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SOME THEOREMS IN MULTI INTUITIONISTIC FUZZY RW-CLOSED AND MULTI INTUITIONISTIC FUZZY RW-OPEN SETS IN MULTI INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

MATHEMATICS

KEYWORDS: Fuzzy subset, multi fuzzy subset, multi fuzzy topological spaces, multi fuzzy rw-closed, multi fuzzy rw-open, intuitionistic fuzzy subset, multi intuitionistic fuzzy subset, multi intuitionistic fuzzy topological spaces, multi intuitionistic fuzzy rw-closed, multi intuitionistic fuzzy rw-open.

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ABSTRACT

In this paper, we study some of the properties of multi intuitionistic fuzzy rw-closed and multi intuitionistic fuzzy rw-open sets in multi intuitionistic fuzzy topological spaces and prove some results on these.

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INTRODUCTION: The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [16] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper. C.L.Chang [5] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Benchalli.S.S, R.S.Wali and Basavaraj M.Ittanagi[3] introduced the fuzzy rw-closed sets and fuzzy rw-open sets in fuzzy topological spaces. Many researchers like R.H.Warren [15], S.R.Malghan and S.S.Benchalli [10] and many others have contributed to the development of fuzzy topological spaces. V.Murugan, U.Karuppiah and M.Marudai [11, 12] have introduced the multi fuzzy rw-closed and multi fuzzy rw-open sets in multi fuzzy topological spaces. We introduce the concept of multi intuitionistic fuzzy rw-closed and multi intuitionistic fuzzy rw-open sets in multi fuzzy topological spaces and established some results.

1.PRELIMINARIES:

1.1 Definition[16]: Let X be a non-empty set. A **fuzzy subset A** of X is a function $A: X \rightarrow [0, 1]$.

1.2 Definition[14]: A **multi fuzzy subset** A of a set X is defined as an object of the form $A = \{ x, A_1(x), A_2(x), A_3(x), \ldots, A_n(x) \ / xX\}$, where A_i: X[0, 1] for all i. It is denoted as $A = A_1, A_2, A_3, \ldots, A_n$.

1.3 Definition: Let X be a set and be a family of multi fuzzy subsets of X. The family is called a multi fuzzy topology on X if and only if satisfies the following axioms

(I) $0,1 \in \Im$,
(ii) If $\{A_i; i\in I\} \subseteq \Im$, then $\bigcup_{i\in I} A_i \in \Im$,
(iii) If $A_1, A_2, A_3, \ldots An \in \Im$, then $\bigcap_{i=1}^{i=n} A_i \in \Im$.

The pair $(X, \Im)$ is called a multi fuzzy topological space. The members of $\Im$ are called multi fuzzy open sets in X. A multi fuzzy set A in X is said to be multi fuzzy closed set in X if and only if A^c is a multi fuzzy open set in X.

1.4 Definition: Let $(X, \Im)$ be a multi fuzzy topological space. A multi fuzzy set A of X is called multi fuzzy regular w-clsoed (briefly, multi fuzzy rw-closed) if mfcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is multi fuzzy regular semiopen in multi fuzzy topological space X.

1.5 Definition: A multi fuzzy set A of a multi fuzzy topological space X is called a multi fuzzy regular w-open (briefly, multi fuzzy rw-open) set if its complement A^c is a multi fuzzy rw-closed set in multi fuzzy topological space X.

1.6 Definition: An intuitionistic fuzzy subset (IFS) A of a set X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \ / \ x\in X\}$, where $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership of the element x in X respectively and for every x in X satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

1.7 Example: Let X = { a, b, c } be a set. Then $A = \{\langle a, 0.52, 0.34\rangle, \langle b, 0.14, 0.71\rangle, \langle c, 0.25, 0.34\rangle\}$ is an intuitionistic fuzzy subset of X.

1.8 Definition: A multi intuitionistic fuzzy subset(MIFS) A of a set X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x\in X\}$, where $\mu_A(x) = (\mu_{A1}(x), \mu_{A2}(x), \ldots \mu_{An}(x))$, $\mu_{Ai} : X \rightarrow [0, 1]$ for all i and $\nu_A(x) = (\nu_{A1}(x), \nu_{A2}(x), \ldots, \nu_{An}(x))$, $\nu_{Ai} : X \rightarrow [0, 1]$ for all i, define the degree of membership and the degree of non-membership of the element x in X respectively and for every x in X satisfying $0 \leq \mu_{Ai}(x) + \nu_{Ai}(x) \leq 1$ for all I.

