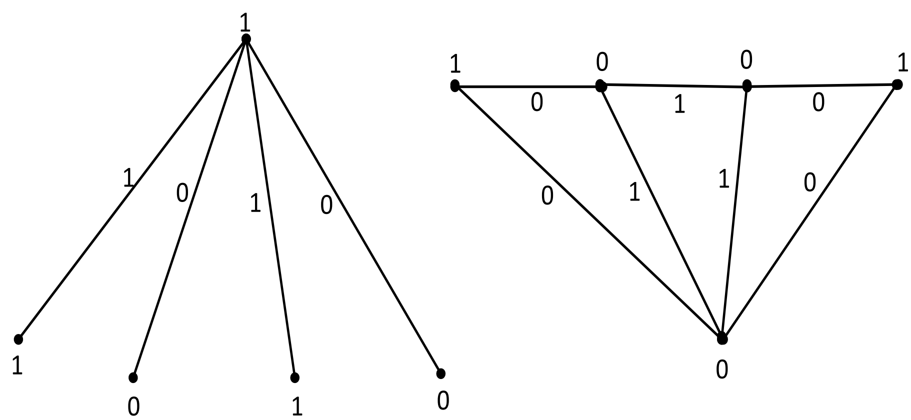


Figure 8

Theorem 12 The graph $K_{1,n} \cup (P_n + 2K_1)$ (n-even) is a homo cordial graph, n >2.

Proof

Let $V(K_{1,n} \cup (P_n + 2K_1)) = \{u, u_i, v_i, w, 1 \le i < n\}$ and

$E(K_{1,n} \cup (P_n + K_1)) = \{uu_i, 1 \le i \le n; v_iv_{i+1}, 1 \le i \le n-1, vv_i; v_iw, 1 \le i \le n\}$

Define $f: V(K_{1,n} \cup (P_n + K_1)) \rightarrow \{0,1\}$ by

$f(u) = 1,$

$f(v) = 0,$

$f(w) = 1,$

$$f(u_i) = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 0 & \text{if } i \text{ is even} \end{cases} \qquad 1 \le i \le n$$

$$f(v_i) = \begin{cases} 0 & i\equiv 2, 3 \pmod 4 \end{cases} \qquad 1 \le i \le n$$

1 i≡0, 1(mod 4)

The induced edge labelings are,

$$f^*(uu_i) = \begin{cases} 1 & \text{if} \quad \text{i is odd} \\ 0 & \text{if} \quad \text{i is even} \end{cases} \qquad 1 \le i \le n$$

$$f^*(vv_i) = \begin{cases} 1 & i \equiv 2, 3 (\text{mod } 4) \\ 0 & i \equiv 0, 1 (\text{mod } 4) \end{cases} \qquad 1 \le i \le n$$

$$f^*(v_i v_{i+1}) = \begin{cases} 1 & i \equiv 0, 2 (\text{mod } 4) \\ 0 & i \equiv 1, 3 (\text{mod } 4) \end{cases} \qquad 1 \le i \le n-1$$

$$f^*(wv_i) = \begin{cases} 1 & i \equiv 0, 1 (\text{mod } 4) \\ 0 & i \equiv 2,3 (\text{mod } 4) \end{cases} \qquad 1 \le i \le n$$

Here,

$v_f(1) = v_f(0) + 1$ and

$e_f(0) = e_f(1) + 1.$

Therefore the graph $K_{1,n} \cup (P_n + 2K_1)$ (n-even) satisfies the condition $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence $K_{1,n} \cup (P_n + 2K_1)$ (n-even) is a homo cordial graph $n > 2$. ∎

Example 13 The homo cordial labeling of $K_{1,4} \cup (P_4 + 2K_1)$ is shown in Figure 9 .

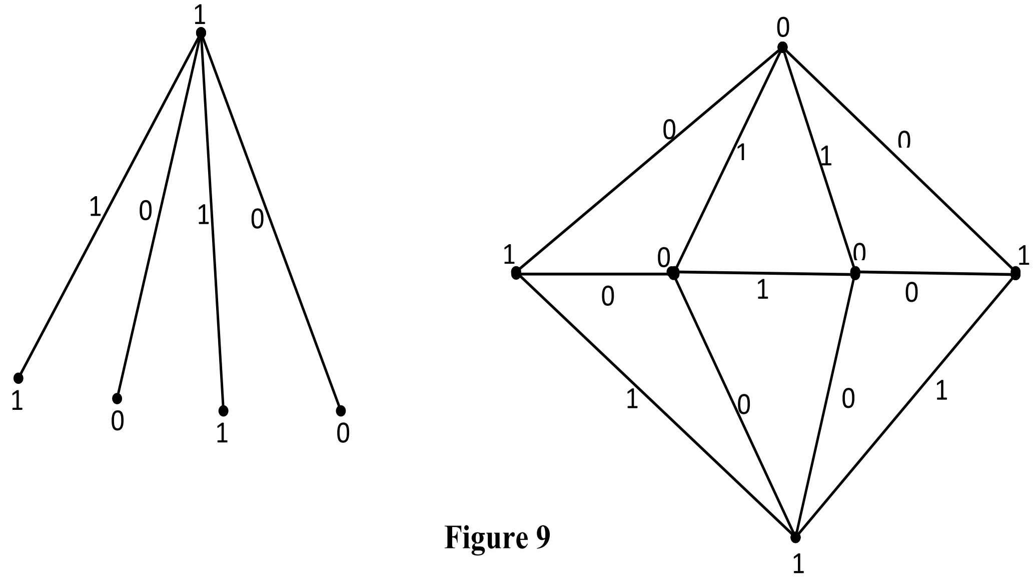


Figure 9

Lemma 14 The graph $K_{1,n} \cup (C_n \odot K_1)$ (n-odd) is a homo cordial graph, n $\geq$3.

