Original Research Paper

Volume : 5 | Issue : 11 | November 2016 • ISSN No 2277 - 8179 | IF : 3.508 | IC Value : 69.48

Economic Order Quantity Model for Multi-Item with Shortages and Trade Credit Policy

Management

KEYWORDS: Economic order quantity, Trade credit period, Shortages, Multi-item

Mukesh Kumar Jain

Associate Professor & Head, Department of Commerce & Business Administration M.M.H. College, Ghaziabad (U.P), India

ABSTRACT

In this paper, an economic order quantity model is developed for multi-item with constant demand under the effect of trade credit policy. Shortages are considered which are fully backlogged. It is assumed that trade credit periods for different items are known and fixed. Model is developed for two different cases depending on the values of trade credit period and the duration of positive inventory. To obtain the optimal policy two different theorems has been developed. Models are illustrated with the help of numerical example. Sensitive analysis is also carried out with respect to different key parameters.

1. Introduction: From the literature of inventory, it is observed that inventory model related to economic order quantity is one of the oldest inventory models. **Harris (1913)** was the first who obtained the expression for economic order quantity. He proposed a formula $q_0 = \sqrt{\frac{2KD}{h}}$ to obtain the optimal order quantity q_0 where D is the rate of demand per unit time, h is carrying cost per unit item per unit time, and K is the cost of replenishment inventory. **Ritchie (1980)** developed an economic order quantity model by considering appropriate policies for a linear demand followed by a period of steady demand. In literature, it is found that many inventory practitioners such as **Hariga (1996), Chang and Dye (1999), Ouyang et al., (2005), Mahata (2011, 2012),** and **Yadav et al., (2015)** developed an economic order quantity inventory model by taking different assumption.

From literature, it is observed that in most of the economic order quantity model it is assumed that retailers have to settle the account with supplier as soon as he/she received it. But now the market scenario gradually changes. To attract the retailers and to increase their demand, supplier offer the trade credit period to retailer to settle the account. During this trade credit period, retailer can earn interest on the revenue getting by selling the items. **Goyal (1985)** was the first who first time in literature used the phenomenon of permissible period while developing the inventory model. After that, several inventory practitioners such as **Aggarwal and Jaggi (1995), Chung (2000), Sana and Chaudhuri (2008), Khanra et al., (2011), and Yadav et al., (2015)** enrich the literature through their work by incorporating this phenomenon.

In this paper, an economic order quantity is developed for multi-items by considering trade credit period with shortages. Here, it is assumed that shortages are fully backlogged. On the bases of trade credit period and time of positive inventory, two different scenarios have been discussed. And for each scenario theorem has been developed to obtain the optimal policy. Rest of the paper is organized as follows. Section 2 contains the assumptions and notations. In section 3 mathematical formulations have been carried out. Solution methodology to obtain the optimal solution is provided in section 4. Model is illustrated with the help of numerical example in section 5. It also includes sensitive analysis with respect to key parameters. The conclusion and recommendations for future research are given in section 6.

2. Assumptions and Notations: Following assumptions and notations has been used for the development of proposed model.

Assumptions:

1. Replenishment rate is infinite i.e., lead time is zero.
2. The inventory system involves 'n' different items.
3. Demand rate of each type of item is constant.
4. Shortages are allowed and completely backlogged.
5. Trade credit period is considered.
6. No interest is to be charged during shortage period.
7. No interest is to be earned after trade credit period.

Notations:

$I_i(t)$ Inventory level of i^th item at any time t
T Length of replenishment period for i^th item (decision variable)
T_1 Time when inventory level becomes zero for i^th item (decision variable)
M_i Trade credit period for the i^th item
K_i Ordering cost for i^th item per order
H_i Holding cost for i^th item per unit
S_i Shortage cost for i^th item per unit
P_i Purchasing cost for i^th item per unit
D_i Demand rate of i^th item
Q_i Initial inventory level of i^th item
I_{ei} Interest earned per year for i^th item
I_{ri} Interest paid per year for i^th item, $I_{ei} > I_{ri}$

3. Mathematical Modeling: The inventory level $I_i(t)$ for the i^th item at any time t decreases due to meet the demand of the customer. Thus, the inventory level at any time t for i^th item can be described by the following differential equation

$$\frac{dI_i(t)}{dt} = -D_i, \quad 0 \le t \le T \qquad .... (1)$$

With boundary condition $I_i(T_1) = 0,\ I_i(0) = Q_i$

Now the inventory level for the i^th item becomes zero at T_1. Shortages accumulated during $T_1 \le t \le T$.

