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An Approach for Solving Transportation Problem With FUZZY Parameters

Mathematics

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ABSTRACT

This paper deals with Transportation problem under fuzzy optimization method by using ranking of Fuzzy numbers. In many Fuzzy decision problems, the quantities are represented informs of fuzzy numbers. Fuzzy numbers usually classified as Triangular Trapzezordal, any L.R Fuzzy number. This some fuzzy numbers are not directly comparable first. We transform the fuzzy quantities by using classical algorithims then we solve and obtain the optimal solution of the problem. The new methods systematic procedure, easy to apply and can be utilized for all types of Transportation illustrated the new method.

A fuzzy Transportation problem is a Transportation problem in which Transportation cost, supply and demand quantities are fuzzy numbers. The objective of the fuzzy transportations problem in to determine the shipping schedule that minimizes the total fuzzy transport cost while satisfying fuzzy supply and demand limits. In this paper, we investigate more realistic problems, namely the Transport problem fuzzy costs α_{ij}, suppose that m origins are to supply n destinations with a certain product. Let a_i be the amount of the product available at origin i and b_j the amount product required at destination. Most of the existing techniques provides crisp solutions for the fuzzy transport problem. Shiang-Tai Lnn and Chiang Kao [11], Chanas and Kuchta [10] proposed a method for solving fuzzy transportation problems. Nagoar Gani and Adbul Razak [1] obtained a fuzzy solution for a two stage cost minimizing fuzzy transport problems in which supply and demands are Trapezoidal fuzzy numbers. Pandian et al [2] proposed a method namely, Fuzzy Zero point methods, for finding a fuzzy optimal solution for a fuzzy transportation problems where all the parameters are the Trapezoidal fuzzy numbers. Ranking fuzzy numbers is one of the important subjects when comparing two or multi fuzzy has been one of the main problems. Several methods are introduced for ranking fuzzy numbers. Here, we introduced this ranking method to obtain the fuzzy optimal Researcher, Basirzadeh et al [5] introduce the ranking fuzzy method. Now, we want to apply this method for all fuzzy Transportation problems, where all the parameters can be Trapezoidal fuzzy numbers. Triangular fuzzy numbers and L.R type fuzzy numbers, normal or abnormal fuzzy numbers. This method is very useful and easy to apply. At the end, a numerical example is provided to illustrate the ranking method.

2. MATHEMATICAL DESCRIPTION

A transportation problem can be stated mathematically as follows:

$$\text{Minimise } Z=\sum_{i=1}^{m}\sum_{i=1}^{n} C_{ij}\, x_{ij} \qquad (1)$$

$$\text{Subject to } \left.\begin{aligned} \sum_{j=1}^{m} x_{ij} &= a_i,\ i = 1, 2\ldots m \\ \sum_{i=1}^{m} x_{ij} &= b_i,\ j = 1, 2\ldots m \end{aligned}\right\} \qquad (2)$$

$x_{ij} \geq 0$, i = 1,2 ….m, j = 1,2….m

where C_{ij} the cost of the Transportation of an unit from the i^{th} source to the j^{th} destination and the quantity x_{ij} is to be some positive integer or zero which is to be transported from the i^{th} origin to j^{th} destinations. An obvious necessary and sufficient conditions for the linear programming problem given in (i) to have a solution is that

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_j .....$$

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(ie) assume that the total available is equal to the total required. If it is not true, a fictitious source or destination can be added. It should be noted that problem has a feasible solution if and only if the condition (2) satisfied. Now the problem is to determine x_{ij}, in such a way that the total Transportation cost is minimum.

A fuzzy Transportation problem can be stated mathematically as follows:

$$\text{Minimise } Z=\sum_{i=1}^{m} \sum_{i=1}^{m} C_{ij}\, x_{ij} \ldots. \qquad (4)$$

$$\text{Subject to } \left.\begin{array}{l} \sum_{j=1}^{m} x_{ij} = a_i,\ i = 1, 2 \ldots m \\ \sum_{i=1}^{m} x_{ij} = b_i,\ j = 1, 2 \ldots m \end{array}\right\} \qquad (5)$$

$x_{ij} \geq 0$, i = 1,2 ….m, j = 1,2….m

where

C_{ij} = Transports costs, the quantities

a_i = supply ; b_j = demand

are fuzzy quantities.