1.9 Definition: Let A and B be any two multi intuitionistic fuzzy subsets of a set X. We define the following relations and operations:

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, for all x in X.
(ii) A = B if and only if $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$, for all x in X.
(iii) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x\in X\}$.
(iv) $A\cap B = \{\langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\}\rangle / x\in X\}$.
(v) $A\cup B = \{\langle x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\nu_A(x), \nu_B(x)\}\rangle / x\in X\}$.

1.10 Definition: Let X be a set and $\Im$ be a family of multi intuitionistic fuzzy subsets of X. The family $\Im$ is called a multi intuitionistic fuzzy topology on X if and only if $\Im$ satisfies the following axioms (i) $0_X, 1_X \in \Im$, (ii) If $\{Ai ; i\in I\} \subseteq \Im$, then $\bigcup_{i\in I} A_i \in \Im$, (iii) If $A_1, A_2, A_3, \ldots\ldots A_n \in \Im$, then $\bigcap_{i=1}^{i=n} A_i \in \Im$. The pair $(X, \Im)$ is called a multi intuitionistic fuzzy topological space. The members of $\Im$ are called multi intuitionistic fuzzy open sets in X. A multi intuitionistic fuzzy set A in X is said to be multi intuitionistic fuzzy closed set in X if and only if A^c is a multi intuitionistic fuzzy open set in X.

1.11 Definition: Let $(X, \Im)$ be a multi intuitionistic fuzzy topological space and A be a multi intuitionistic fuzzy set in X. Then $\cap\{B : B^c \in \Im$ and $B \supseteq A\}$ is called multi intuitionistic fuzzy closure of A and is denoted by mifcl (A).

1.12 Theorem: Let A and B be two multi intuitionistic fuzzy sets in multi intuitionistic fuzzy topological space $(X, \Im)$. Then the following results are true, (i) mifcl(A) is a multi intuitionistic fuzzy closed set in X,

(ii) mifcl(A) is the least multi intuitionistic fuzzy closed set containing A,

(iii) A is a multi intuitionistic fuzzy closed if and only if A = mifcl(A),
(iv) $mifcl(0_X) = 0_X$, 0_X is the empty multi intuitionistic fuzzy set,
(v) mifcl(mifcl(A)) = mifcl(A),
(vi) mifcl(A) $\cup$ mifcl (B) = mifcl(A $\cup$ B),
(vii) mifcl(A) $\cap$ mifcl (B) $\supseteq$ mifcl(A $\cap$ B),

1.13 Definition: Let (X, $\Im$) be a multi intuitionistic fuzzy topological space and A be a multi intuitionistic fuzzy set in X. Then $\cup$\{ B : B $\in \Im$ and B $\subseteq$ A \} is called multi intuitionistic fuzzy interior of A and is denoted by mifint(A).

1.14 Theorem: Let (X, $\Im$) be a multi intuitionistic fuzzy topological space, A and B be two multi intuitionistic fuzzy sets in X. The following results hold,

(i) mifint(A) is an multi intuitionistic fuzzy open set in X,
(ii) mifint(A) is the largest multi intuitionistic fuzzy open set in X which is less than or equal to A,
(iii) A is a multi intuitionistic fuzzy open set if and only if A = mifint(A),
(iv) A $\subseteq$ B implies mifint(A) $\subseteq$ mifint(B),
(v) mifint(mifint(A)) = A,
(vi) mifint(A) $\cap$ mifint(B) = mifint(A $\cap$ B),
(vii) mifint(A) $\cup$ mifint(B) $\subseteq$ mifint(A $\cup$ B),
(viii) mifint(Ac) = $(mifcl(A))^c$,
(ix) mifcl(Ac) = $(mifint(A))^c$.

1.15 Definition: Let (X, $\Im$) be a multi intuitionistic fuzzy topological space and A be multi intuitionistic fuzzy set in X. Then A is said to be

(i) multi intuitionistic fuzzy semiopen if and only if there exists a multi intuitionistic fuzzy open set V in X such that V $\subseteq$ A $\subseteq$ mifcl(V),

(ii) multi intuitionistic fuzzy semiclosed if and only if there exists a multi intuitionistic fuzzy closed set V in X such that mifint(V) $\subseteq$ A $\subseteq$ V,

(iii) multi intuitionistic fuzzy regular open set of X if mifint(mifcl(A)) = A,

(iv) multi intuitionistic fuzzy regular closed set of X if mifcl(mifint(A)) = A,

(v) multi intuitionistic fuzzy regular semiopen set of X if there exists a multi intuitionistic fuzzy regular open set V in X such that V $\subseteq$ A $\subseteq$ mifcl(V).