On solving equation (1) by applying condition, we get

$$I_i(t) = D_i(T_1 - t), \quad 0 \le t \le T \qquad .... (2)$$

On putting t=0 in equation (2), we get

$$Q_i = D_i T_1 \qquad .... (3)$$

Now, we calculate different cost associated with inventory one by one.

Holding cost for i^th item (Hc_i) = $H_i \int_0^{T_1} I_i(t)dt = \frac{H_i D_i T_1^2}{2}$

Ordering cost for i^th item (OC_i) = K_i

Shortage cost for i^th item (Sc_i) = $S_i \int_{T_1}^{T} \left(-I_i(t)\right)dt = \frac{S_i(T-T_1)^2}{2}$

Here, we assumed that trade credit period offered by the retailer to the customer. So, two different cases arises on depending the values of T_1, and M_i

Case-1: $M_i \le T_1$ (See Fig.1)

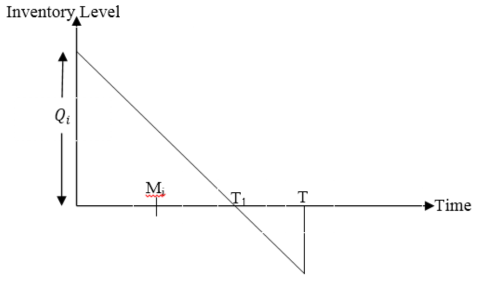

Fig.1 Inventory Lavel for i[th] Item in Case of $M_i \leq T_1$

In this case,

Interest earned for the i[th] item (IE_{1i}) = $P_i I_{ei} \int_0^{M_i} t D_i dt = \frac{P_i I_{ei} D_i M_i^2}{2}$

Interest paid for the i[th] item (Ip_{1i}) = $P_i I_{ri} \int_{M_i}^{T_1} I_i(t) dt = \frac{P_i I_{ri} D_i (T_1 - M_i)^2}{2}$

Total average cost for i[th] item per unit

$$= \frac{1}{T}[HC_i + OC_i + SC_i + IP_{1i} - IE_{1i}]$$

$$= \frac{1}{T}\left[\frac{H_i D_i T_1^2}{2} + K_i + \frac{S_i(T-T_1)^2}{2} + \frac{P_i I_{ri} D_i (T_1 - M_i)^2}{2} - \frac{P_i I_{ei} D_i M_i^2}{2}\right]$$

Thus,
Total average cost for the system per unit

$$TC_1 = \sum_{i=1}^{n} \frac{1}{T}\left[\frac{H_i D_i T_1^2}{2} + K_i + \frac{S_i(T-T_1)^2}{2} + \frac{P_i I_{ri} D_i (T_1 - M_i)^2}{2} - \frac{P_i I_{ei} D_i M_i^2}{2}\right] \quad .... (4)$$

Case-2 $T_1 \leq M_i$ (See Fig.2)

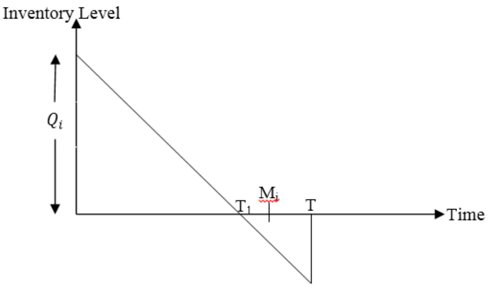

Fig.2 Inventory Lavel for i[th] Item in Case of $T_1 \leq M_i$

In this case,
Interest earned for the i[th] item (IE_{2i})

$$= P_i I_{ei} \int_0^{T_1} t D_i dt + P_i I_{ei}(M_i - T_1) \int_0^{T_1} D_i \, dt$$

$$= P_i I_{ei}\left(M_i D_i T_1 - \frac{D_i T_1^2}{2}\right)$$

Interest paid for the i[th] item (IP_{2i}) = 0

Total average cost for i[th] item per unit

$$= \frac{1}{T}[HC_i + OC_i + SC_i + IP_{2i} - IE_{2i}]$$

$$= \frac{1}{T}\left[\frac{H_i D_i T_1^2}{2} + K_i + \frac{S_i(T-T_1)^2}{2} - P_i I_{ei}\left(M_i D_i T_1 - \frac{D_i T_1^2}{2}\right)\right]$$

Thus, total average cost for the system per unit

$$TC_2 = \sum_{i=1}^{n} \frac{1}{T}\left[\frac{H_i D_i T_1^2}{2} + K_i + \frac{S_i(T-T_1)^2}{2} - P_i I_{ei}\left(M_i D_i T_1 - \frac{D_i T_1^2}{2}\right)\right] \quad .... (5)$$

Thus, the annual average cost for the retailer can be expressed as

$$TC = \begin{cases} TC_1, & M_i \leq T_1 \\ TC_2, & T_1 \leq M_i \end{cases} \quad .... (6)$$

4. Solution Methodology: In this section, we develop efficient decision rules to obtain the optimal values of T_1 and T in different case so that the inventory cost of retailer is minimum.