An obvious necessary and sufficient condition for the fuzzy Linear programming problem in (4) and (5) to hour a solution is that

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_j \ldots\ldots \qquad (6)$$

In fuzzy Transportation problem. Two factors are very important in Fuzzy decision system.

(i) Contribution of the decision maker in the decision making process.
(ii) Simplicity of calculation

In this paper we propose a method for ranking and comparing Fuzzy numbers to account for the above mentioned factors as much as possible.

Definition

A measure of a fuzzy number P is a function T : F(x) → R+ where F(x) denotes the set of all fuzzy numbers on X. For each fuzzy number P, this function assigns a non-negative real number T(P) that expresses the measure of P.

The measure of a fuzzy number is obtained by using the following axioms and the average of two sides, left side area and right side area from member function to α-axis.

(i) T(P) = A if and only if P is a crisp number.
(ii) P ≤ Q if and only if T(P) ≤ T(Q)
(iii) If α ≥ β, when $T_\alpha (P_\beta) = 0$

In order clarify the idea of the above mentioned quantity, consider the following

$$P = \left\{ \underline{P}(r), \overline{P}(r) \right\}, \text{ where } 0 \leq r \leq 1.$$

It is clear that if α ≥ β → $T\alpha (P_\beta) = 0$

Now

$$T_\alpha (P_\beta) = \frac{1}{2}\int_{\alpha}^{\beta} \left\{ \underline{P}(r) + \overline{P}(r) \right\} dr$$

$$= \frac{1}{2}\int_{\alpha}^{\beta} \underline{P}(r)dr + \frac{1}{2}\int_{\alpha}^{\beta} \overline{P}(r)dr$$

Definition

If P_β and Q_β are two arbitrary fuzzy number and $\beta_1, \beta_2 \in [0,1]$, then we have

$$(i) \quad P_{\beta_1} \leq Q_{\beta_2} \Leftrightarrow \forall \alpha \in [0,1] \;\; T_\alpha(P_{\beta_1}) \leq T_\alpha(Q_{\beta_2})$$

$$(ii) \quad \bar{P}_{\beta_1} = \bar{Q}_{\beta_2} \Leftrightarrow \forall \alpha \in [0,1] \;\; T_\alpha(P_{\beta_1}) = T_\alpha(Q_{\beta_2})$$

$$(iii) \quad \bar{P}_{\beta_1} \geq \bar{Q}_{\beta_2} \Leftrightarrow \forall \alpha \in [0,1] \;\; T_\alpha(P_{\beta_1}) \geq T_\alpha(Q_{\beta_2})$$

Definition: Triangular Fuzzy Number

A Triangular Fuzzy Number P is specified by three parameters x_1, x_2, x_3 and its membership function is given by,

$$\mu_{p(x)} = \begin{cases} 0 & x \le x_1 \\ \dfrac{x - x_1}{x_2 - x_1} & x_1 \le x \le x_2 \\ \dfrac{x_3 - x}{x_3 - x_2} & x_2 \le x \le x_3 \\ 1 & x_3 \le x \end{cases}$$

From the above definition,

Let $P = \{\underline{P}(r), \overline{P}(r)\}, 0 \le r \le 1$ be a fuzzy number, then the value of T(P) is calculated as follows,

$$\mathrm{T_0(P)} = \frac{1}{2}\int_0^1 \{\underline{P}(r) + \mathrm{P}(r)\}\ dr \frac{1}{4}[x_1 + 2x_2 + x_3]$$

Definition: Trapezoidal Fuzzy Number

A Trapezoidal Fuzzy Number P is specified by four parameters (x_1, x_2, x_3, x_4) and its membership function is given by,

$$\mu_{p(x)} = \begin{cases} 0 & x \le x_1 \\ \dfrac{x - x_1}{x_2 - x_1} & x_1 \le x \le x_2 \\ \dfrac{x_3 - x}{x_3 - x_4} & x_3 \le x \le x_4 \\ 1 & x_4 \le x \end{cases}$$

From the above definition,

Let $\overline{P} = \{\underline{P}(r), \overline{P}(r)\}, 0 \le r \le 1$ be a fuzzy number, then the value of T(P) is calculated as follows.

$$\mathrm{T_0(P)} = \frac{1}{2}\int_0^1 \{\underline{P}(r) + \mathrm{P}(r)\}\ dr \frac{1}{4}[x_1 + 2x_2 + x_3 + x_4]$$

SOLUTION PROCEDURE FOR SOLVING FUZZY TRANSPORT PROBLEM

Here, we introduce on approach for solving transport costs, supply and demand quantities are fuzzy numbers. The fuzzy numbers in each problem may be Triangular, or Trapazoidal or any fuzzy numbers in or mixture of them. The optimal solution for the fuzzy transportation problem can be obtain as a crisp or fuzzy form.