Proof

Let V($K_{1,n} \cup (C_n \odot K_1)$) = $\{u, u_i, v_i, w_i; 1 \leq i \leq n\}$ and

E($K_{1,n} \cup (C_n \odot K_1)$) = {[($uu_i$),$1 \leq i \leq n$];[($v_i\ v_{i+1}$),$1 \leq i \leq n-1$]∪ [($v_1v_n$)] ∪ [($v_i\ w_i$);1≤i≤n]}

Define $f: V(K_{1,n} \cup (C_n \odot K_1)) \rightarrow \{0,1\}$ by

$$f(u) = 1,$$

$$f(u_i) = \begin{cases} 0 & \text{if} \quad \text{i is odd} \\ 1 & \text{if} \quad \text{i is even} \end{cases} \qquad 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} 1 & i \equiv 0 (\text{mod } 2) \\ 0 & i \equiv 1 (\text{mod } 2) \end{cases} \qquad 1 \leq i \leq n$$

$$f(w_1) = 1,$$

$$f(w_i) = \begin{cases} 1 & i \equiv 0 (\text{mod } 2) \\ 0 & i \equiv 1 (\text{mod } 2) \end{cases} \qquad 2 \leq i \leq n$$

The induced edge labelings are,

$$f^*(uu_i) = \begin{cases} 1 & \text{if} \quad \text{i is even} \\ 0 & \text{if} \quad \text{i is odd} \end{cases} \qquad 1 \leq i \leq n$$

$$f^*(v_i v_{i+1}) = 0, \qquad 1 \leq i \leq n-1$$

$$f^*(v_1 w_1)) = 0,$$

$$f^*(v_i w_i) = 1, \qquad 2 \leq i \leq n$$

Here,

$$v_f(1) = v_f(0) \text{ and}$$

$$e_f(0) = e_f(1) + 1.$$

Therefore the graph ($K_{1,n} \cup (C_n \odot K_1)$ (n-odd) satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence ($K_{1,n} \cup (C_n \odot K_1)$ (n-odd) is a homo cordial graph n$\geq$ 3. ∎

Example 15 The homo cordial labelings of $K_{1,5} \cup (C_5 \odot K_1)$ are shown in Figure 10.

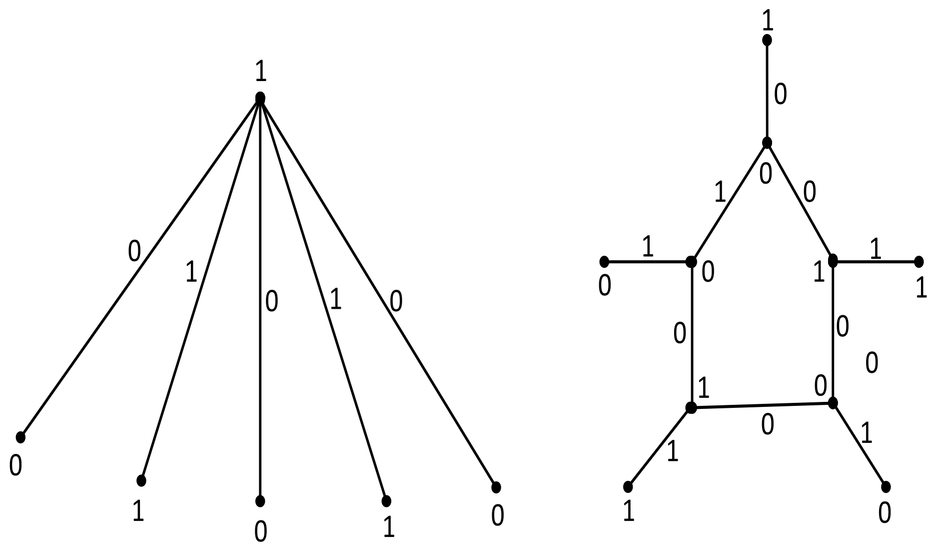


Figure 10

Lemma 16 The graph $K_{1,n} \cup (C_n \odot K_1)$ (n-even) is a homo cordial graph, $n > 2$.

