Volume : 5 | Issue : 10 | October 2016 • ISSN No 2277 - 8179 | IF : 3.508 | IC Value : 69.48

Summerization of Effect of Irregularity and Initial Stresses with Normal and Tangential Loading in Elastic Medium

Mathematics

KEYWORDS : Static deformation, Irregularity, Initial stress, Transversely isotropic medium, Orthotropic medium.

DINESH KUMAR MADAN Department of Mathematics, C.B.L.U. Bhiwani-127021, India

AGIN KUMARI Department of Mathematics, C.B.L.U. Bhiwani-127021, India

ABSTRACT

In the present paper, we have summarized the effect of presence of rectangular irregularity on displacements and stresses due to normal and tangential loading in transversely isotropic elastic medium and initially stressed orthotropic elastic medium. Due to their closeness to natural environmental conditions, irregular boundaries on the elastic medium have gained much importance in the study of elastic deformation field.

I. Introduction

The solution of problem of the deformation of elastic materials due to loading has been finding wide applications in geophysics, soil mechanics and engineering when the source surface is very long in on direction in comparison to the others, the use of two-dimensional is justified and consequently calculations are simplified to a great extent and we gets a closed form analytical solution. A very long strip-source and a very long line source are examples of such two-dimensional sources. The deformation due to loading such as inclined line load, strip-load, continuous line load, etc., is useful in analyzing the field around mining tremors and drilling in to the crust of the earth. It can also contribute to the theoretical consideration of the seismic and volcanic sources, since it can account for the deformation fields in the entire volume surrounding the source region.

Kuo [11] studied the static response of a stratified elastic half space due to surface loads. He used Thomson-Haskell matrix method which was earlier used in elastodynamics. Garg et al. [17] discussed the general plain- strain problem of an infinite unstressed orthotropic elastic medium due to 2-D sources. They obtained the deformation due to inclined line load by using eigen value approach with distinct eigen values. By considering distinct eigen values Selim and Ahmed [15] used same eigen value approach and obtained closed form analytical expression for displacement and stresses at any point of initially stressed orthotropic elastic medium due to an inclined line load.

A transverse isotropic has an axis of cylindrical symmetry. This is usually the result of parallel crack with coplanar normal or aligned grains. Pan [7] provided unified solution of the static deformation of the transversely isotropic and layered elastic half space by seismic sources. Singh et al. [12] obtained that in a transversely isotropic elastic medium and in an isotropic elastic medium the eigen values become equal and Chugh et al. [19] obtained the corresponding deformation field in isotropic [unstressed] elastic medium.

The problem of static deformation with irregularity present in the elastic medium due to continental margin, mountain roots etc. is very important to study. The problem of irregular thickness of the medium was discussed by Sato [20], De Noyer [10], Mal [3], Kar et al. [4], Chhattopadhyay et al. [2], Chhattopadhyay and Pal [1] and others. Selim [14] studied the two dimensional static problem of an isotropic elastic half space with irregularity present in the medium. Madan et al. [5], [6] obtained the closed form expressions on the deformation fields at any point of irregular transversely isotropic elastic medium. In order to understand better the behavior of media at continental margins the static deformation with irregular thickness under initial stress is very important. Since, in fact the earth is an initially stressed medium. Tolstoy [1982] derived explicit solutions of the dynamical equations for a pre-stressed solid under horizontal compression is a gravity field. Selim and Ahmed [15] discuss the plane strain problem of initially stressed orthotropic elastic medium. Here we have reproduced collectively displacements and stresses due to normal and tangential loading in irregular transversely isotropic elastic medium and irregular initially stressed orthotropic elastic medium earlier discussed by Madan et al. [5], [6] and Selim [16].

II. Basic Equations

(a) Unstressed Elastic Medium

The stress strain relations in matrix form for a medium with hexagonal or transverse isotropic elastic symmetry is

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{13}\\ \sigma_{12}\end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0\\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0\\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & c_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & c_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{c_{11}-c_{12}}{2}\end{bmatrix}\begin{bmatrix}e_{11}\\ e_{22}\\ e_{33}\\ 2e_{23}\\ 2e_{13}\\ 2e_{12}\end{bmatrix} \quad (1)$$

where the two-suffix quantity c_{ij} denotes the elastic constants of the medium. So in transversely isotropic elastic medium we have five elastic constants. For an elastic isotropic medium, these constants reduce to just two as given below

$$c_{11} = c_{33} = \lambda + 2\mu,$$
$$c_{12} = c_{13} = \lambda,$$
$$c_{44} = \frac{c_{11}-c_{12}}{2} = \mu \quad (2)$$

where λ and μ are the Lame's constants.

