INTRODUCTION
Sampling inspection consists of procedure for taking decisions on one or more lots of finished products which have been submitted for inspection. The decision of either acceptance or rejection of the lots is usually taken by adopting suitable sampling inspection procedures, called sampling plans. Sampling plans are generally categorized into two types, namely, lot-by-lot sampling by attributes and lot-by-lot sampling by variables. In lot-by-lot inspection by attributes, one or more samples of items are drawn from a given lot of manufactured items; each item in the sample(s) is classified as conforming or nonconforming and the decision of acceptance or rejection of the lot is made based on a specific rule. In lot-by-lot inspection by variables, one or more samples of items are drawn from a given lot; the measurement of a quality characteristic in each sampled item is recorded; and the decision of acceptance or rejection of the lot is made as a function of such measurements. The theory of inspection by variables is applicable when the quality characteristic of sampled items is measurable on a continuous scale and the functional form of the probability distribution is assumed to be known. A variables sampling is advantageous in the sense that it generates more information from each item inspected, requires less inspection units, and lot-by-lot sampling by variables. In lot-by-lot inspection procedures of this kind are defined in general under the condition that the quality characteristic, which is considered as a random variable, is measurable on a continuous scale and the functional form of the probability distribution of the random variable must be known. Several works in variable sampling plan have been done by many researchers under the assumption that the quality characteristics is modeled by a normal distribution and are found in the literature of acceptance sampling. However, the assumption of normality may not be often realized in practice and hence, it is quite inevitable to make an investigation about the properties of sampling inspection plans when the quality variables are modeled by non-normal distributions. In this paper a single sampling plan by variables is devised and evaluated under the assumption that the quality characteristic follows an Inverse Gaussian distribution. A procedure is developed for determining the parameters of the proposed plan when the requirements in terms of quality levels so as to provide protection to the producer and the consumer are specified.

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ABSTRACT
Sampling inspection by variables is one of the major classifications of acceptance sampling and consists of procedures for taking decisions about the disposition of a lot of individual units based on sample measurements of units on a quality characteristic under study. Inspection procedures of this kind are defined in general under the condition that the quality characteristic, which is considered as a random variable, is measurable on a continuous scale and the functional form of the probability distribution of the random variable must be known. Several works in variable sampling plan have been done by many researchers under the assumption that the quality characteristic is modeled by a normal distribution and are found in the literature of acceptance sampling. However, the assumption of normality may not be often realized in practice and hence, it is quite inevitable to make an investigation about the properties of sampling inspection plans when the quality variables are modeled by non-normal distributions. In this paper a single sampling plan by variables is devised and evaluated under the assumption that the quality characteristic follows an Inverse Gaussian distribution. A procedure is developed for determining the parameters of the proposed plan when the requirements in terms of quality levels so as to provide protection to the producer and the consumer are specified.

SINGLE SAMPLING INSPECTION PLANS BY VARIABLES
A single sampling inspection plan by variables is defined under the following assumptions:

(a) The quality characteristic, denoted by, X is measurable on a continuous scale and has a known form of probability distribution, represented by \( F_x(x; \mu, \sigma) \), which is the distribution function of \( X \) with mean \( \mu \) and variance \( \sigma^2 \).

(b) Each individual unit in a submitted lot has a one-sided specification, say, lower specification, \( L \) or upper specification, \( U \). If, for a unit, \( X > U \) (or \( X < L \)), the unit is classified as a non-conforming unit.

The operating procedure of a variable sampling plan is as follows:

Step 1: Draw a random sample of \( n \) units from a lot and observe the measurements \( x_1, x_2, ..., x_n \) of the quality characteristic, \( X \).

Step 2: Whensis known, accept the lot if \( \tau + k_2 \sigma \leq U \) (or \( \tau + k_2 \sigma \geq L \)); otherwise reject the lot, where \( \tau \) is the sample mean. When \( \sigma \) is unknown, accept the lot, if \( \tau + k_\sigma \leq U \) (or \( \tau + k_\sigma \geq L \)), where

\[
\hat{\sigma} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2
\]

is an unbiased estimate of \( \sigma^2 \).