Proof

Let $V(K_{1,n} \cup (C_n \odot K_1)) = \{u, u_i, v_i, w_i\ ; 1 \leq i \leq n\}$ and

$E(K_{1,n} \cup (C_n \odot K_1)) = \{(uu_i,), 1 \leq i \leq n; [(v_i v_{i+1}), 1 \leq i \leq n-1] \cup$

$[(v_1 v_n)] \cup [(v_i\, w_i); 1 \leq i \leq n]\}$

Define $f: V(K_{1,n} \cup (C_n \odot K_1) \rightarrow \{0,1\}$ by

$f(u) = 1,$

$$f(u_i) = \begin{cases} 0 & \text{if} \quad \text{i is odd} \\ 1 & \text{if} \quad \text{i is even} \end{cases}$$

$$f(vi) = \begin{cases} 1 & i \equiv 0 \pmod 2 \\ 0 & i \equiv 1 \pmod 2 \end{cases} \qquad 1 \leq i \leq n$$

$$f(wi) = \begin{cases} 1 & i \equiv 0 \pmod 2 \\ 0 & i \equiv 1 \pmod 2 \end{cases} \qquad 1 \leq i \leq n$$

The induced edge labelings are,

$$f^*(uu_i) = \begin{cases} 1 & \text{if} \quad \text{i is even} \end{cases} \qquad 1 \leq i \leq n$$

0 if i is odd

$f^*(v_iv_{i+1}) = 0$, $1 \le i \le n-1$

$f^*(v_1v_n) = 0$,

$f^*(v_iw_i) = 1$, $1 \le i \le n$

Here,

$v_f(1) = v_f(0) + 1$ and

$e_f(0) = e_f(1)$.

Therefore the graph$(K_{1,n} \cup (C_n \odot K_1)$ (n-even) satisfies the condition $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence $(K_{1,n} \cup (C_n \odot K_1)$ (n-even) is a homo cordial graph n>2.

Example 17 The homo cordial labelings of $K_{1,6} \cup (C_6 \odot K_1)$ are shown in Figure 11.

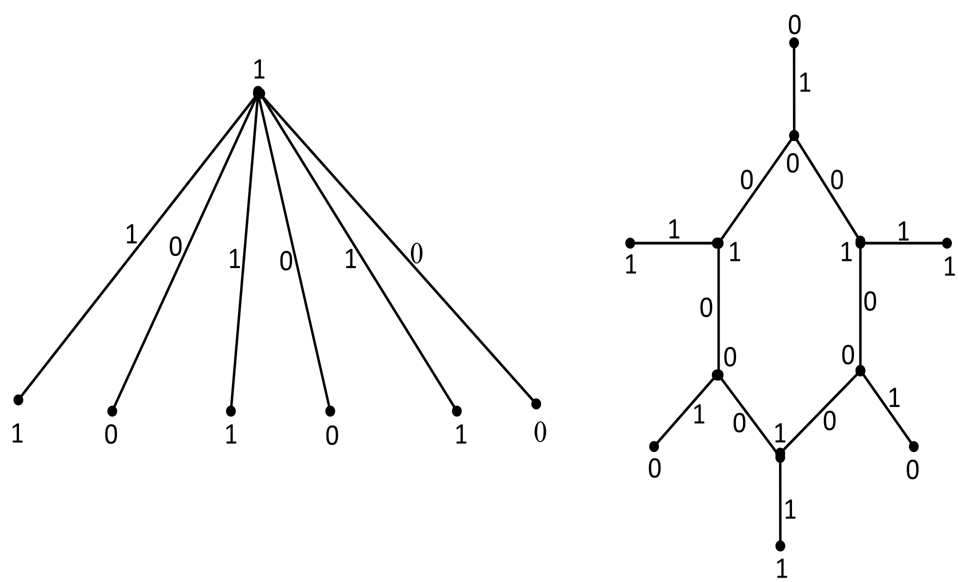


Figure 11

Combining lemmas 14 and 15, we have the following theorem:

Theorem 18 The graph$(K_{1,n} \cup (C_n \odot K_1))$ is a homo cordial graph. ■

Theorem 19 The graph $(C_n \odot K_1) \cup P_n$ is a homo cordial graph.

Proof

Let $V((C_n \odot K_1) \cup P_n)) = \{u_i, v_i, w_i\ ; 1 \le i \le n\}$ and

$E((C_n \odot K_1) \cup P_n)) = \{uu_{i+1};\ 1 \le i \le n-1\ ; u_1u_n; u_iv_i, w_iw_{i+1}, 1 \le i \le n-1\}$

Define $f: V((C_n \odot K_1) \cup P_n)) \to \{0,1\}$ by

Case 1: n is even, $n > 2$.

$$f(u_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases}$$

$$f(v_i) = \begin{cases} 1 & i \equiv 0 \bmod 2 \\ 0 & i \equiv 1 \bmod 2 \end{cases} \qquad 1 \le i \le n$$

$$f(w_i) = \begin{cases} 1 & i \equiv 2,3 \bmod 4 \\ 0 & i \equiv 0,1 \bmod 4 \end{cases} \qquad 1 \le i \le n$$

The induced edge labelings are,

$$f^*(u_i v_i) = 1, \qquad 1 \le i \le n$$

$$f^*(w_i w_{i+1}) = \begin{cases} 1 & i \equiv 0,2 \bmod 4 \\ 0 & i \equiv 1,3 \bmod 4 \end{cases} \qquad 1 \le i \le n-1$$

$$f^*(u_i u_{i+1}) = 0,$$

$$f^*(u_1 u_n) = 0,$$

Here,

$$v_f(0) = v_f(1) \text{ and}$$

$$e_f(0) = e_f(1) + 1.$$

Therefore the graph $(C_n \odot K_1) \cup P_n$ (n-even) satisfies the condition $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence $((C_n \odot K_1) \cup P_n$ (n-even) is homo cordial graph n>2.