In the absence of body forces, the equilibrium equations in the Cartesian coordinate system (x,y,z) are

$$\sigma_{ij,j} = 0 \quad (3)$$

Where σ_{ij} (i, j=1,2,3) are the stress components for transversely isotropic medium. The strain displacement relations are

$$e_{ij} = e_{ji} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \qquad 1 \le i,j \le 3, \quad (4)$$

where $(u_1,u_2,u_3) = (u, v, w)$ and $(x_1,x_2,x_3) = (x, y, z)$.

The equilibrium equations in terms of displacement components obtained from (1)-(4) are

$$c_{11}\frac{\partial^2 u}{\partial x^2} + \left(\frac{c_{11}-c_{12}}{2}\right)\frac{\partial^2 u}{\partial y^2} + c_{44}\frac{\partial^2 u}{\partial z^2} + \left(\frac{c_{11}+c_{12}}{2}\right)\frac{\partial^2 v}{\partial x\partial y} + (c_{13}+c_{44})\frac{\partial^2 w}{\partial x\partial z} = 0$$

$$\left(\frac{c_{11}-c_{12}}{2}\right)\frac{\partial^2 v}{\partial x^2} + c_{11}\frac{\partial^2 v}{\partial y^2} + c_{44}\frac{\partial^2 v}{\partial z^2} + \left(\frac{c_{11}+c_{12}}{2}\right)\frac{\partial^2 u}{\partial x\partial y} + (c_{13}+c_{44})\frac{\partial^2 w}{\partial y\partial z} = 0$$

$$c_{44}\frac{\partial^2 w}{\partial x^2} + c_{44}\frac{\partial^2 w}{\partial y^2} + c_{33}\frac{\partial^2 w}{\partial z^2} + (c_{13}+c_{44})\frac{\partial^2 u}{\partial x\partial z} + (c_{13}+c_{44})\frac{\partial^2 v}{\partial y\partial z} = \quad (5)$$

(b) Initially Stressed Elastic Medium

In the absence of external forces, the equilibrium equations in the Cartesian co-ordinate system (x,y,z) for the unbounded medium with normal initial stress $S_{11} = -P$ along the horizontal direction (Fig.2) are

$$\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{12}}{\partial y} + \frac{\partial s_{13}}{\partial z} - P\frac{\partial w_z}{\partial y} + P\frac{\partial w_y}{\partial z} = 0,$$

$$\frac{\partial s_{21}}{\partial x} + \frac{\partial s_{22}}{\partial y} + \frac{\partial s_{23}}{\partial z} - P\frac{\partial w_z}{\partial x} = 0$$

$$\frac{\partial s_{31}}{\partial x} + \frac{\partial s_{32}}{\partial y} + \frac{\partial s_{33}}{\partial z} + P\frac{\partial w_y}{\partial x} = 0 \quad (6)$$

where s_{ij} (i, j = 1,2,3) are the incremental stress components and w_x, w_y, w_z are the rational components given by

$$w_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right),$$
$$w_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right),$$
$$w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \quad (7)$$

where u, v and w are the displacement components.

The stress- strain relations for an initially stressed orthotropic elastic medium, with co-ordinate planes as planes of elastic symmetry, are

$S_{11} = B_{11}e_{11} + B_{12}(e_{22}+e_{13}),$
$S_{22} = (B_{12} - P)\ e_{11} + B_{22}e_{22} + B_{23}e_{33},$
$S_{33} = (B_{12} - P)\ e_{11} + B_{23}e_{22} + B_{22}e_{33},$
$S_{23} = 2Q_1\ e_{23}$
$S_{31} = 2Q_2\ e_{31}$
$S_{12} = 2Q_3\ e_{12}$

Volume : 5 | Issue : 10 | October 2016 • ISSN No 2277 - 8179 | IF : 3.508 | IC Value : 69.48

(8)

The incremental strain components e_{ij} (i, j = 1,2,3) are related with the displacement components (u,v,w) through the relations (4).

For the plane-strain deformation, parallel to xy-plane, in which the displacement components are independent of z and are of the type

u= u(x,y), v= v(x,y), w=0.