Case-1 $M_i \leq T_1, \forall i$

$$\frac{\partial TC_1}{\partial T} = -\frac{1}{T^2}\left[\frac{H_i D_i T_1^2}{2} + K_i + \frac{S_i(T-T_1)^2}{2} + \frac{P_i I_{ri} D_i (T_1 - M_i)^2}{2} - \frac{P_i I_{ei} D_i M_i^2}{2}\right] + \frac{1}{T}[S_i(T - T_1)]$$

$$\frac{\partial TC_1}{\partial T_1} = \frac{1}{T}[H_i D_i T_1 - S_i(T - T_1) + P_i I_{ri} D_i (T_1 - M_i)]$$

$$\frac{\partial^2 TC_1}{\partial T^2} = \frac{2}{T^3}\left[\frac{H_i D_i T_1^2}{2} + K_i + \frac{S_i(T-T_1)^2}{2} + \frac{P_i I_{ri} D_i (T_1 - M_i)^2}{2} - \frac{P_i I_{ei} D_i M_i^2}{2}\right] - \frac{2}{T^2}[S_i(T - T_1)] + \frac{S_i}{T}$$

$$\frac{\partial^2 TC_1}{\partial T_1^2} = \frac{1}{T}[H_i D_i + S_i + P_i I_{ri} D_i]$$

$$\frac{\partial^2 TC_1}{\partial T_1 \partial T} = \frac{\partial^2 TC_1}{\partial T \partial T_1} = -\frac{1}{T^2}[H_i D_i T_1 - S_i(T - T_1) + P_i I_{ri} D_i (T_1 - M_i)] - \frac{S_i}{T}$$

Lemma-1 $TC_1(T, T_1)$ has minimum value for those values of T and T_1 which satisfies the following equations

(i) $\left[\frac{H_i D_i T_1^2}{2} + K_i + \frac{S_i(T-T_1)^2}{2} + \frac{P_i I_{ri} D_i (T_1 - M_i)^2}{2} - \frac{P_i I_{ei} D_i M_i^2}{2}\right] = T[S_i(T - T_1)]$
(ii) $H_i D_i T_1 - S_i(T - T_1) + P_i I_{ri} D_i (T_1 - M_i) = 0$

Provided

$$\left[\frac{H_i D_i T_1^2}{2} + K_i + \frac{S_i(T-T_1)^2}{2} + \frac{P_i I_{ri} D_i (T_1 - M_i)^2}{2} - \frac{P_i I_{ei} D_i M_i^2}{2}\right] - 2T[S_i(T - T_1)] + S_i T^2 > 0$$

and

$$[H_i D_i + S_i + P_i I_{ri} D_i]\left\{2\left[\frac{H_i D_i T_1^2}{2} + K_i + \frac{S_i(T-T_1)^2}{2} + \frac{P_i I_{ri} D_i (T_1 - M_i)^2}{2} - \frac{P_i I_{ei} D_i M_i^2}{2}\right] - 2T[S_i(T - T_1)] + S_i T^2\right\} - \{[H_i D_i T_1 - S_i(T - T_1) + P_i I_{ri} D_i (T_1 - M_i)] - S_i T\}^2 > 0$$

Theorem-1: If a function $TC_1(T, T_1)$ have continuous partial derivative of second order, then $TC_1(T, T_1)$ is minimum at $T = T^*, T_1 = T_1^*$ if

$$\begin{vmatrix} \frac{\partial^2 TC_1}{\partial T^2} & \frac{\partial^2 TC_1}{\partial T \partial T_1} \\ \frac{\partial^2 TC_1}{\partial T_1 \partial T} & \frac{\partial^2 TC_1}{\partial T_1^2} \end{vmatrix} > 0 \text{ and } \frac{\partial^2 TC_1}{\partial T^2} > 0 \left(or \frac{\partial^2 TC_1}{\partial T_1^2} > 0\right)$$

Proof: Using lemma-1, we can easily prove it.