Case 2: n is odd, $n \ge 3$.

$$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \qquad 1 \le i \le n$$

$$f(v_1) = 0,$$

$$f(v_i) = \begin{cases} 1 & i \equiv 1 \bmod 2 \\ 0 & i \equiv 0 \bmod 2 \end{cases} \qquad 1 \le i \le n$$

$$f(w_i) = \begin{cases} 0 & i \equiv 2,3 \bmod 4 \end{cases} \qquad 1 \le i \le n$$

1 i≡0,1 mod 4

The induced edge labelings are,

$$f^*(u_iu_{i+1}) = 0, \qquad 1 \le i \le n$$

$$f^*(w_iw_{i+1}) = \begin{cases} 1 & i\equiv 0,2 \bmod 4 \\ 0 & i\equiv 1,3 \bmod 4 \end{cases} \qquad 1 \le i \le n-1$$

$$f^*(u_1u_n) = 1,$$

$$f^*(u_1v_1) = 0,$$

$$f^*(u_iv_i) = 1, \qquad 2 \le i \le n$$

Here,

$v_f(0) = v_f(1) + 1$ when n≡ 3 (mod 4).

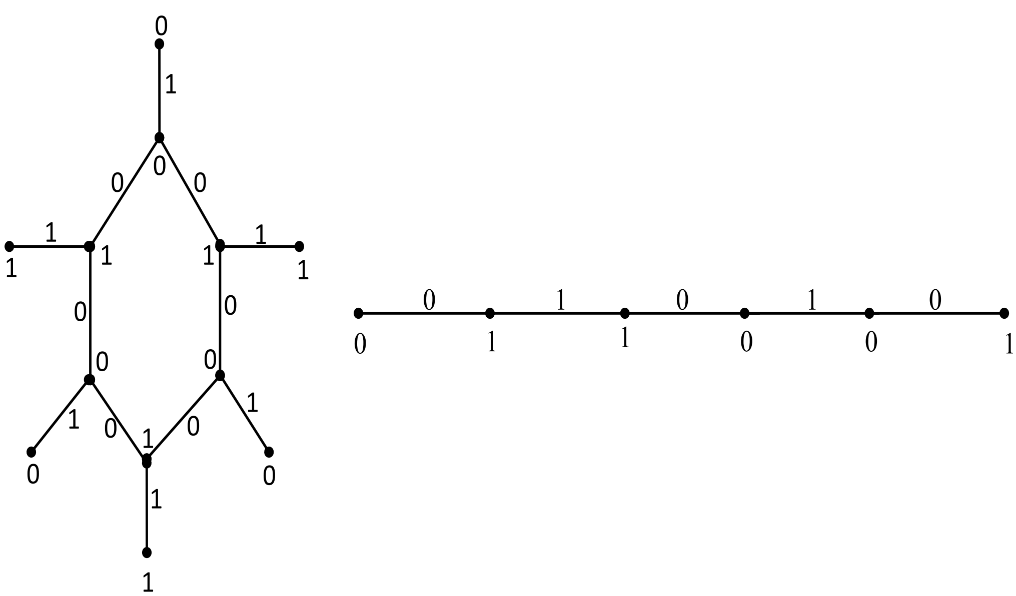
$v_f(1) = v_f(0) + 1$ when n≡ 1 (mod 4) and

$e_f(0) = e_f(1) + 1.$

Therefore the graph $(C_n \odot K_1) \cup P_n$ (n-odd) satisfies the condition $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence $((C_n \odot K_1) \cup P_n$ (n-odd) is a homo cordial graph, $n \ge 3$. ∎

Example 20 The homo cordial labelings of $(C_6 \odot K_1) \cup P_6$ and $(C_5 \odot K_1) \cup P_5$ are shown in Figure 12.



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$(C_6 \odot K_1) \cup P_6$

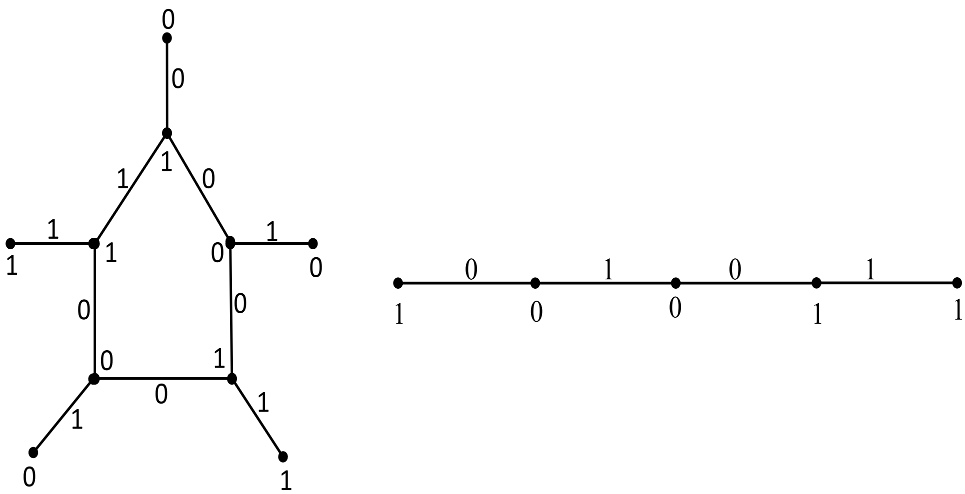


Figure 12

Theorem 21 The graph $(C_n \odot K_1) \cup S(P_n)$ is a homo cordial graph.

