Original Research Paper

Volume : 5 | Issue : 10 | October 2016 • ISSN No 2277 - 8179 | IF : 3.508 | IC Value : 69.48

Algebraic Reasoning at Elementary Level: Filling the Gaps Between Arithmetic and Algebra

Education

KEYWORDS : Algebraic Reasoning, Equalities, Algebra, Transition.

Prateek Chaurasia Research Scholar, Faculty of Education,BHU, Varanasi, U.P India , 221010

ABSTRACT

Algebra has been characterized by the most important part of mathematics learning at elementary level, as it opens the doors of abstract thinking as well as reasoning abilities among the new eager minds those who have just transit from the world of numbers and computations to the sphere of equalities, sings, and symbols/letters. The present article focuses on the fact that how arithmetic and algebra are related to each other and it highlights how arithmetic plays a role of scaffolding for elementary algebra. It drags the attention towards the approach of algebraic reasoning and its utilities in filling the gaps between arithmetic and early algebra at elementary grades. It also signifies the importance of algebraic reasoning in overcoming the challenges of transition phase of arithmetic to early algebra to enhance and make the algebraic learning joyful and learners friendly. The discussions of the present article will be fruitful to mathematic teachers for applying themselves to classroom teaching of algebra and to develop the interest and involve the leaner's inside the classroom during algebra teaching.

Introduction

Algebra is one of the most difficult areas of mathematics in elementary schools curriculum, with the use of letters (e.g. X, Y, Z etc.) for Unknown numbers and presence of variables offers a serious challenge to students. Especially students in phase of beginning algebra or we can say it the phase of transition from arithmetic to algebra, this transition becomes difficult as it requires some adjustments that should be adopted to enhance the algebra learning (Kieran 2004, Kilpatrick et al. 2001). In this phase child starts to change its style of working with mathematical properties like working with numbers shifts with working with letter and symbols and to become competent in this a well designed instruction that facilitates the transition is required (Banerjee, 2008). Apart from that some other major challenges hinders the algebra learning, such as ways to perceive the sign of equality, working with letters, making mathematical generalisations, and adopting and understanding the process of doing and undoing (adaptive towards the reversibility).

The major aim of early algebra is to educate elementary students such that to cultivates and develops habits of mind that focus on the deeper, underlying structure of mathematics. These "habits of mind" include two essential features: (1) generalizing, or identifying, expressing and justifying mathematical structure, properties, and relationships and (2) reasoning and actions based on promoting generalizations. Two develop these central features algebraic reasoning is necessary to fill and link the gaps between arithmetic and algebra

Algebraic thinking in arithmetic involves viewing arithmetic with ***"Algebra Eyes"*** (Subramaniam & Banerjee, 2011). Cai and Knuth (2011) call this *algebraization*, and they describe it as the nature of the thinking that is essential to algebra learning related to the conceptual areas within elementary and middle school mathematics. In India, children are first exposed to algebra in Grade 6 and algebra begins with discussion on the foundation of some basic arithmetic properties (like closure, commutativity, associativity, and distributive property, identity) with the use of variables. In Grade 7, solving algebraic equations (the presence of x, y, z in the form of unknown) becomes a major topics and solving and framing equations from word problems is introduced. In Grade 8 students enter into the world of quadratic equations, mononomials, binomials and polynomials. The emphasis is on solving algebraic equations by the use of algebraic notation. To develop the children's algebraic reasoning, how to scaffold their thinking abilities towards algebraic reasoning becomes key and this is the actual phase that requires the development of algebraic reasoning and tools for thinking algebraically.

Linkage of Arithmetic and Algebra at Elementary Level

It is normally accepted that the concept of a variable is difficult for students beginning in algebra. Elementary Algebra is the basic form which is taught to students those who were just entered to the arena of algebra having the knowledge of basic principles of arithmetic. The major concepts of algebra include variables, relations, and functions, inequalities, graphs, and equations all requires some certain level of abstraction in the form of algebraic reasoning, and with the introduction of this abstraction the major challenge that emerges is that students arriving at this juncture are unable to make the association between algebra and arithmetic. This inability leads to the downfall in understanding algebra. As arithmetic involves Chains of successive calculations where as algebra involves Chains of successive equalities, in arithmetic learners usually get the solutions by multiple calculations while in algebra working with unknowns quantities will lead to the solutions.

Algebraic reasoning with the help of some basic principles like Doing –Undoing (Reversibility), Abstracting from Computation, Building Rules to Represent the Function, (Driscolle, 1999) Structural Understanding, Coordination of Numeric and Spatial Structures unable the learners to solve the problems of algebra.

Approaches to link Arithmetic with Algebra by means of Algebraic Reasoning.

Link the early numeral computation with the use of symbols and letters.

Developing the ability to form the expressions with unknowns/symbols.

Developing the clear understanding of the equals sign

Developing the habit of using mathematical operations on integers.

Promoting the use of conjectures to reach at some certain conclusions/ solutions.

Developing the understanding of inverse relation.

Developing the understanding of growing patterns.

Understanding the Challenges of Transition Phase from Arithmetic to Algebra

This is very important aspect of algebra learning. To promote algebraic reasoning firstly it is very important to understand

the algebraic reasoning that what it is actually. Many studies have proved the importance of instructional strategies in developing algebraic reasoning and promoting algebra learning at early grades. In our country algebra has been introduced at grade 6 as per the NCERT curriculum, before coming to the algebra child has an arithmetical bent of mind which faces new challenges in the form of unknown quantities and variables which creates a mental haze in the mind of learners. Apart from this the major challenge that emerges is the inadequate effort to use arithmetic in teaching algebra, and this is the phase where role of algebraic reasoning comes into play. It requires a kind of bridging between arithmetic and algebra (*Banerjee & Subramaniam 2008).* Therefore it becomes very important to understand the challenges of this transition phase and provide remedies to overcome the problems and enhance algebra learning at elementary level. Some key issues of transition phase are discussed in sub heading as follows.