We denote the class of multi intuitionistic fuzzy regular semiopen sets in multi intuitionistic fuzzy topological space X by MIFRSO(X).

(vi) multi intuitionistic fuzzy generalized closed (mifg-closed) if mifcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is multi intuitionistic fuzzy open set and A is multi intuitionistic fuzzy generalized open set if A^c is multi intuitionistic fuzzy generalized closed,

(vii) multi intuitionistic fuzzy rg-closed if mifcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is multi intuitionistic fuzzy regular open set in X,

(viii) multi intuitionistic fuzzy rg-open if its complement Ac is multi intuitionistic fuzzy rg-closed set in X,

(ix) multi intuitionistic fuzzy w-closed if mifcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is multi intuitionistic fuzzy semi open set in X,

(x) multi intuitionistic fuzzy w-open if its complement A^c is multi intuitionistic fuzzy w-closed set in X,

(xi) multi intuitionistic fuzzy gpr-closed if mifpcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is multi intuitionistic fuzzy regular open set in X,

(xii) multi intuitionistic fuzzy gpr-open if its complement A^c is multi intuitionistic fuzzy gpr-closed set in X.

1.16 Theorem: The following are equivalent:
(i) A is a multi intuitionistic fuzzy semiclosed set,
(ii) A^C is a multi intuitionistic fuzzy semiopen set,
(iii) mifint(mifcl(A)) $\subseteq$ A,
(iv) mifcl(mifint(A^C)) $\supseteq$ A.

1.17 Theorem: Any union of multi intuitionistic fuzzy semiopen sets is a multi intuitionistic fuzzy semiopen set and any intersection of multi intuitionistic fuzzy semiclosed sets is a multi intuitionistic fuzzy semiclosed.

1.18 Remark: (i) Every multi intuitionistic fuzzy open set is a multi intuitionistic fuzzy semiopen but not conversely.

(ii) Every multi intuitionistic fuzzy closed set is an multi intuitionistic fuzzy semi-closed set but not conversely.

(iii) The closure of a multi intuitionistic fuzzy open set is multi intuitionistic fuzzy semiopen set.

(iv) The interior of a multi intuitionistic fuzzy closed set is multi intuitionistic fuzzy semi-closed set.

1.19 Theorem: A multi intuitionistic fuzzy set A of a multi intuitionistic fuzzy topological space X is a multi intuitionistic fuzzy regular open if and only if A^C is multi intuitionistic fuzzy regular closed set.

1.20 Remark: (i) Every multi intuitionistic fuzzy regular open set is a multi intuitionistic fuzzy open set but not conversely. (ii) Every multi intuitionistic fuzzy regular closed set is a multi intuitionistic fuzzy closed set but not conversely.

1.21 Theorem: (i) The closure of a multi intuitionistic fuzzy open set is a multi intuitionistic fuzzy regular closed. (ii) The interior of a multi intuitionistic fuzzy closed set is a multi intuitionistic fuzzy regular open set.

1.22 Theorem: (i) Every multi intuitionistic fuzzy regular semiopen set is a multi intuitionistic fuzzy semiopen set but not conversely.

(ii) Every multi intuitionistic fuzzy regular closed set is a multi intuitionistic fuzzy regular semiopen set but not conversely.

(iii) Every multi intuitionistic fuzzy regular open set is a multi intuitionistic fuzzy regular semiopen set but not conversely.

1.23 Theorem: Let (X, $\Im$) be a multi intuitionistic fuzzy topological space and A be multi intuitionistic fuzzy set in X. Then the following conditions are equivalent:

(i) A is multi intuitionistic fuzzy regular semiopen,

(ii) A is both multi intuitionistic fuzzy semiopen and multi intuitionistic fuzzy semi-closed,

(iii) A^c is multi intuitionistic fuzzy regular semiopen in X.