Proof

Let $V((C_n \odot K_1) \cup S(P_n)) = \{u_i, v_i, w_i, w_i' \,; 1 \le i \le n\}$ and

$$E((C_n \odot K_1) \cup S(P_n)) = \{u_i u_{i+1}; 1 \le i \le n-1; u_1 u_n; u_i v_i, 1 \le i \le n$$

$$w_i w_{i+1}, 1 \le i \le n-1,; w_i w'_{i+1}, w'_i w_{i+1}, 1 \le i \le n\}.$$

Define $f: V(C_n \odot K_1) \cup S(P_n)) \to \{0,1\}$ by

Case 1: n is even, $n > 2$.

$$f(u_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \qquad 1 \le i \le n$$

$$f(v_i) = \begin{cases} 1 & i \equiv 0 \bmod 2 \\ 0 & i \equiv 1 \bmod 2 \end{cases} \qquad 1 \le i \le n$$

$$f(w_i) = \begin{cases} 0 & i \equiv 2,3 \bmod 4 \\ 1 & i \equiv 0,1 \bmod 4 \end{cases} \qquad 1 \le i \le n$$

$$f(w_i') = \begin{cases} 1 & i \equiv 2,3 \bmod 4 \\ 0 & i \equiv 0,1 \bmod 4 \end{cases} \qquad 1 \le i \le n$$

The induced edge labelings are

$$f^*(u_i u_{i+1}) = 0,$$

$$f^*(u_1 u_n) = 0,$$

$$f^*(u_i\, v_i) \quad = \quad 1, \qquad\qquad 1 \le i \le n$$

$$f^*(w_i w_{i+1}) \quad = \begin{cases} 1 & i\equiv 0,1 \bmod 4 \\ 0 & i\equiv 2,3 \bmod 4 \end{cases} \qquad 1 \le i \le n-1$$

$$f^*(w_i w'_{i+1}) \quad = \begin{cases} 1 & i\equiv 1 \bmod 2 \\ 0 & i\equiv 0 \bmod 2 \end{cases} \qquad 1 \le i \le n$$

$$f^*(w'_i w_{i+1}) \quad = \begin{cases} 1 & i\equiv 1 \bmod 2 \\ 0 & i\equiv 0 \bmod 2 \end{cases} \qquad 1 \le i \le n$$

Here,

$v_f(0) = v_f(1)$ and

$e_f(1) = e_f(0) + 1.$

Therefore the graph $(C_n \odot K_1) \cup S(P_n)$ (n-even) satisfies the condition $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1.$

Hence $(C_n \odot K_1) \cup S(P_n)$ (n-even) is a homo cordial graph n>2. ∎

Case 2: n is odd, $n \ge 3$.

$$f(u_i) = \begin{cases} 0 & i\equiv 0 \bmod 2 \\ 1 & i\equiv 1 \bmod 2 \end{cases} \qquad 1 \le i \le n$$

$$f(v_1) = 0,$$

$$f(v_i) = \begin{cases} 0 & i\equiv 0 \bmod 2 \\ 1 & i\equiv 1 \bmod 2 \end{cases} \qquad 2 \le i \le n$$

$$f(w_i) = \begin{cases} 0 & i\equiv 2,3 \bmod 4 \\ 1 & i\equiv 0,1 \bmod 4 \end{cases} \qquad 1 \le i \le n$$

$$f(w'_i) = \begin{cases} 1 & i\equiv 2,3 \bmod 4 \\ 0 & i\equiv 0,1 \bmod 4 \end{cases} \qquad 1 \le i \le n$$

The induced edge labelings are,

$$f^*(u_i u_{i+1}) = \begin{cases} 0 & i\equiv 1 \pmod 2 \\ 1 & i\equiv 0 \pmod 2 \end{cases} \qquad 1 \le i \le n\text{-}1$$

$$f^*(u_1u_n) = 1,$$

$$f^*(u_1v_1) = 0,$$

$$f^*(u_i\,v_i) = 1, \qquad 2 \le i \le n$$

$$f^*(w_iw_{i+1}) = \begin{cases} 1 & i\equiv 0,1 \pmod 4 \\ 0 & i\equiv 2,3 \pmod 4 \end{cases} \qquad 1 \le i \le n-1$$

$$f^*(w_iw'_{i+1}) = \begin{cases} 1 & i\equiv 1 \pmod 2 \\ 0 & i\equiv 0 \pmod 2 \end{cases} \qquad 1 \le i \le n$$

$$f^*(w'_iw_{i+1}) = \begin{cases} 1 & i\equiv 1 \pmod 2 \\ 0 & i\equiv 0 \pmod 2 \end{cases} \qquad 1 \le i \le n$$