Thus, a single sampling plan by variables is designated by two parameters, namely, the sample size, \( n \), and the acceptability constant, \( k \). When these parameters are known, the plan could be implemented. The explicit expressions for \( n \) and \( k \) can be derived by specifying two points on the operating characteristic curve of the plan, namely, \( (p_a, 1-\alpha) \), and \( (p_b, \beta) \) where \( p_a \) and \( p_b \) are termed as acceptable quality level (AQL) and the limiting quality level (LQL), associated with the producer’s risk, \( \alpha \) and the consumer’s risk, \( \beta \), respectively. A sampling plan by variables is termed as a known \( \sigma \) or unknown \( \sigma \) plan according as \( \sigma \) is known or unknown.

KEYWORDS: Inverse Gaussian distribution, Normal distribution, Operating characteristic function, Quality level, Single sampling plan.
OPERATING CHARACTERISTIC FUNCTION

An important measure of performance of a variables sampling plan is its operating characteristic function, which is a function of the proportion, \( p \), of non-conforming units, called incoming lot quality, and provides the probability, \( P_1(p) \), of acceptance of a lot. The plot of \( P_1(p) \) against \( p \) results in a curve, called operating characteristic (OC) curve. For a given upper specification limit, \( u \), when \( \sigma \) is known, \( p \) and \( P_1(p) \) are defined by

\[
p = P(X > U | \mu) \quad (1)
\]

\[
P_1(p) = P(\bar{x} + k\sigma \leq U | \mu) \quad (2)
\]

AQL and LQL, using (1), are defined by

\[
AQL = P_1 = P(\bar{x} > U | \mu_1) \quad (3)
\]

and

\[
LQL = P_2 = P(\bar{x} > U | \mu_2) \quad (4)
\]

where \( \mu_1 \) and \( \mu_2 \) are the means of the underlying distribution which results in AQL and LQL, respectively.

Assume that \( X \) be a random variable representing a specific quality characteristic of manufactured items and be distributed according to a Inverse Gaussian distribution with two parameters, namely, the mean, \( \theta \), and the shape parameter, \( \lambda \). The probability density function of the Inverse Gaussian distribution is given by

\[
f(x; \theta, \lambda) = \frac{\lambda}{\sqrt{2\pi x^3}} \exp\left(-\frac{\lambda(x - \theta)^2}{2\theta^2x}\right), \quad 0 > \lambda > 0, x > 0
\]

Therefore, the distribution function of the Inverse Gaussian variable whose density function is given by (5) is expressed by

\[
F(x; \theta, \lambda) = \Phi\left(\frac{\sqrt{\lambda} - \frac{\theta}{x}}{\sqrt{\lambda}}\right) + \exp\left(\frac{2\sqrt{\lambda}}{\theta}\right)\Phi\left(\frac{\sqrt{\lambda}}{\theta\sqrt{x}} - 1\right), \quad 0 > \lambda > 0, x > 0
\]

where \( \Phi \) is the cumulative distribution function of the standard normal variable.

The mean and the variance of the underlying distribution are given by \( \mu = \theta \) and \( \sigma^2 = \frac{\theta^2}{\lambda} \). Further, the measures of skewness and kurtosis, which are the measures for identifying the asymmetric nature of the distribution, are expressed as \( \alpha_1 = \frac{\theta}{\lambda} \) and \( \alpha_2 = \frac{\theta^2}{\lambda^2} - 3 \), respectively. From (1), (3) and (4), the lot quality levels, \( p \), AQL and LQL using Inverse Gaussian distribution are, respectively, defined by

\[
p = P(\bar{x} > U | \mu) \quad (5)
\]

\[
AQL = P_1 = P(\bar{x} > U | \mu_1) \quad (6)
\]

and

\[
LQL = P_2 = P(\bar{x} > U | \mu_2) \quad (7)
\]

where \( \mu_1 \) and \( \mu_2 \) are the standardized deviates exceeded with the probabilities \( \alpha \) and \( \beta \), respectively. The known standard deviation plan is expressed as

\[
P_1(p) = P_1(\bar{x} + k\sigma \leq U | \mu) = P_1(\bar{x} \leq U | \mu) \quad (8)
\]