The non-zero stresses for plane strain problem are

$$S_{11} = B_{11}e_{11} + B_{12}\, e_{22}$$
$$S_{22} = (B_{11} - P)\, e_{11} + B_{22}e_{22}$$
$$S_{12}=2Q_3e_{12} \qquad (8a)$$

where B_{ij} (i,j=1,2,3) and Q_3 are the incremental elastic coefficients and shear modulus, respectively. These incremental elastic coefficients are related to Lame's coefficients of the isotropic unstressed state. For this case these are

$$B_{11} = [\lambda + 2\mu(1+\zeta)], \qquad B_{12} = (\lambda + 2\mu\zeta),$$
$$B_{21} = \lambda,\ B_{12} = (\lambda + 2\mu),\ Q_3 = \mu \qquad (8b)$$

where $\zeta = \frac{P}{2\mu}$ is the initial stress parameter.

The equilibrium equations in terms of displacement components are

$$B_{11}\frac{\partial^2 u}{\partial x^2} + \left[Q_3 + \frac{P}{2}\right]\frac{\partial^2 u}{\partial y^2} + \left[Q_2 + \frac{P}{2}\right]\frac{\partial^2 u}{\partial z^2} + \left[B_{12} + Q_3 - \frac{P}{2}\right]\frac{\partial^2 u}{\partial x \partial y} + \left[B_{12} + Q_2 - \frac{P}{2}\right]\frac{\partial^2 w}{\partial x \partial z} = 0,$$

$$\left[B_{12} + Q_3 - \frac{P}{2}\right]\frac{\partial^2 u}{\partial x \partial y} + \left[Q_3 - \frac{P}{2}\right]\frac{\partial^2 v}{\partial x^2} + B_{22}\frac{\partial^2 v}{\partial y^2} + Q_1\frac{\partial^2 v}{\partial z^2} + [B_{23} + Q_1]\frac{\partial^2 w}{\partial y \partial z} = 0,$$

$$\left[B_{12} + Q_2 - \frac{P}{2}\right]\frac{\partial^2 u}{\partial x \partial z} + [B_{23} + Q_1]\frac{\partial^2 v}{\partial y \partial z} + \left[Q_2 - \frac{P}{2}\right]\frac{\partial^2 w}{\partial x^2} + Q_1\frac{\partial^2 w}{\partial y^2} + B_{22}\frac{\partial^2 w}{\partial z^2} = 0 \qquad (9)$$

III. Rectangular Irregularity

The origin of Cartesian coordinates is situated at x=0. When the rectangular irregularity is present in the elastic medium length 2a and depth h,

the equation of irregularity at x=$\in$ f(y) is

$$\in f(y) = \begin{cases} h & for\ |y| \le a \\ 0 & for\ |y| > a \end{cases}$$

and $\in$ is a small parameter $\in = \frac{h}{2a} << 1$

IV. Initially Unstressed Transversely isotropic Elastic Half-Space

(a) Normal Line-Load in An Irregular Transversely isotropic Elastic Half-Space

For the irregular-medium consisting of region x<$\in$ f (y) (Medium 1) and regions x> $\in$ f (y) (Medium II) of identical elastic properties,

the displacement and stress components due to normal line load R_1 per unit length acting on z-axis, Madan [6] are,

Medium I

$$u^I(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} i\left[\{B|k| + C(x|k| - M_1^I)\}e^{|k|x}\right]e^{-iky}dk$$

$$v^I(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} k\left[\left\{B + C\left(x - \frac{1}{|k|}\right)\right\}e^{|k|x}\right]e^{-iky}dk \qquad (11)$$

$$\sigma_{11}^I(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} i\Big[\{Bk^2(C_{11} - C_{12}) + C\left(xk^2(C_{11} - C_{12}) + |k|(C_{12} - M_2^1 C_{11})\right)\}e^{|k|x}e^{-iky}\Big]dk$$

$$\sigma_{12}^I(x,y) = \frac{1}{2\pi}\left(\frac{C_{11} - C_{12}}{2}\right)\int_{-\infty}^{\infty} k\left[\{2B|k| + C(2|k|x - M_1^I)\}e^{|k|x}\right]e^{-iky}dk$$

(9) (12)

Medium II

$$u^{II}(x,y) = \frac{1}{2\pi} i \int_{-\infty}^{\infty} \left[-\{D|k| + G(x|k| - M_1^{II})\} e^{-|k|x} \right] e^{-iky} dk$$

$$v^{II}(x,y) = \frac{1}{2\pi} k \int_{-\infty}^{\infty} \left[\left\{ D + Gk\left(x + \frac{1}{|k|}\right) \right\} e^{-|k|x} \right] e^{-iky} dk \qquad (13)$$