Case-2 $T_1 \leq M_i$

$$\frac{\partial TC_2}{\partial T} = -\frac{1}{T^2}\left[\frac{H_i D_i T_1^2}{2} + K_i + \frac{S_i(T-T_1)^2}{2} - P_i I_{ei}\left(M_i D_i T_1 - \frac{D_i T_1^2}{2}\right)\right] + \frac{1}{T}[S_i(T - T_1)]$$

$$\frac{\partial TC_2}{\partial T_1} = \frac{1}{T}[H_i D_i T_1 - S_i(T - T_1) - P_i I_{ei}(M_i D_i - D_i T_1)]$$

$$\frac{\partial^2 TC_2}{\partial T^2} = \frac{2}{T^3}\left[\frac{H_i D_i T_1^2}{2} + K_i + \frac{S_i(T-T_1)^2}{2} - P_i I_{ei}\left(M_i D_i T_1 - \frac{D_i T_1^2}{2}\right)\right] - \frac{2}{T^2}[S_i(T - T_1)] + \frac{S_i}{T}$$

$$\frac{\partial^2 TC_2}{\partial T_1^2} = \frac{1}{T}[H_i D_i + S_i + P_i I_{ei} D_i]$$

$$\frac{\partial^2 TC_2}{\partial T_1 \partial T} = \frac{\partial^2 TC_2}{\partial T \partial T_1} = -\frac{1}{T^2}[H_i D_i T_1 - S_i(T - T_1) - P_i I_{ei}(M_i D_i - D_i T_1)] - \frac{S_i}{T}$$

Lemma-2 $TC_2(T, T_1)$ has the minimum value for those values of T and T_1 which satisfies the following equations

(i) $\left[\frac{H_i D_i T_1^2}{2} + K_i + \frac{S_i(T-T_1)^2}{2} - P_i I_{ei}\left(M_i D_i T_1 - \frac{D_i T_1^2}{2}\right)\right] = T[S_i(T - T_1)]$
(ii) $H_i D_i T_1 - S_i(T - T_1) - P_i I_{ei}(M_i D_i - D_i T_1) = 0$

Provided

$$\left[\frac{H_i D_i T_1^2}{2} + K_i + \frac{S_i(T-T_1)^2}{2} - P_i I_{ei}\left(M_i D_i T_1 - \frac{D_i T_1^2}{2}\right)\right] - 2T[S_i(T - T_1)] + S_i T^2 > 0$$

and

Volume : 5 | Issue : 11 | November 2016 • ISSN No 2277 - 8179 | IF : 3.508 | IC Value : 69.48

$$[H_iD_i + S_i + P_iI_{ei}D_i]\left\{2\left[\frac{H_iD_iT_1^2}{2} + K_i + \frac{S_i(T-T_1)^2}{2} - P_iI_{ei}\left(M_iD_iT_1 - \frac{D_iT_1{}^2}{2}\right)\right] - 2T[S_i(T-T_1)] + S_iT^2\right\} >$$
$$\{[H_iD_iT_1 - S_i(T-T_1) - P_iI_{ei}(M_iD_i - D_iT_1)] + S_iT\}^2$$

Theorem-2: If a function $TC_2(T,T_1)$ have continuous partial derivative of second order, then $TC_2(T,T_1)$ is minimum at $T = T^*$, $T_1 = T_1^*$ if

$$\begin{vmatrix} \frac{\partial^2 TC_2}{\partial T^2} & \frac{\partial^2 TC_2}{\partial T \partial T_1} \\ \frac{\partial^2 TC_2}{\partial T_1 \partial T} & \frac{\partial^2 TC_2}{\partial T_1^2} \end{vmatrix} > 0 \text{ and } \frac{\partial^2 TC_2}{\partial T^2} > 0 \left(or\ \frac{\partial^2 TC_2}{\partial T_1^2} > 0\right)$$

Proof: Using lemma-1, we can easily prove it.

5. Numerical Example and Sensitive Analysis:

ABC Company produces three types of product and sold it to the retailer. The values of different parameters associated with the model are as follows in appropriate units.

Item	M_i	K_i	H_i	S_i	P_i	D_i	I_{ei}	I_{ri}
1st	0.15	78	0.14	0.25	9	195	0.12	0.14
2nd	0.15	80	0.16	0.30	11	200	0.13	0.15
3rd	0.15	75	0.13	0.28	8	180	0.11	0.13

Solving equation (4), we get $T_1^* = 0.32$, $T^* = 1.45$, $TC_1 = 250.89$

Solving equation (5), we get $T_1^* = 0.12$, $T^* = 1.24$, $TC_2 = 209.02$

Hence, inventory cost is minimum in case 2. In this case

$T_1^* = 0.12$, $T^* = 1.24$ and $TC_2 = 209.02$

In this situation, economic order quantities of the items are $Q_1^* = 23.4$, $Q_2^* = 24$ and $Q_3^* = 21.6$ respectively.