Here,

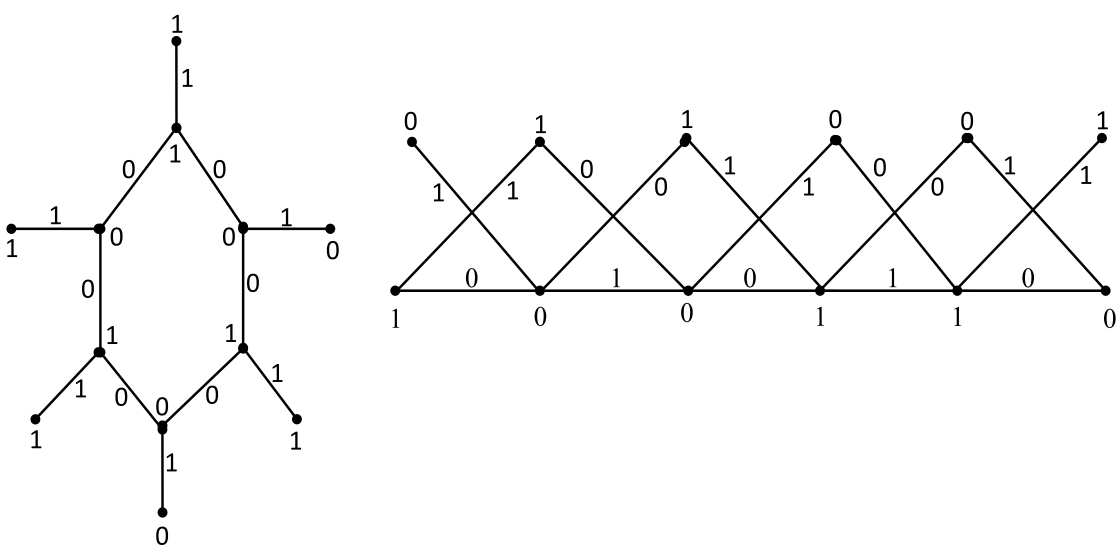
$v_f(0) = v_f(1)$ and

$e_f(1) = e_f(0)$.

Therefore the graph $(C_n \odot K_1) \cup S(P_n)$ (n-odd) satisfies the condition $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence $(C_n \odot K_1) \cup S(P_n)$ (n-odd) is a homo cordial graph,$n \ge 3$.

Example 22 The homo cordial labelings of $(C_6 \odot K_1) \cup S(P_6)$ and $(C_5 \odot K_1) \cup S(P_5)$ are shown in Figure 13.



$(C_6 \odot K_1) \cup S(P_6)$

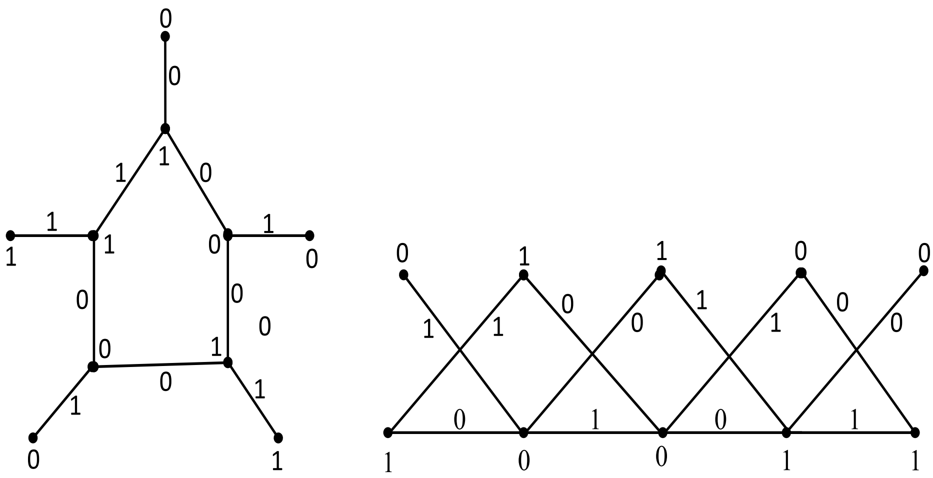


Figure 13

Theorem 23 The graph $(C_n \odot K_1) \cup (P_n \odot K_1)$ is a homo cordial graph.

Proof

Let $V((C_n \odot K_1) \cup (P_n \odot K_1)) = \{u_i, v_i, w_i, w_i'; 1 \leq i \leq n\}$

$$E((C_n \odot K_1) \cup (P_n \odot K_1)) = \{[(u_i u_{i+1}); 1 \leq i \leq n-1] \cup [(u_1 u_n)];[(w_i w_{i+1}); 1 \leq i \leq n-1, (w_i w_i'); 1 \leq i \leq n]\}$$

Define $f: v((C_n \odot K_1) \cup (P_n \odot K_1)) \rightarrow \{0,1\}$ by

Case 1: n is even, $n > 2$.

$$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

$f(w_i) = 0,$

$f(w_i') = 1,$

The induced edge labelings are,

$f^*(u_i u_{i+1}) = 0, \quad 1 \leq i \leq n-1$

$f^*(u_1 u_n) = 0,$

$f^*(u_i v_i) = 1, \quad 1 \leq i \leq n$

$f^*(w_i w_{i+1}) = 1, \quad 1 \leq i \leq n-1$

$f^*(w_i w_i') = 0, \quad 1 \leq i \leq n$

Here,

$v_f(0) = v_f(1)$ and

$e_f(0) = e_f(1) + 1.$

Therefore the graph $(C_n \odot K_1) \cup (P_n \odot K_1)$ (n-even) satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$

Hence $(C_n \odot K_1) \cup (P_n \odot K_1)$ (n-even) is a homo cordial graph, $n > 2$.