The operating characteristic function \( P_1(p) \), for an unknown standard deviation plan is expressed as

\[
P_1(p) = P_1(\bar{x} + k\sigma \leq U | \mu) = P_1(\bar{x} \leq U | \mu) \quad (9)
\]

Assume that \( p \) and \( \mu \) are specified with the risks, \( \alpha \), of rejecting the lot and \( \beta \), of accepting the lot. Then, corresponding to \( \alpha \) and \( \beta \), the normal deviates \( K_\alpha \) and \( K_\beta \) are, respectively, defined by

\[
K_\alpha = U - \left(\frac{\mu + k\sigma}{\sigma}\right) \quad (10)
\]

and

\[
K_\beta = U - \left(\frac{\mu - k\sigma}{\sigma}\right) \quad (11)
\]

On substituting the expressions (12) and (13) for \( K_\alpha \) and \( K_\beta \), equations (14) and (15) would result in the following:

\[
K_\alpha^* = \frac{K_\alpha}{K_\alpha + K_\beta} \quad (16)
\]

and

\[
K_\beta^* = \frac{K_\beta}{K_\alpha + K_\beta} \quad (17)
\]

The mathematical expressions for nd of an unknown standard deviation plan for specified requirements in terms of the points \((\alpha, \beta, \mu, \sigma)\) and \((\alpha, \beta, \mu, \sigma)\) on the OC curve such that \( P_1(p) = 1 - \alpha \) and \( P_1(p) = \beta \) when \( \mu \) is specified, are derived from (16) and (17) as

\[
n_s = n_1 + \left(\frac{k^2}{2}\right) \quad (18)
\]

and

\[
k_s = \frac{k^2}{\left(n_1 + n_2\right)} \quad (19)
\]

where \( K_\alpha \) and \( K_\beta \) are the standardized deviates exceeded with the probabilities \( \alpha \) and \( \beta \), respectively. The known standard deviation plans is obtained with parameters \( n_1 \) and \( k_s \) given by

\[
n_s = n_1 + \left(\frac{k_s^2}{2}\right) \quad (20)
\]

and

\[
k_s = \frac{k_s^2}{n_1 + n_2} \quad (21)
\]

The known standard deviation plan with parameters \( n_s \) and \( k_s \) could be derived as

\[
n_s = n_1 + \left(\frac{k_s^2}{2}\right) \quad (22)
\]

and

\[
k_s = k_s \quad (23)
\]
NUMERICAL EXAMPLE
Suppose that it is desired to institute a single sampling plan by variables giving protection to the producer and consumer in terms of quality levels $p_l = 0.01$ and $p_u = 0.06$ associated with producer's risk, $\alpha = 0.05$ and $\beta = 0.10$ when the quality characteristic is distributed according to Inverse Gaussian distribution having the mean $\theta = 5$ and the shape parameter fixed at $\lambda = 10$. Corresponding to $p_l$ and $p_u$, the values of $K_p$ and $K_d$ are determined respectively from equations (7) and (8) as $3.6458$ and $1.7621$. The normal deviates $K_p$ and $K_d$ are obtained as $1.6485$ and $1.28155$ by satisfying (9) and (10) for specified values of $\alpha$ and $\beta$. Substituting these values in (18) and (19), the parameters of the desired plan are determined as $n = 10.4896$ which, when rounded, becomes $11$ and $k = 5.2870$. Thus, the parameters of a known standard deviation plan are computed as

$$n = n_0 \begin{pmatrix} \frac{K_p}{2} \\ \frac{K_d}{2} \end{pmatrix} = 2.4134 \approx 3$$

and $k = 2.5870$

CONCLUSION
A single sampling plan by variables under the assumption of Inverse Gaussian distribution is proposed and its performance measure, called the operating characteristic function is derived. A procedure for determining the parameters of such plans is discussed through an illustration.

REFERENCES