1.24 Definition: A multi intuitionistic fuzzy set A of a multi intuitionistic fuzzy topological space (X, $\Im$) is called:

(i) multi intuitionistic fuzzy g-closed if mifcl (A) $\subseteq$ V whenever A $\subseteq$ V and V is multi intuitionistic fuzzy open set in X,

(ii) multi intuitionistic fuzzy g-open if its complement A^c is multi intuitionistic fuzzy g-closed set in X,

(iii) multi intuitionistic fuzzy rg-closed if mifcl(A) $\subseteq$ V whenever A $\subseteq$

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V and V is multi intuitionistic fuzzy regular open set in X,

(iv) multi intuitionistic fuzzy rg-open if its complement A^c is multi intuitionistic fuzzy rg-closed set in X,

(v) multi intuitionistic fuzzy w-closed if mifcl(A) ⊆ V whenever A ⊆ V and V is multi intuitionistic fuzzy semi open set in X,

(vi) multi intuitionistic fuzzy w-open if its complement A^c is multi intuitionistic fuzzy w-closed set in X,

(vii) multi intuitionistic fuzzy gpr-closed if mifpcl(A) ⊆ V whenever A ⊆ V and V is multi intuitionistic fuzzy regular open set in X,

(viii) multi intuitionistic fuzzy gpr-open if its complement A^c is multi intuitionistic fuzzy gpr-closed set in X.

1.25 Definition: Let (X, ℑ) be a multi intuitionistic fuzzy topological space. A multi intuitionistic fuzzy set A of X is called multi intuitionistic fuzzy regular w-closed (briefly, mifrw-closed) if mifcl(A) ⊆ U whenever A ⊆ U and U is multi intuitionistic fuzzy regular semiopen in multi intuitionistic fuzzy topological space X.

1.26 NOTE: We denote the family of all multi intuitionistic fuzzy regular w-closed sets in multi intuitionistic fuzzy topological space X by MIFRWC(X).

1.27 Definition: A multi intuitionistic fuzzy set A of a multi intuitionistic fuzzy topological space X is called a multi intuitionistic fuzzy regular w-open (briefly, mifrw-open) set if its complement A^C is a multi intuitionistic fuzzy rw-closed set in multi intuitionistic fuzzy topological space X.

1.28 NOTE: We denote the family of all multi intuitionistic fuzzy rw-open sets in multi intuitionistic fuzzy topological space X by MIFRWO(X).

2. SOME THEOREMS:

2.1 Theorem: Every multi intuitionistic fuzzy closed set is a multi intuitionistic fuzzy rw-closed set in multi intuitionistic fuzzy topological space X.

Proof: Let A be a multi intuitionistic fuzzy closed set in multi intuitionistic fuzzy topological space X. Let B be a multi intuitionistic fuzzy regular semiopen set in X such that A ⊆ B. Since A is multi intuitionistic fuzzy closed, mifcl(A) = A. Therefore mifcl(A) = A ⊆ B. Hence A is a multi intuitionistic fuzzy rw-closed in multi intuitionistic fuzzy topological space X.

2.2 Remark: The converse of the above Theorem 2.1 need not be true in general.

Proof: Consider the example, let X = { 1, 2, 3 }. Define a multi intuitionistic fuzzy set A in X by A = { < 1, (1, 1, 1), (0, 0, 0) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (0, 0, 0), (1, 1, 1) > }. Let ℑ = { 0_X, 1_X, A }. Then (X, ℑ) is a multi intuitionistic fuzzy topological space. Define a multi intuitionistic fuzzy set B in X by B = { < 1, (0, 0, 0), (1, 1, 1) >, < 2, (1, 1, 1), (0, 0, 0) >, < 3, (0, 0, 0), (1, 1, 1) > }. Then B is a multi intuitionistic fuzzy rw-closed set but it is not a multi intuitionistic fuzzy closed set in multi intuitionistic fuzzy topological space X.

2.3 Remark: Multi intuitionistic fuzzy generalized closed sets and multi intuitionistic fuzzy rw-closed sets are independent.