Case 2: n is odd, $n \geq 3$.

$$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

$f(v_i) = 0,$

$$f(v_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 2 \leq i \leq n$$

$f(w_i) = 0,$

$f(w_i') = 1,$

The induced edge labelings are,

$f^*(u_i u_{i+1}) = 0, \quad 1 \leq i \leq n-1$

$f^*(u_1 u_n) = 0,$

$f^*(u_i v_i) = 1, \quad 1 \leq i \leq n$

$f^*(w_i w_{i+1}) = 1, \quad 1 \leq i \leq n-1$

$f^*(w_i w_i') = 0, \quad 1 \leq i \leq n$

Here,

$v_f(0) = v_f(1)$ and

$e_f(0) = e_f(1) + 1.$

Therefore the graph $(C_n \odot K_1) \cup (P_n \odot K_1)$ (n-odd) satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1.$

Hence $(C_n \odot K_1) \cup (P_n \odot K_1)$ (n-odd) is a homo cordial graph, $n \geq 3$.

Example 24 The homo cordial labelings of $(C_4 \odot K_1) \cup P_4$ and $(C_5 \odot K_1) \cup P_5$ are shown in Figure 14 and Figure 15.

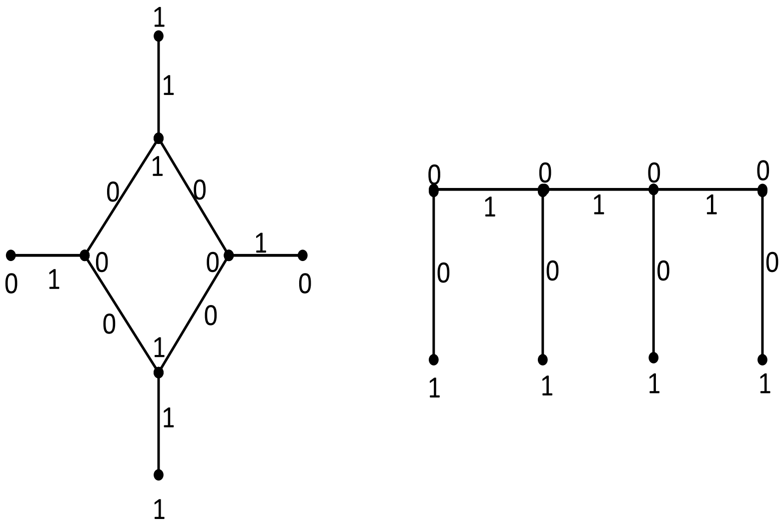


Figure 14

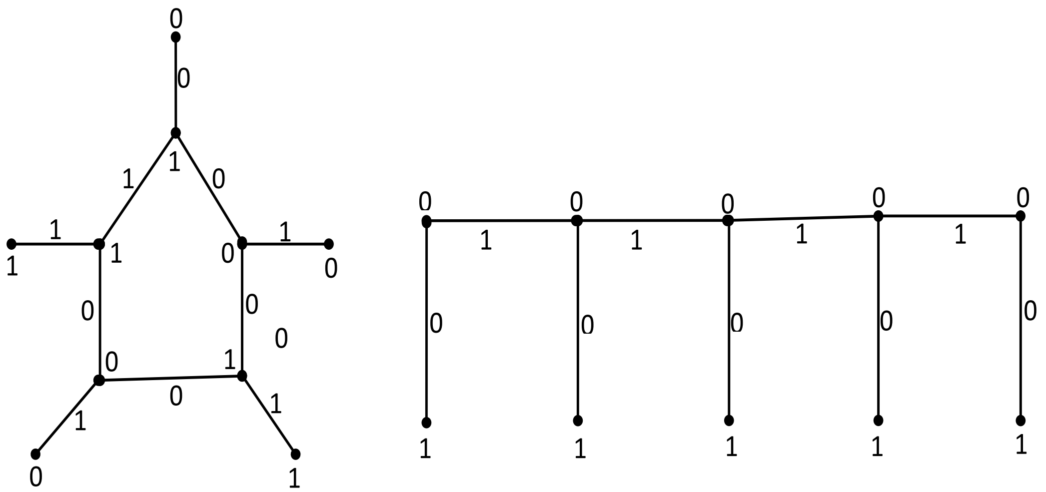


Figure 15

Theorem 25 The graph $(C_n \odot K_1) \cup (C_n \odot K_1)$ is a homo cordial graph.

Proof

Let $V((C_n \odot K_1) \cup (C_n \odot K_1)) = \{u_i, u_i', v_i, v_i'; 1 \le i \le n\}$

$E((C_n \odot K_1) \cup (C_n \odot K_1))=\{u_i u_{i+1}, 1 \le i \le n-1; u_1 u_n; u_i u_i', 1 \le i \le n; v_i v_{i+1},$

$1 \le i \le n-1; v_1 v_n; v_i v_i';\ 1 \le i \le n]\}.$

Define $f: V((C_n \odot K_1) \cup (C_n \odot K_1)) \to \{0,1\}$ by

Case 1: n is odd, $n \ge 3$

$$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \qquad 1 \le i \le n$$

$$f(u_i') = 0,$$

$$f(u_i') = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \qquad 2 \le i \le n$$

$$f(v_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \qquad 1 \le i \le n$$

$$f(v_i') = 0,$$

$$f(v_i') = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \qquad 2 \le i \le n$$

The induced edge labelings are,

$$f^*(u_i u_{i+1}) = 0, \qquad 1 \le i \le n-1$$

$$f^*(u_1 u_n) = 1,$$

$$f^*(u_1 u_1') = 0,$$

$$f^*(u_i u_i') = 1, \qquad 2 \le i \le n$$

$$f^*(v_i v_{i+1}) = 0, \qquad 1 \le i \le n-1$$

$$f^*(v_1 v_n) = 1,$$

$$f^*(v_1 v_1') = 0,$$

$$f^*(v_i v_i') = 1, \qquad 2 \le i \le n$$

Here,

$v_f(0) = v_f(1)$ and

$e_f(0) = e_f(1)$.