Algorithm for Optimal Solution

Step: 1 Calculate the values of T(.) for each fuzzy data, the transportation costs Cij, supply ai, and demand bj which are fuzzy quantities.

Step: 2 By replacing T(Cij), T(ai), T(bj) insteady of Cij, ai, bj respectively, which are fuzzy quantities, define a new crisp transportation problem.

Step: 3 Solve the new crisp transportation problem by usual method and obtain the crisp optimal solution of the problem.

Step: 4 Determine the locations of non-zero basic feasible solutions in transportation problem. The basis is rooted spanning tree, that is there must be atleast one basic cell in each row and in each column of the Transportation table an. Also, the basis, must be a tree, that is (m + n-1) basic cells should not contain a cycle. Therefore, there exist some rows and columns which have only one basic cell. Then, by starting from these cells, calculate the fuzzy basic. Solutions, continue until we obtain (m + n – 1) basic solutions. In the optimam solution of the fuzzy Transportation problem is given by occupied cells = (m rows + n columns – 1) = (m + n – 1) is the required occupied cell.

Numerical Example

Consider the following fuzzy transportation problem with Trapezoidal numbers.

	1	2	3	4	Supply
1	(2,4,5,6)	(2,5,6,8)	(11,13,14,16)	(7,9,10,13	(3,8,9,14)
2	(2,3,4,6)	(1,2,3,4)	(7,8,9,10)	(2,3,4,5)	(2,3,4,5)
3	(5,7,8,10)	(7,10,11,13)	(14,17,18,21)	(9,11,12,14)	(7,12,14,19)
Demand	(7,9,10,12)	(3,7,8,12)	(3,5,6,8)	(3,4,5,6)	

From the table values of Trapezoidal Numbers and according to the definition of a Trapezoidal fuzzy number, the values of T(P) is assigned to P calculated as follows.

$$T_0(P) = \frac{1}{2}\int_0^1 \left\{\underline{P}(r) + P(r)\right\} dr \frac{1}{4}\left[x_1 + x_2 + x_3 + x_4\right]$$

Then, we calculate the values of T(C_{ij}), T(a_i) and T(b_j) fuzzy Transportation costs.

$A_{11} = (2,4,5,6)$	$T_0(A_{11}) = \frac{1}{4}(2+4+5+6) = 4.5$
$A_{12} = (2,5,6,8)$	$T(A_{12}) = \frac{1}{4}(2+5+6+8) = 5.5$
$A_{13} = (11,13,14,16)$	$T(A_{13}) = \frac{1}{4}(11+13+14+16) = 13.5$
$A_{14} = (7,9,10,13)$	$T(A_{14}) = \frac{1}{4}(7+9+10+13) = 9.5$
$A_{21} = (2,3,4,6)$	$T(A_{21}) = \frac{1}{4}(2+3+4+6) = 3.75$
$A_{22} = (1,2,3,4)$	$T(A_{22}) = \frac{1}{4}(1+2+3+4) = 2.5$
$A_{23} = (7,8,9,10)$	$T(A_{23}) = \frac{1}{4}(7+8+9+10) = 8.5$
$A_{24} = (2,3,4,5)$	$T(A_{24}) = \frac{1}{4}(2+3+4+5) = 3.5$
$A_{31} = (5,7,8,10)$	$T(A_{31}) = \frac{1}{4}(5+7+8+10) = 7.5$
$A_{32} = (7,8,10,13)$	$T(A_{32}) = \frac{1}{4}(7+8+10+13) = 10.5$
$A_{33} = (14,17,18,21)$	$T(A_{33}) = \frac{1}{4}(14+17+18+21) = 17.5$
$A_{34} = (9,11,12,14)$	$T(A_{34}) = \frac{1}{4}(9+11+12+14) = 11.5$