Proof: Consider the example, let X = { 1, 2, 3, 4 } and the multi intuitionistic fuzzy sets A, B, C be defined as A = { < 1, (1, 1, 1), (0, 0, 0) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (0, 0, 0), (1, 1, 1) >, < 4, (0, 0, 0), (1, 1, 1) > }, B = { < 1, (0, 0, 0), (1, 1, 1) >, < 2, (1, 1, 1), (0, 0, 0) >, < 3, (0, 0, 0), (1, 1, 1) >, < 4, (0, 0, 0), (1, 1, 1) > } and C = { <1, (1, 1, 1), (0, 0, 0) >, < 2, (1, 1, 1), (0, 0, 0) >, < 3, (0, 0, 0), (1, 1, 1) >, < 4, (0, 0, 0), (1, 1, 1) > }. Consider ℑ = { 0_X, 1_X, A, B, C }. Then (X, ℑ) is a multi intuitionistic fuzzy topological space. In this multi intuitionistic fuzzy topological space X, the multi intuitionistic fuzzy set D is defined by D = { < 1, (0, 0, 0), (1, 1, 1) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (1, 1, 1), (0, 0, 0) >, < 4, (0, 0, 0), (1, 1, 1) > }. Then D is a multi intuitionistic fuzzy generalized closed set in multi intuitionistic fuzzy topological space X. In this multi intuitionistic fuzzy topological space, the multi intuitionistic fuzzy set E is defined by E = { < 1, (1, 1, 1), (0, 0, 0) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (1, 1, 1), (0, 0, 0) >, < 4, (0, 0, 0), (1, 1, 1) > }. Then E is a multi intuitionistic fuzzy regular semiopen set containing D, but E does not contain mifcl(D) which is CC. Therefore E is not a multi intuitionistic fuzzy rw-clsoed set in multi intuitionistic fuzzy topological space X.

Consider the another example, let X = I = [0, 1]. Define a multi intuitionistic fuzzy set D in X by $\mu_D(x)$ = (0.5, 0.5, 0.5) if x = 2/3 and $\mu_D(x)$ = (0, 0, 0) otherwise; $\gamma_D(x)$ = (0.5, 0.5, 0.5) if x = 2/3 and $\gamma_D(x)$ = (1, 1, 1) otherwise. Let ℑ = { 0_X, 1_X, D }. Then (X, ℑ) is a multi intuitionistic fuzzy topological space. Let A be multi intuitionistic fuzzy set by μA(x) = (0.3, 0.3, 0.3) if x = 2/3 and $\mu_A(x)$ = (0, 0, 0) otherwise; $\gamma_A(x)$ = (0.3, 0.3, 0.3) if x = 2/3 and $\gamma_A(x)$ = (1, 1, 1) otherwise. Then A is a multi intuitionistic fuzzy rw-closed set in multi intuitionistic fuzzy topological space X. Now mifcl (A) = D^C and D is a multi intuitionistic fuzzy open set containing A but D does not contain mifcl (A) which is D^C. Therefore A is not a multi intuitionistic fuzzy generalized closed.

2.4 Remark: Multi intuitionistic fuzzy rw-closed sets and multi intuitionistic fuzzy semi-closed sets are independent.

Proof: Consider the example, let X = { 1, 2, 3 }. Define a multi intuitionistic fuzzy set A in X by A = { < 1, (1, 1, 1), (0, 0, 0) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (0, 0, 0), (1, 1, 1) > }. Let ℑ = { 0_X, 1_X, A }.Then the multi intuitionistic fuzzy set A is a multi intuitionistic fuzzy rw-closed but it is not a multi intuitionistic fuzzy semi-closed set in multi intuitionistic fuzzy topological space X.

Consider the another example, let X = { 1, 2, 3, 4 } and the multi intuitionistic fuzzy sets A, B, C be defined as A = {< 1, (1, 1, 1), (0, 0, 0) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (0, 0, 0), (1, 1, 1) >, < 4, (0, 0, 0), (1, 1, 1) > }, B = { < 1, (0, 0, 0), (1, 1, 1) >, < 2, (1, 1, 1), (0, 0, 0) >, < 3, (0, 0, 0), (1, 1, 1) >, < 4, (0, 0, 0), (1, 1, 1) > } and C = { <1, (1, 1, 1), (0, 0, 0) >, < 2, (1, 1, 1), (0, 0, 0) >, < 3, (0, 0, 0), (1, 1, 1) >, < 4, (0, 0, 0), (1, 1, 1) > }. Consider ℑ = { 0_X, 1_X, A, B, C }. The multi intuitionistic fuzzy set S is defined by S = { < 1, (1, 1, 1), (0, 0, 0) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (1, 1, 1), (0, 0, 0) >, < 4, (0, 0, 0), (1, 1, 1) > }. Then S is a multi intuitionistic fuzzy semi-closed in multi intuitionistic fuzzy topological space X. S is also multi intuitionistic fuzzy regular semiopen set containing S which is does not contain mifcl(S) = Bc = { < 1, (1, 1, 1), (0, 0, 0) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (1, 1, 1), (0, 0, 0) >, < 4, (1, 1, 1), (0, 0, 0) > }. Therefore S is not a multi intuitionistic fuzzy rw-closed set in multi intuitionistic fuzzy topological space X.