(13)

$$\sigma_{11}^{II}(x,y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} i\left[\{Dk^2(C_{11} - C_{12}) + G\left(xk^2(C_{11} - C_{12}) - |k|(C_{12} - M_2^{11}C_{11})\right)\} e^{-|k|x} \right] e^{-iky} dk$$

(b)

$$\sigma_{12}^{II}(x,y) = \frac{1}{2\pi} \left(\frac{C_{11} - C_{12}}{2} \right) \int_{-\infty}^{\infty} k\left[-\{2D|k| + G(2|k|x - M_1^{11})\} e^{-|k|x} \right] e^{-iky} dk \qquad (14)$$

Boundary Conditions

For a normal line-load R_1 per unit length is acting vertically downwards on the interface irregularity x = h along z-axis (Fig.1) Then the boundary conditions at x= h are

$u^{1}(h,y) = u^{11}(h,y)$,

$v^{1}(h,y) = v^{11}(h,y)$,

$$\sigma_{12}^{I}(x = h, y) = \sigma_{12}^{II}(x = h, y)$$

$$\sigma_{11}^{II}(x = h, y) - \sigma_{11}^{I}(x = h, y) = -R_1\delta(y)$$

Where h = $\in$f(y) and δ(y) is the Dirac – Delta satisfying the following properties:

$$\int_{-\infty}^{\infty} \delta(y)dy = 1, \ \delta(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\eta y} d\eta = 1$$

The closed form expressions for displacement at any point of an irregular transversely isotropic elastic half space due to normal line force acting at (h,0) Madan [6] are

$$u^{n}(x,y) = \frac{R_0}{2\pi}\left[\left\{ (k_2M_1^I + k_3M_1^{II} - 2k_1)log(x^2+y^2) - (k_2+k_3)\frac{2x^2}{x^2+y^2} \right\} - \gamma\left\{ \left(\frac{(\delta_1 - \delta_3)}{2} + (\delta_4M_1^{II} - M_1^I) \right) \frac{2x^2}{x^2+y^2} + (\delta_2+\delta_4)\frac{x^2(x^2-y^2)}{(x^2+y^2)^2} \right\} \right] \qquad (15)$$

$$v^{n}(x,y) = -\frac{R_0}{2\pi}\left[\left\{ 2(k_2 - 2k_1 + k_3)\tan^{-1}\frac{x}{y} + (k_2+k_3)\frac{2xy}{x^2+y^2} \right\} + \gamma\left\{ \left(\frac{\delta_1+\delta_3}{2}(\delta_4 - \delta_2) \right) \frac{2xy}{x^2+y^2} + (\delta_4 - \delta_2)\frac{4x^3y}{(x^2+y^2)^2} \right\} \right]$$

(b) Tangential Line-Load in An Irregular Transversely isotropic Elastic Half-Space

Boundary Conditions

For a tangential line-load R_2 per unit length is acting in the positive y-direction on the interface irregularity x = h along z-axis . Then the boundary conditions at x= h are

$u^{1}(h,y) = u^{11}(h,y)$,

$v^{1}(h,y) = v^{11}(h,y)$

$$\sigma_{11}^{I}(x = h, y) = \sigma_{11}^{II}(x = h, y)$$

$$\sigma_{12}^{I}(x = h, y) - \sigma_{12}^{II}(x = h, y) = -R_2\delta(y) \qquad (16)$$

where h = $\in$f(y) and δ(y) is the Dirac – Delta satisfying the following properties:

$$\int_{-\infty}^{\infty} \delta(y)dy = 1, \qquad \delta(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\eta y} d\eta = 1$$

The closed form expressions for displacement at any point of an irregular transversely isotropic elastic half space due to normal line force acting at (h,0) Madan [6] are

$$u^{t}(x,y) = -\frac{R_2}{2\pi}\left[\left\{ (k_1 + k_2 - k_3M_1^I + k_4M_1^{II})\frac{2x}{x^2+y^2} - \left((k_3+k_4)\frac{2x(x^2-y^2)}{(x^2+y^2)^2} \right) \right\} - \gamma\left\{ \frac{2\delta_1(x^2-y^2)}{x(x^2+y^2)^2} + \frac{4x\delta_3(x^2-3y^2)}{(x^2+y^2)^2} + \frac{2M_1^I\delta_3(x^2-y^2)}{x(x^2+y^2)^2} \right\} \right] \qquad (17)$$