5.1 Sensitivity Analysis: Now, we analyze the effect of different key parameters on the optimal solution with the help of sensitive analysis. The sensitivity analysis is performed by changing the parameters by -50%, -25%, 25% and 50% keeping other parameters fixed.

Sensitivity w.r.t Demand:-

Fig.3 shows the effect of demand on the optimal inventory cost. It is observed that as the demand increases total inventory cost of the system also increases.

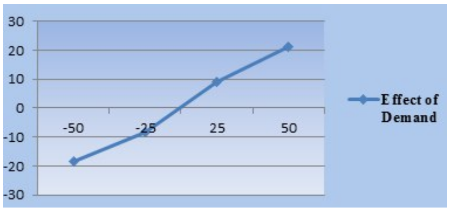

Fig.3 Effect of Demand on Total Inventory Cost

Fig.4 shows the effect of trade credit period on the total inventory cost of the system. It shows that as the trade credit period increases total inventory cost of the system decreases. Longer trade credit period means more incentive for the retailer.

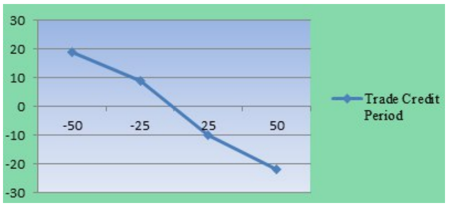

Fig. 4 Effect of Trade Credit Period on Total Inventory Cost

6. Conclusion: In practical situation hardly any system is found that dealing with single item. So, it is more practical to consider multi-item in place of single items. Due to stiff competition, supplier offers trade credit period to their retailer to enhance their demand. It is also practical problem that shortages cannot be avoided due in many uncertainties. Thus, the major contribution of the proposed model is to incorporate all of the above phenomenon simultaneously. With the help of sensitive analysis it is observed that total inventory cost is highly sensitive with respect to demand parameter and trade credit period. So while making the inventory policy, decision-makers pay special attention on these parameters. We can extend the proposed model by considering inflation, learning effect, partial backlogging, and partial delay in payment.

References:

1. Aggarwal, S.P., and Jaggi, C.K. (1995). Ordering policies of deteriorating items under permissible delay in payments. Journal of the Operational Research Society, 36, 658-662.
2. Chang, H., and Dye, C. (1999). An EOQ model for deteriorating items with time varying demand and partial backlogging. Journal of the Operational Research Society, 50(11), 1176-1182.
3. Chung, K.J. (2000). The inventory replenishment policy for deteriorating items under conditions of permissible delay in payments. Opsearch, 37, 267-281.
4. Goyal, S.K. (1985). EOQ under conditions of permissible delay in payments. Journal of the Operational Research Society, 36, 335-338.
5. Hariga, M. (1996). Optimal EOQ models for deteriorating items with time-varying demand. Journal of the Operational Research Society, 47(10), 1228-1246.
6. Harris, F.W. (1913). How many parts to make at once. Factory Management Manage, 10(2), 135-136.
7. Khanra, S., Ghosh, S.K., and Chaudhuri, K.S. (2011). An EOQ model for a deteriorating item with time dependent quadratic demand. Computers and Operations Research, 30, 1901-1916.
8. Mahata G.C. (2012). An EPQ-based inventory model for exponentially deteriorating items under retailer partial trade credit policy in supply chain. Expert Systems with Applications, 39(3), 3537-3550.
9. Mahata, G.C. (2011). EOQ model for items with exponential distribution deterioration and linear trend demand under permissible delay in payments. International Journal of Soft Computing, 6(3), 46-53
10. Ouyang, L., Chang, C., Teng, J. (2005). An EOQ model for deteriorating items under trade credits. Journal of the Operational Research Society, 56(6), 719-726.
11. Ritchie, E. (1980). Practical inventory replenishment policies for a linear trend in demand followed by a period of steady demand. Journal of the Operational Research Society, 31(7), 605-613.
12. Sana, S.S., and Chaudhuri, K.S. (2008). A deterministic EOQ model with delays in payment and price discount offers. European Journal of Operational Research, 184, 509-533.
13. Yadav, D., Singh, S.R., and Kumari, R. (2015). Retailer's Optimal Policy under Inflation in Fuzzy Environment with Trade Credit. International Journal of Systems Science, 42, 754-762.
14. Yadav, D., Singh, S.R., and Meenu. (2015). EOQ Model with Partial Backordering for Imperfect Items under the Effect of Inflation and Learning with Selling Price Dependent Demand. International Journal of Computer Applications, 111 (17), 25-31.