2.5 Theorem: Every multi intuitionistic fuzzy w-closed set is multi intuitionistic fuzzy rw-closed.

Proof: The proof follows from the Definition multi intuitionistic fuzzy rw-closed and the fact that every multi intuitionistic fuzzy regular semi open set is multi intuitionistic fuzzy semi open.

2.6 Remark: The converse of the above Theorem 2.5 need not be true. Proof: Consider the example, let X = { 1, 2 } and ℑ = { 0_X, 1_X, A } be a multi intuitionistic fuzzy topology on X, where A = { < 1, (0.7, 0.7, 0.7), (0.2, 0.2, 0.2) >, < 2, (0.6, 0.6, 0.6), (0.3, 0.3, 0.3) > }. Then the multi intuitionistic fuzzy set B = { < 1, (0.7, 0.7, 0.7), (0.2, 0.2, 0.2) >, < 2, (0.8, 0.8, 0.8), (0.1, 0.1, 0.1) >} is multi intuitionistic fuzzy rw-closed but it is not multi intuitionistic fuzzy w-closed.

2.7 Theorem: Every multi intuitionistic fuzzy rw-closed set is multi intuitionistic fuzzy rg-closed.

Proof: The proof follows from the Definition multi intuitionistic fuzzy rw-closed and the fact that every multi intuitionistic fuzzy regular open set is multi intuitionistic fuzzy regular semi open.

2.8 Remark: The converse of above Theorem 2.7 need not be true.

Proof: Consider the example, let X = { 1, 2, 3, 4 } and multi intuitionistic fuzzy sets A, B, C, D defined as follows A = { <1, (0.9, 0.9, 0.9), (0.1, 0.1, 0.1) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (0, 0, 0), (1, 1, 1) >, < 4, (0, 0, 0), (1, 1, 1) > }, B = { < 1, (0, 0, 0), (1, 1, 1) >, < 2, (0.8, 0.8, 0.8), (0.1, 0.1, 0.1) >, < 3, (0, 0, 0), (1, 1, 1) >, < 4, (0, 0, 0), (1, 1, 1) >}, C = {< 1, (0.9, 0.9, 0.9), (0.1, 0.1, 0.1) >, < 2, (0.8, 0.8, 0.8), (0.1, 0.1, 0.1) >, < 3, (0, 0, 0), (1, 1, 1) >, < 4, (0, 0, 0), (1, 1, 1) > }, D = { < 1, (0.9, 0.9, 0.9), (0.1, 0.1, 0.1) >, < 2, (0.8, 0.8, 0.8), (0.1, 0.1, 0.1) >, < 3, (0.7, 0.7, 0.7), (0.2, 0.2, 0.2) >, < 4, (0, 0, 0), (1, 1, 1) > }, $\Im = \{ 1_X, 0_X$, A, B, C, D } be a multi intuitionistic fuzzy topology on X. Then the multi intuitionistic fuzzy set E = { < 1, (0, 0, 0), (1, 1, 1) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (0.7, 0.7, 0.7), (0.2, 0.2, 0.2) >, < 4, (0, 0, 0), (1, 1, 1) > } is multi intuitionistic fuzzy rg-closed but it is not multi intuitionistic fuzzy rw-closed.

2.9 Theorem: Every multi intuitionistic fuzzy rw-closed set is multi intuitionistic fuzzy gpr-closed.

Proof: Let A be a multi intuitionistic fuzzy rw closed set in multi intuitionistic fuzzy topological space (X, $\Im$). Let A $\subseteq$ O, where O is multi intuitionistic fuzzy regular open in X. Since every multi intuitionistic fuzzy regular open set is multi intuitionistic fuzzy regular semi open and A is multi intuitionistic fuzzy rw-closed set, we have mifcl (A) $\subseteq$ O. Since every multi intuitionistic fuzzy closed set is multi intuitionistic fuzzy pre closed, mifpcl (A) $\subseteq$ mifcl (A). Hence mifpcl(A) $\subseteq$ O which implies that A is multi intuitionistic fuzzy gpr-closed.