Volume : 5 | Issue : 10 | October 2016 • ISSN No 2277 - 8179 | IF : 3.508 | IC Value : 69.48

$$v^t(x,y) = -\frac{R_2}{2\pi}\left[\left\{(k_1+k_2-k_3+k_4)\frac{2x}{x^2+y^2}\right\} + y\left\{\left(\delta_1+\delta_2+\delta_3-\delta_4-\frac{3x^2-y^2}{x^2+y^2}\right)+(k_4-k_3)\frac{4x^3y}{(x^2+y^2)^2}\right\}\right]$$

(c) Inclined Line load

For inclined line load R_0, per unit length, $R_1 = R_0 \cos\beta$, $R_2 = R_0 \sin \beta$

The displacement subjected to inclined line load can be obtained by superposition of the normal and tangential cases.

$$U^{inc}(x,y) = u^n(x,y) + u^t(x,y)$$
$$V^{inc}(x,y) = v^n(x,y) + v^t(x,y) \qquad (18)$$

where $u^n(x,y)$, $v^n(x,y)$ & $u^t(x,y)$, $v^t(x,y)$ are the deformation due to a normal line load R_1 and a tangential line load R_2 respectively.

(d) Particular Case

By using equation (2), the corresponding results in isotropic elastic medium can be obtained from equation (11)-(15) & (17)-(18) as a particular case.

V. Initially Stressed Elastic Medium
Normal line-load

For infinite medium as consisting of region x<h (Medium II) and region x> h (Medium I), h=εf(y).

The displacement and stress components for the medium I are Selim [16]

$$u^I(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(-C_3P_1^Ie^{-\xi_1^I|\eta|x} - C_4P_2^Ie^{-\xi_2^I|\eta|x}\right)e^{-i\eta y}d\eta$$
$$v^I(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(C_3e^{-\xi_1^I|\eta|x} + C_4e^{-\xi_2^I|\eta|x}\right)e^{-i\eta y}d\eta$$
$$S_{11}^I = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(H_1^IC_3e^{-\xi_1^I|\eta|x} + H_2^IC_4e^{-\xi_2^I|\eta|x}\right)e^{-i\eta y}d\eta \qquad (19)$$
$$S_{12}^I = \frac{1}{2\pi}\int_{-\infty}^{\infty}Q_3^I\left(-H_1'^IC_3e^{-\xi_1^I|\eta|x} - H_2'^IC_4e^{-\xi_2^I|\eta|x}\right)e^{-i\eta y}d\eta$$
(19)

and for the medium II are

$$u^{II}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(C_1P_1^{II}e^{\xi_1^{II}|\eta|x} + C_2P_2^{II}e^{\xi_2^{II}|\eta|x}\right)e^{-i\eta y}d\eta$$
$$v^{II}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(C_1e^{\xi_1^{II}|\eta|x} + C_2e^{\xi_2^{II}|\eta|x}\right)e^{-i\eta y}d\eta$$
$$S_{11}^{II} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(H_1^{II}C_1e^{\xi_1^{II}|\eta|x} + H_2^{II}C_2e^{\xi_2^{II}|\eta|x}\right)e^{-i\eta y}d\eta$$
$$S_{12}^{II} = \frac{1}{2\pi}\int_{-\infty}^{\infty}Q_3^{II}\left(-H_1'^{II}C_1e^{\xi_1^{II}|\eta|x} + H_2'^{II}C_2e^{\xi_2^{II}|\eta|x}\right)e^{-i\eta y}d\eta$$
(20)

where

$$(\xi_K^L)^2 = \frac{B_0^L \pm\sqrt{(B_0^L)^2-4C_0^L}}{2}, B_o^L = \frac{(B_{11}^L)^2+N_1^LN_3^L-(N_2^L)^2}{B_{11}^LN_3^L}, C_0^L = \frac{N_1^L}{N_3^L},$$
$$N_1^L = \mu^L(1+\zeta), N_2^L = [\lambda^L+\mu^L(1+\zeta)], N_3^L = \mu^L(1-\zeta), B_{11}^L = [\lambda^L+2\mu^L(1+\zeta)]$$
$$B_{22}^L = [\lambda^L+2\mu^L], Q_3^L = \mu^L, B_{12}^L = [\lambda^L+2\mu^L\zeta], P_k^L = i\frac{\xi_k^LN_2^L}{(\xi_K^L)^2B_{11}^L-N_1^L},$$
$$H_k^L = B_{11}^L P_k^L\xi_K^L|\eta| - iB_{12}^L\eta, H_k'L = \xi_K^L|\eta| - iP_k^L\eta, L = I, II \text{ and } K = 1,2. \quad (21)$$