Fuzzy supplies quantities

$a_1 = (3,8,9,14)$	$T_0(a_1) = \frac{1}{4}(3+8+9+14) = 8.5$
$a_2 = (2, 3, 4, 5)$	$T_0(a_2) = \frac{1}{4}(2+3+4+6) = 3.5$
$a_3 = (7,12,14,19)$	$T_0(a_{13}) = \frac{1}{4}(7+12+14+19) = 13$

TABLE 3
FUZZY DEMAND QUANTITIES

$\ddot{b}_1 = (6, 8, 9, 11)$	$T_0(\ddot{b}_1) = \frac{1}{4}(6 + 8 + 9 + 11) = 9.5$
$\ddot{b}_2 = (2, 6, 7, 11)$	$T_0(\ddot{b}_2) = \frac{1}{4}(2 + 6 + 7 + 11) = 6.5$
$\ddot{b}_3 = (3, 5, 6, 8)$	$T_0(\ddot{b}_{13}) = \frac{1}{4}(3 + 5 + 6 + 8) = 5.5$
$\ddot{b}_4 = (3, 4, 5, 6)$	$T_0(\ddot{b}_4) = \frac{1}{4}(3 + 4 + 5 + 6) = 4.5$

Table 4

From the above tables (1), (2), (3), (4), we have the total fuzzy supply is $\ddot{S} = (12, 23, 27, 38)$ and the total fuzzy Demand is $\ddot{D} = (14, 25, 29, 38)$ (ie) $\ddot{S} = (12, 23, 27, 38)$ $T_0\left(\ddot{S}\right) = 25$

$$\ddot{D} = (14, 25, 27, 38)$$

$$T_0\left(\ddot{D}\right) = 25$$

Since $T_0\left(\ddot{S}\right) = T_0\left(\ddot{D}\right) = 25$, the given problem is balanced problem.

Now, using this proposed approach we change the fuzzy Transportation problem in to a crisp transportation problem. So, we have the following reduced fuzzy transportation problem.

	1	2	3	4	Supply
1	4.5	5.5	13.5	9.75	8.5
2	3.75	2.5	8.5	3.5	3.5
3	7.5	10.5	17.5	11.5	13
Demand	8.5	6.5	5.5	4.5	25

As shown in the table 6, the result of defuzzification of the fuzzy numbers obtaining the measures which are not all integer, So, existing of a non-integer value in a Transportation problem follows this fact that the solution of the crisp transportation problem is not integer. Since the transportations problems is an integer, because its matrix is an unimodulur matrix. If we solve the new problem, we obtain solutions as follows.

$X_{12} = 7.5$, $X_{13} = 3$, $X_{23} = 3.5$, $X_{31} = 7.5$, $X_{33} = 3$, $X_{34} = 4.5$

	1 X_{11}	2 X_{12}	3 X_{13}	4 X_{14}	Supply
1		7.5	3		10.5
2			3.5		3.5
3	9.5		3	4.5	17
Demand	9.5	7.5	9.5	4.5	31

Table 7

Now, we can return to initial problem and obtain the fuzzy solution of the fuzzy Transportation problem based on the above table.

	1	2	3	4	Supply
1		(3,7,8,12)	(-7,2,4,13)		(3,8,9,14)
2			(2,3,4,5)		(2,3,4,5)
3	(7,9,10,12)		(-7,1,5,13)	(3,4,5,6)	(7,12,14,19)
Demand	(7,9,10,12)	(3,7,8,12)	(3,5,6,8)	(3,4,5,6)	

Table 8

We conclude that the fuzzy optimal values of the given fuzzy Transportation problem are,

$X_{12} = (3,7,8,12)$; $X_{13} = (-7, 2, 4, 13)$; $X_{33} = (-7, 1,5,13)$
$X_{23} = (2,3,4,5)$; $X_{31} = (7,9,10,12)$; $X_{34} = (3,4,5,6)$

The crisp value of the optimum fuzzy transportation cost for given problem is 129. (ie) We conclude that the solution method is vary for every method when comparing the fuzzy numbers.

CONCLUSION

In this paper, we proposed an optimum solution for fuzzy Transportation problem by using a simple effective parametric method. This method can be used for all kinds of fuzzy transportation problem, whether triangular, and Trapezoidal fuzzy numbers with normal and abnormal data. Further, this method is a symmetric procedure, easy to apply and can be utilize for all types of Transportation problem whether maximize or minimize objection functions. Finally, we can conclude that any particular method is not an unique manners for solving the fuzzy transportation problem when comparing the fuzzy numbers.