2.10 Remark: The converse of above Theorem 2.9 need not be true.

Proof: Consider the example, let X = { 1, 2, 3, 4, 5 } and multi intuitionistic fuzzy sets A, B, C defined as follows A = { < 1, (0.9, 0.9, 0.9), (0.1, 0.1, 0.1) >, < 2, (0.8, 0.8, 0.8), (0.1, 0.1, 0.1) >, < 3, (0, 0, 0), (1, 1, 1) >, < 4, (0, 0, 0), (1, 1, 1) >, < 5, (0, 0, 0), (1, 1, 1) > }, B = { < 1, (0, 0, 0), (1, 1, 1) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (0.8, 0.8, 0.8), (0.1, 0.1, 0.1) >, < 4, (0.7, 0.7, 0.7), (0.2, 0.2, 0.2) >, < 5, (0, 0, 0), (1, 1, 1) > }, C = { < 1, (0.9, 0.9, 0.9), (0.1, 0.1, 0.1) >, < 2, (0.8, 0.8, 0.8), (0.1, 0.1, 0.1) >, < 3, (0.8, 0.8, 0.8), (0.1, 0.1, 0.1) >, < 4, (0.7, 0.7, 0.7), (0.2, 0.2, 0.2) >, < 5, (0, 0, 0), (1, 1, 1) > }. Let $\Im$ = { 1X, 0X, A, B, C } be a multi intuitionistic fuzzy topology on X. Then the multi intuitionistic fuzzy set D = { < 1, (0.9, 0.9, 0.9), (0.1, 0.1, 0.1) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (0, 0, 0), (1, 1, 1) >, < 4, (0, 0, 0), (1, 1, 1) >, < 5, (0, 0, 0), (1, 1, 1) > } is multi intuitionistic fuzzy gpr-closed but it is not multi intuitionistic fuzzy rw-closed.

2.11 Theorem: If A is a multi intuitionistic fuzzy regular open and multi intuitionistic fuzzy rg-closed in multi intuitionistic fuzzy topological space (X, $\Im$), then A is multi intuitionistic fuzzy rw-closed in X.

Proof: Let A be a multi intuitionistic fuzzy regular open and multi intuitionistic fuzzy rg-closed in X. We prove that A is a multi intuitionistic fuzzy rw-closed in X. Let U be any multi intuitionistic fuzzy regular semi open set in X such that A $\subseteq$ U. Since A is multi intuitionistic fuzzy regular open and multi intuitionistic fuzzy rg-closed, we have mifcl(A) $\subseteq$ A. Then mifcl(A) $\subseteq$ A $\subseteq$ U. Hence A is multi intuitionistic fuzzy rw-closed in X.

2.12 Theorem: If A and B are multi intuitionistic fuzzy rw-closed sets in multi intuitionistic fuzzy topological space X, then union of A and B is multi intuitionistic fuzzy rw-closed set in multi intuitionistic fuzzy topological space X.

Proof: Let C be a multi intuitionistic fuzzy regular semiopen set in multi intuitionistic fuzzy topological space X such that A$\cup$B $\subseteq$ C. Now A $\subseteq$ C and B $\subseteq$ C. Since A and B are multi intuitionistic fuzzy rw-closed sets in multi intuitionistic fuzzy topological space X, mifcl(A) $\subseteq$ C and mifcl(B) $\subseteq$ C. Therefore (mifcl(A) $\cup$ mifcl(B)) $\subseteq$ C. But mifcl(A) $\cup$ mifcl(B) = mifcl(A$\cup$ B). Thus mifcl(A$\cup$ B) $\subseteq$ C. Hence A$\cup$B is a multi intuitionistic fuzzy rw-closed set in multi intuitionistic fuzzy topological space X.

2.13 Theorem: If A and B are multi intuitionistic fuzzy rw-closed sets in multi intuitionistic fuzzy topological space X, then the intersection of A and B need not be a multi intuitionistic fuzzy rw-closed set in multi intuitionistic fuzzy topological space X.