Boundary conditions

When a normal line-load F_0 per unit length, is acting vertically downwards on the interface irregularity x = h along z-axis (Fig.2). Then the boundary conditions at x = h are

$$u^1(h,y) = u^{11}(h,y),$$
$$v^1(h,y) = v^{11}(h,y)$$
$$s_{12}^I(x=h,y) = s_{12}^{II}(x=h,y)$$

$$s_{11}^{II}(x=h,y) - s_{11}^{II}(x=h,y) = -F_0\delta(y)$$
(22)

where h= εf(y) and δ(y)is the Dirac-delta satisfying the following properties.

$$\int_{-\infty}^{\infty}\delta(y)dy = 1, \qquad \delta(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\eta y}d\eta = 1$$

The expressions for displacement and stress at any point of an initially stresses, irregular, isotropic elastic half-space due to normal line force acting at (h,0) are

$$u(x,y)=-\frac{P}{\pi}\left[\frac{-\xi_1^{II}P_1^{II}y_4x}{[y^2+(\xi_1^{II}x)^2]}-\frac{-\xi_2^{II}P_2^{II}y_2x}{[y^2+(\xi_2^{II}x)^2]}-\frac{-\xi_1^{I}P_1^{I}y_1x}{[y^2+(\xi_1^{I}x)^2]}-\frac{\xi_2^{I}P_2^{I}x}{[y^2+(\xi_2^{I}x)^2]}\right]$$
$$-\frac{p\delta x}{\pi}\left(\frac{\varphi_1P_1^{II}[(\xi_1^{II}x)^2-y^2]}{[y^2+(\xi_1^{II}x)^2]^2}+\frac{\varphi_2P_2^{II}[(\xi_2^{II}x)^2-y^2]}{[y^2+(\xi_2^{II}x)^2]^2}-\frac{\varphi_3P_1^{I}[(\xi_1^{I}x)^2-y^2]}{[y^2+(\xi_1^{I}x)^2]^2}-\frac{\varphi_4P_2^{I}[(\xi_2^{I}x)^2-y^2]}{[y^2+(\xi_2^{I}x)^2]^2}\right)$$

$$v(x,y)=-\frac{P}{\pi}\left[\frac{-\xi_1^{II}y_4x}{[y^2+(\xi_1^{II}x)^2]}-\frac{\xi_2^{II}y_2x}{[y^2+(\xi_2^{II}x)^2]}+\frac{\xi_1^{I}y_1x}{[y^2+(\xi_1^{I}x)^2]}+\frac{\xi_2^{I}x}{[y^2+(\xi_2^{I}x)^2]}\right]$$
$$-\frac{p\delta x}{\pi}\left(\frac{\varphi_1[(\xi_1^{II}x)^2-y^2]}{[y^2+(\xi_1^{II}x)^2]^2}+\frac{\varphi_2[(\xi_2^{II}x)^2-y^2]}{[y^2+(\xi_2^{II}x)^2]^2}+\frac{\varphi_3[(\xi_1^{I}x)^2-y^2]}{[y^2+(\xi_1^{I}x)^2]^2}+\frac{\varphi_4[(\xi_2^{I}x)^2-y^2]}{[y^2+(\xi_2^{I}x)^2]^2}\right) \quad (23)$$

$$S_{11}(x,y)=-\frac{P}{\pi}\left[\frac{-\xi_1^{II}H_1^{II}y_4x}{[y^2+(\xi_1^{II}x)^2]}-\frac{\xi_2^{II}H_2^{II}y_2x}{[y^2+(\xi_2^{II}x)^2]}+\frac{\xi_1^{I}H_1^{I}y_1x}{[y^2+(\xi_1^{I}x)^2]}+\frac{\xi_2^{I}H_2^{I}x}{[y^2+(\xi_2^{I}x)^2]}\right]$$
$$-\frac{p\delta x}{\pi}\left(\frac{\varphi_1H_1^{II}[(\xi_1^{II}x)^2-y^2]}{[y^2+(\xi_1^{II}x)^2]^2}+\frac{\varphi_2H_2^{II}[(\xi_2^{II}x)^2-y^2]}{[y^2+(\xi_2^{II}x)^2]^2}+\frac{\varphi_3H_1^{I}[(\xi_1^{I}x)^2-y^2]}{[y^2+(\xi_1^{I}x)^2]^2}+\frac{\varphi_4H_2^{I}[(\xi_2^{I}x)^2-y^2]}{[y^2+(\xi_2^{I}x)^2]^2}\right)$$