Therefore the graph $(C_n \odot K_1) \cup (C_n \odot K_1)$ (n-odd) satisfies the condition

$\left|v_f(0) - v_f(1)\right| \le 1$ and $\left|e_f(0) - e_f(1)\right| \le 1$.

Hence $(C_n \odot K_1) \cup (C_n \odot K_1)$ (n-odd) is a homo cordial graph, $n \geq 3$.

Case 2: n is even,$n > 2$.

$$f(u_i) = \begin{cases} 0 & \text{i} \equiv 0 \text{ mod } 2 \\ 1 & \text{i} \equiv 1 \text{ mod } 2 \end{cases} \qquad 1 \leq i \leq n$$

$$f(u_i') = \begin{cases} 0 & \text{i} \equiv 0 \text{ mod } 2 \\ 1 & \text{i} \equiv 1 \text{ mod } 2 \end{cases} \qquad 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} 0 & \text{i} \equiv 1 \text{ mod } 2 \\ 1 & \text{i} \equiv 0 \text{ mod} \end{cases} \qquad 1 \leq i \leq n$$

$$f(v_i') = \begin{cases} 0 & \text{i} \equiv 1 \text{ mod } 2 \\ 1 & \text{i} \equiv 0 \text{ mod } 2 \end{cases} \qquad 1 \leq i \leq n$$

The induced edge labeling are,

$$f^*(u_i \text{u}_{\text{i+1}}) = 0, \qquad 1 \leq \text{i} \leq \text{n-1}$$

$$f^*(u_1 \text{u}_\text{n}) = 0,$$

$$f^*(u_i \text{u}_\text{i}') = 1, \qquad 1 \leq i \leq n$$

$$f^*(v_i v_{\text{i+1}}) = 0, \qquad 1 \leq \text{i} \leq \text{n-1}$$

$$f^*(v_1 v_\text{n}) = 0,$$

$$f^*(v_i \text{v}_\text{i}') = 1, \qquad 1 \leq i \leq n$$

Here,

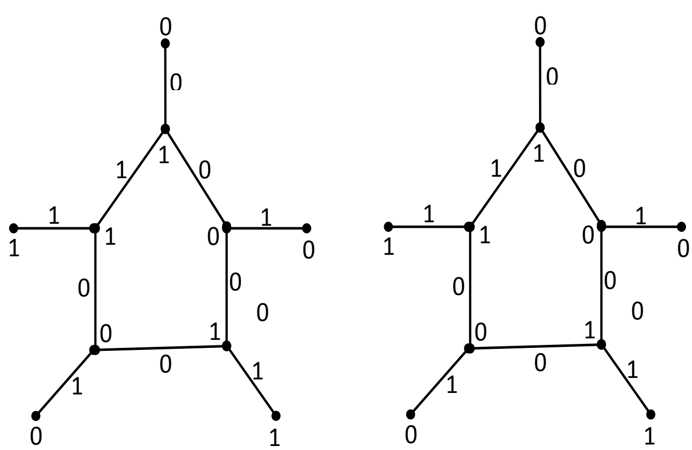
$v_f(0) = v_f(1)$ and

$e_f(0) = e_f(1)$.

Therefore the graph $(C_n \odot K_1) \cup (C_n \odot K_1)$ (n-even) satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence $(C_n \odot K_1) \cup (C_n \odot K_1)$ (n-even) is a homo cordial graph, $n > 2$. ∎

Example 26 The homo cordial labelings of $(C_5 \odot K_1) \cup (C_5 \odot K_1)$ and $(C_6 \odot K_1) \cup (C_6 \odot K_1)$ are shown in Figure 17.



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$(C_5 \odot K_1) \cup (C_5 \odot K_1)$

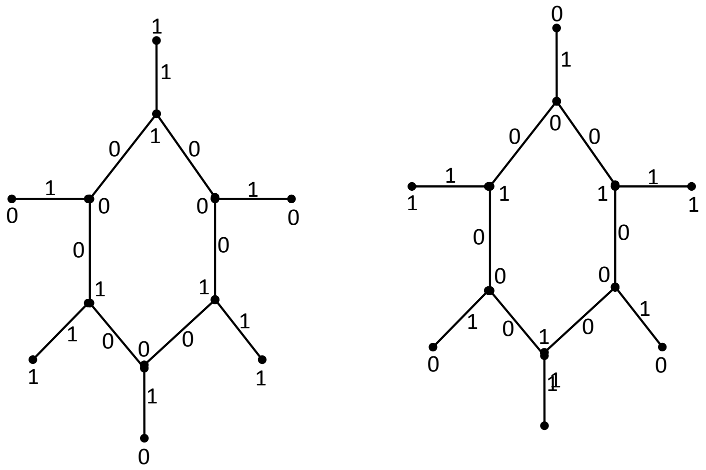

Figure 17

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