Proof: Consider the example, let X = { 1, 2, 3, 4 } and the multi intuitionistic fuzzy sets A, B, C be defined as A = { < 1, (1, 1, 1), (0, 0, 0) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (0, 0, 0), (1, 1, 1) >, < 4, (0, 0, 0), (1, 1, 1) > }, B = { < 1, (0, 0, 0), (1, 1, 1) >, < 2, (1, 1, 1), (0, 0, 0) >, < 3, (0, 0, 0), (1, 1, 1) >, < 4, (0, 0, 0), (1, 1, 1) > } and C = { <1, (1, 1, 1), (0, 0, 0) >, < 2, (1, 1, 1), (0, 0, 0) >, < 3, (0, 0, 0), (1, 1, 1) >, < 4, (0, 0, 0), (1, 1, 1) > }. Consider $\Im$ = { 0_X, 1_X, A, B, C }. Then (X, $\Im$) is a multi intuitionistic fuzzy topological space. In this multi intuitionistic fuzzy topological space X, the multi intuitionistic fuzzy sets K and L are defined by K={< 1, (0, 0, 0), (1, 1, 1) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (1, 1, 1), (0, 0, 0) >, < 4, (1, 1, 1), (0, 0, 0) > }, L = { < 1, (1, 1, 1), (0, 0, 0) >, < 2, (1, 1, 1), (0, 0, 0) >, < 3, (1, 1, 1), (0, 0, 0) >, < 4, (0, 0, 0), (1, 1, 1) > }. Then K and L are the multi intuitionistic fuzzy rw-closed sets in multi intuitionistic fuzzy topological space X. Let D = K$\cap$L. Then D = { < 1, (0, 0, 0), (1, 1, 1) >, < 2, (0, 0, 0), (1, 1, 1) >, < 3, (1, 1, 1), (0, 0, 0) >, < 4, (0, 0, 0), (1, 1, 1) > }. Then D = K $\cap$ L is not a multi intuitionistic fuzzy rw-closed set in multi intuitionistic fuzzy topological space X.

2.14 Theorem: If a multi intuitionistic fuzzy subset A of multi intuitionistic fuzzy topological space X is both multi intuitionistic fuzzy regular open and multi intuitionistic fuzzy rw-closed, then A is a multi intuitionistic fuzzy regular closed set in multi intuitionistic fuzzy topological space X.

Proof: Suppose a multi intuitionistic fuzzy subset A of multi intuitionistic fuzzy topological space X is both multi intuitionistic fuzzy regular open and multi intuitionistic fuzzy rw-closed. As every multi intuitionistic fuzzy regular open set is a multi intuitionistic fuzzy regular semiopen set and A $\subseteq$ A, we have mifcl(A) $\subseteq$ A. Also A $\subseteq$ mifcl(A). Therefore mifcl(A) = A. That is A is a multi intuitionistic fuzzy closed. Since A is a multi intuitionistic fuzzy regular open, mifint(A) = A. Now mifcl(mifint(A)) = mifcl(A) = A. Therefore A is a multi intuitionistic fuzzy regular closed set in multi intuitionistic fuzzy topological space X.

2.15 Theorem: If a multi intuitionistic fuzzy subset A of a multi intuitionistic fuzzy topological space X is both multi intuitionistic fuzzy regular semiopen and multi intuitionistic fuzzy rw-closed, then A is a multi intuitionistic fuzzy closed set in multi intuitionistic fuzzy topological space X.

Proof: Suppose a multi intuitionistic fuzzy subset A of a multi intuitionistic fuzzy topological space X is both multi intuitionistic fuzzy regular semiopen and multi intuitionistic fuzzy rw-closed. Now A $\subseteq$ A, we have mifcl(A) $\subseteq$ A. Also A $\subseteq$ mifcl(A). Therefore mifcl(A) = A and hence A is a multi intuitionistic fuzzy closed set in multi intuitionistic fuzzy topological space X.

2.16 Corollary: If A is an multi intuitionistic fuzzy regular semi open and multi intuitionistic fuzzy rw-closed in multi intuitionistic fuzzy topological space (X, $\Im$). Suppose that F is multi intuitionistic fuzzy closed in X then A$\cap$F is multi intuitionistic fuzzy rw-closed in X.

Proof: Suppose A is both multi intuitionistic fuzzy regular semi open and multi intuitionistic fuzzy rw-closed set in X and F is multi intuitionistic fuzzy closed in X. By Theorem 2.15, A is multi intuitionistic fuzzy closed in X. So A $\cap$ F is multi intuitionistic fuzzy closed in X. Hence A $\cap$ F is multi intuitionistic fuzzy rw-closed in X.

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CONCLUSION

The study of intuitionistic fuzzy sets, throws more light on the inprecise, uncertain and vague concepts. In real life situations many of the problems are dominated by these concepts. Hence ti is necessary to introduced fuzzy concepts in topology also. The class of fuzzy topological sets and functions redefine the structure of topological spaces which are more approximate to the problems concerned with real life situations. Many of the systems are now designed using the concept of fuzzy sets. In this respect the concepts introduced in this thesis are more useful to practical applications and designing smart and soft systems.

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