$$S_{12}(x,y)=-\frac{P}{\pi}\left[\frac{-\xi_1^{II}Q_3^{II}H_1^{\prime II}y_4x}{[y^2+(\xi_1^{II}x)^2]}-\frac{\xi_2^{II}Q_3^{II}H_2^{\prime II}y_2x}{[y^2+(\xi_2^{II}x)^2]}-\frac{\xi_1^{I}Q_3^{I}H_1^{\prime I}y_1x}{[y^2+(\xi_1^{I}x)^2]}-\frac{\xi_2^{I}Q_3^{I}H_2^{\prime I}x}{[y^2+(\xi_2^{I}x)^2]}\right]$$
$$-\frac{p\delta x}{\pi}\left(\frac{\varphi_1H_1^{\prime II}Q_3^{II}[(\xi_1^{II}x)^2-y^2]}{[y^2+(\xi_1^{II}x)^2]^2}+\frac{\varphi_2H_2^{\prime II}Q_3^{II}[(\xi_2^{II}x)^2-y^2]}{[y^2+(\xi_2^{II}x)^2]^2}-\frac{\varphi_3H_1^{\prime I}Q_3^{I}[(\xi_1^{I}x)^2-y^2]}{[y^2+(\xi_1^{I}x)^2]^2}-\frac{\varphi_4Q_3^{I}H_2^{\prime I}[(\xi_2^{I}x)^2-y^2]}{[y^2+(\xi_2^{I}x)^2]^2}\right) \quad (24)$$

where $\delta = \frac{h}{x}$ is the parameter of irregularity. When the medium is initially unstressed, the parameter $\zeta = \frac{P}{2\mu}$ will become zero and medium become isotropic elastic. The corresponding expression for the displacement and stresses can be obtained from equation (23) and (24) which coincides with the results obtained by Selim [14] a particular case discussed here in section IV(c).

V. Observations

It has been observed that there is significant effect of irregularities present at the interface of elastic media. In an unstressed elastic medium, the distance in magnitude between the corresponding normal displacements and corresponding tangential displacement for transversely isotropic and isotropic medium increases as the size of irregularity increases. The angle of inclination increases, the displacement in magnitude increases. In, an initially stressed medium an increase in compressive initial stresses in the medium decrease the value of normal and tangential displacement for the same value of horizontal distance. Further it has been observed that the results for unstressed isotropic elastic medium can be obtained from the initially stressed orthotropic elastic medium. In the study of the propagation of seismic waves, the problems of static deformation of an initially stressed and unstressed irregular elastic medium play an important role. The effect of initial stress on incident surface waves in prestressed elastic half spaces has been discussed by Singh [18]. He observed that maximum effect of the initial stresses on reflection and refraction coefficients occur near 30^o of angle of incidence and normal angle of incidence respectively.

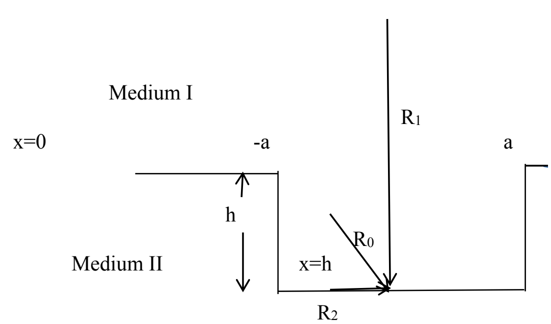
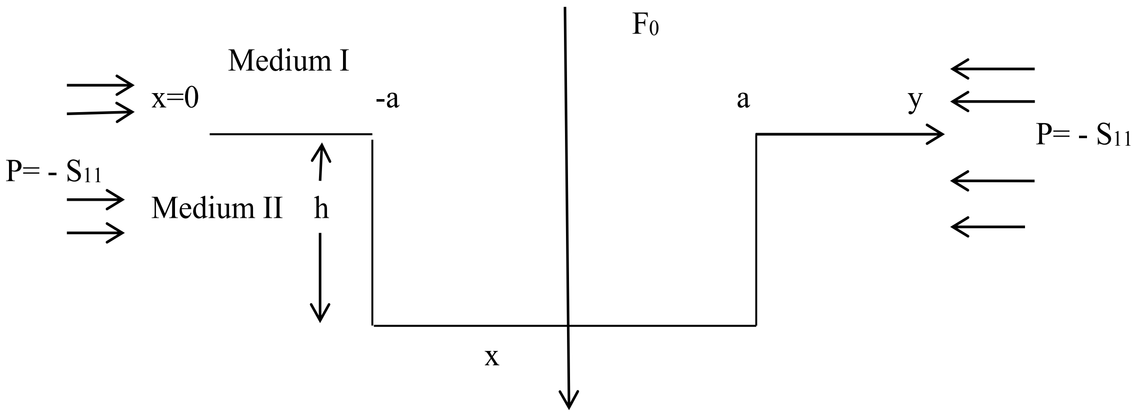