Problems of Adjustment

In the transition from arithmetic to algebra students need to do lot of adjustments even those students who are reasonably capable in dealing with arithmetical problems. When the students reaches the early algebra the adjustment problems begins with the signals to compute and deals with the signs such as + (addition), - (subtraction),*(multiplication) ÷ (division) and = (equality). Suppose a sum such as 6 + 8, is a signal to compute then students tries to solve it and then write 14, but when similar addition comes with variables like $3x + x = ?$; $x+ y+ 2 - (3) =$___. Then they cannot solve such equations only by handling number they need to make adjustment with the letters also as well as focus on representation of letters. Such question comes very frequently at the stage of early algebra, therefore to tackle with this issue adjustment with symbols and letters becomes very crucial.

Similarly, Kieran (2004) suggested the following adjustments.

A focus on relations and not merely on the calculation of a numerical answer.

A focus on both numbers and letters, rather than on numbers alone.

A focus on operations as well as their inverses, and on the related idea of doing/undoing.

A focus on both representing and solving a problem rather than on merely solving it.

A refocusing of the meaning of the equal sign.

These five adjustments surely come under the area of arithmetic, yet they also stand for a shift toward developing those ideas which are fundamental to the study of algebra.

Refocusing on the Meaning of Equality Sign "="

Representing and analyzing mathematical situations and structures is a major component of algebraic reasoning. As per the Kieran (2004) Learners in transition phase from arithmetic to algebra need to make some adjustment in their ways of thinking. This thinking needs to be introduced and matched with the symbols like equality. Students develop the misconception about notion of equality (operational instead of relational) since (Smith, 2011). Suppose an example, 5+2 = __ here students will write the correct answer 7. Now considered the another situation 7+2 = __ + 5, here instead of writing 4 in the space as a correct answer students assumes sign of equality as a separator and write the answer 9. This situation also explained by the (Smith, 2011) i.e. Learners tend to imagine equal sign as a separator instead of mathematical relation. In the present situation learners and teachers need to adopt the ways of algebraic reasoning to tackle with such kind of symbol in algebra.

Dilemma of Swing from Numbers With Symbols /Letters

Students oriented toward computation are also perplexed by an expression such as $x + 5 = 0$; they think they should be able to do something with it (solve), but are unsure as to what that might be. Problems related to an expression such as $x + 5 = 0$; is that learners relate it with the arithmetic problem such as 5+ 4 = ___ as mostly learners writes answer like $x5$ or $5x$ or put them self in the fuzzy situations, were learners become unsure to handle the expression with the presence of "x"(variable), and "="(sign of equality) as a sign of separator.

Conclusion

Most of the students generally face problems in their early grades of algebra. They come with some arithmetical bent of mind and make an effort to make adjustment with the unknown quantities, symbols and variables they encounter at the stage of early algebra. This is the most important phase of transition from arithmetic to algebra and algebraic reasoning can play a vital role in filling the gaps between this two. Just because it links the ways of thinking of algebra with arithmetic and develops a habit of mind among the learners to think algebraically.

If we want to enhance the children's algebraic learning at elementary stage then it is essential for our teachers to encourage early algebraic reasoning in the early years in a age specific ways and also promoting the ways to think algebraically, especially when students mind were growing and searching the existence of unknown quantities in the real world. If students were familiar with the experiences of algebraic reasoning, tasks and activities related to algebra in elementary school, there approach to the algebra in middle and high school will be healthy and they will be equipped with an already developed ability to think algebraically. With these insights in hand, students will find that algebra is not a mystery, but a useful and productive component of mathematics learning.

References

1. Banerjee, R. (2008). Developing a Learning Sequence for Transiting from Arithmetic to Algebra. Thesis Submitted to Tata Institute of Fundamental Research, Mumbai.
2. Banerjee, R., & Subramaniam, K. (2008). Bridging arithmetic and algebra: Evolution of a teaching sequence. In O. Figueras et al. (Eds.), International Group of the Psychology of Mathematics Education: Proceedings of the Joint Meeting of PME 32 and PMENA XXX (PME29) (Vol. 2, pp. 121–128). Morelia, Mexico.
3. Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers, grades 6-10.* Portsmouth: Heinemann.
4. Kieran, C. (2004): Algebraic thinking in the early grades: What is it? – In: The Mathematics Educator (Singapore) 8(No. 1), p. 139-151
5. Kilpatrick, J.; Bradford, F.; Swafford, J. (2001): Adding it up: Helping children learn mathematics. – Washington, DC: National Academy Press
6. Mathematics: Text book for class VI (2015). New Delhi: National Council of Educational Research and Training.
7. NCTM (National Council of Teachers of Mathematics). 2000. *Principles and standards for school mathematics.* Reston, VA: NCTM.
8. Smith, M. (2011). A Procedural Focus and a Relationship Focus to Algebra: How U. S.Teachers and Japanese Teachers Treat Systems of Equations. In Cai, J. & Knuth, E.(Ed.). Early Algebraization A Global Dialogue from Multiple Perspectives (pp. 511- 528). London: Springer Heidelberg.
9. Subramaniam, K., and Rakhi Banerjee. (2011). The arithmetic algebra connection: A historical-pedagogical perspective. In Jinfa Cai and Eric Knuth (Eds.), Early algebraization (pp. 87–107). New York: Springer