Fig.-1



Reference:

[1] A. Chattopadhyay and A. Pal, Dispersion curves of SH waves caused by irregularity in the prestressed internal stratum, Acta Geophys. Pol. XXX(1) 37-48, 1983.

[2] A. Chattopadhyay, M. Chakraborty, A.K. Pal, effect of irregularity on the propagation of guided SH-Waves, J.Mec. theor. Appl. (France) (2) 215, 1983.

[3] A.K. Mal, on the frequency equation for love waves due to abrupt thickening of crustal layer, Geos. Pura Appl. 52 59-68, 1962.

[4] B.K. Kar, A.K. Pal, V.K. Kalyant, Propagation of love waves in an irregular dry sandy layer, Acta Geophys. XXXIV (2) 157, 1986.

[5] D.K. Madan, A. Dahia and S. Chugh, Static response of an anisotropic elastic medium due to inclined line load with irregularity present in the medium, International of Journal Engineering & Science Research, 2(11) 1683-1695, 2012.

[6] D.K. Madan, A. Dahia and S. Chugh, Static response of Transversely isotropic elastic medium with irregularity present in the medium, International of Journal of Mechanical Engineering, 2(3), 2012.

[7] E.Pan, Static Response of a transversely isotropic and layered half-space to general surface loads, Phys. Earth. Planet. Int. 54 353-363, 1989.

[8] I. Tolstoy, on elastic waves in prestressed solids, J. Geophys. Res. (87) 6823-6827, 1982.

[9] I.S. Skolonikoff, Mathematical theory of elasticity, (McGraw-Hill, new York, 1956.

[10] J. De Noyer, The effect of variations in layer thickness of love waves, bull, seisomol. Soc. Am. 51 227-235, 1961.

[11] J.T. Kuo, Static response of a multilayered medium under inclined surface loads, Journal of Geophysical Research, 74(12) 3195-3207, 1969.

[12] K.Singh, D.K. Madan, A.Goel and N.R. Garg, Two-dimensional static deformation of anisotropic medium, Sadhana (India) 30(4), 565, 2005

[13] M.M. Selim, 2-D static deformation of an irregular medium using eigen value approach. J. King Saud Univ. in press.

[14] M.M. Selim, Effect of irregularity on static deformation of elastic half-space, International of Journal of Modern Physics 22(14) 2241-2253, 2008.

[15] M.M. Selim, M.K. Ahmed, Plane strain deformation of an initially stressed orthotropic elastic medium, Appl. Math. Comput. 175 (1) 221-237, 2006.

[16] M.M. Selim, Static deformation of an irregular initially stressed medium, Appl. Math. Comput. 188 1274-1284, 2007.

[17] N.R. Garg, A. Goel and A. Miglani, Plane Strain deformation of an orthotropic elastic medium using eigen value approach, Earth. Planets Space.,55,3-9, 2003.

[18] S.S. Singh, Effect of initial stresses on incident qSV-waves in prestressed elastic half-spaces, ANZIAM J.52, 359-371, 2011.

[19] S. Chugh, K. Singh and D.K. Madan, 2-Dimensional static response of an orthotropic multilayered half-space due to long strike-slip fault,BULL. CAL.MATH.SOC.,103(2) 171-182, 2011.

[20] Y. Sato, Study on surface waves. Vi. Generation of Love and other type of SH waves, Bull. Earthq. Res. Inst. 30 101-120,1952.