The Degree of an Edge in Subdivision, Middle and Total Fuzzy Graphs with Their Edge Regular Properties

Mathematics

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ABSTRACT   *In this paper, the degree of an edge in subdivision, middle and total fuzzy graphs are obtained. In general subdivision and total fuzzy graphs of an edge regular fuzzy graph need not be edge regular. Conditions under which they are edge regular are provided. A necessary and sufficient condition for subdivision fuzzy graph of a regular fuzzy graph to be edge regular is determined.*

1. INTRODUCTION

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975 [11]. Though it is very young, it has been growing fast and has numerous applications in various fields. During the same time Yeh and bang have also introduced various connectedness concepts in fuzzy graphs [12]. A.Nagoorgani and J.Malarvizhi discussed the concept of subdivision, middle and total fuzzy graphs and its properties [2]. K.Radha and N.Kumaravel (2014) introduced the concept of edge regular fuzzy graphs [9]. In this paper, we study about edge regular properties of subdivision, middle and total fuzzy graphs using their edge degrees.

First we go through some basic definitions in the next section from [1], [2], [4], [5] and [8].

2. BASIC CONCEPTS

Let V be a non-empty finite set and $E \subseteq V \times V$. A fuzzy graph [5] $G : (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0, 1]$ and $\mu : E \rightarrow [0, 1]$ such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$. The order and size of a fuzzy graph [4] $G : (\sigma, \mu)$ are defined by $O(G) = \sum_{x \in V} \sigma(x)$ and $S(G) = \sum_{xy \in E} \mu(xy)$. A fuzzy Graph $G : (\sigma, \mu)$ is strong [4], if $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$. A fuzzy Graph $G : (\sigma, \mu)$ is complete [5], if $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$. The underlying crisp graph is denoted by $G^* : (V, E)$.

The degree of a vertex [5] x is $d_G(x) = \sum_{x \neq y} \mu(xy)$. If each vertex in G has same degree k, then G is said to be a regular fuzzy graph or k – regular fuzzy graph [5]. Let $G^* : (V, E)$ be a graph and let $e = uv$ be an edge in G^*. Then the degree of an edge [1] $e = uv \in E$ is defined by $d_{G^*}(uv) = d_{G^*}(u) + d_{G^*}(v) - 2$. If each edge in G^* has same degree, then G^* is said to be edge regular. The degree of an edge [8] $xy \in E$ in $G : (\sigma, \mu)$ is $d_G(xy) = \sum_{x \neq z} \mu(xz) + \sum_{z \neq y} \mu(zy) - 2\mu(xy)$. If each edge in G has same degree k, then G is said to be an edge regular fuzzy graph or k – edge regular fuzzy graph [8].

If every line of G^* is subdivided, the resulting graph is the subdivision graph $S(G^*)$. Let $G : (\sigma, \mu)$ be a fuzzy graph with $G^* : (V, E)$. A subdivision fuzzy graph [2] $sd(G) : (\sigma_{sd}, \mu_{sd})$ is defined as follows: The nodes and edges of G are taken together as node set, in $sd(G)$, each edge ' e ' in G is replaced by a new vertex and that vertex is made as a neighbour of those vertices which lie on ' e ' in G. Let the node set of $sd(G)$ be $V \cup E$.

Here, fuzzy subset σ_{sd} is defined on $V \cup E$ as,

$$\sigma_{sd}(v_i) = \begin{cases} \sigma(v_i), & \text{if } v_i \in V \\ \mu(e_j), & \text{if } e_j \in E \end{cases}.$$

The fuzzy relation μ_{sd} is defined as,

$$\mu_{sd}(v_i e_j) = \begin{cases} \sigma(v_i) \wedge \mu(e_j), & \text{if } v_i \in V, e_j \in E \text{ and the node } v_i \text{ lies on the edge } e_j \\ 0, & \text{otherwise} \end{cases}.$$

The size of subdivision fuzzy graph [2] is $S\big(sd(G)\big)=2S(G)$ and the order of subdivision fuzzy graph is

$O\big(sd(G)\big)=O(G)+S(G)$.

A middle fuzzy graph [2] $M(G):(\sigma_M,\mu_M)$ is defined as follows:

Let the node set of $M(G)$ be $V\cup E$.

The fuzzy subset σ_M is defined on $V\cup E$ as,

$$\sigma_M(v_i)=\begin{cases}\sigma(v_i),\text{ if }v_i\in V\\ \mu(e_j),\text{ if }e_j\in E\end{cases}.$$

The fuzzy relation μ_M is defined as,

$$\mu_M(e_ie_j)=\begin{cases}\mu(e_i)\wedge\mu(e_j),\text{if the edges }e_i\text{ and }e_j\text{ are adjacent}\\ 0,\qquad\qquad\qquad\qquad\qquad\text{otherwise}\end{cases}$$

$\mu_M(v_iv_j)=0,\text{if }v_i,v_j\in V$,

$$\mu_M(v_ie_j)=\begin{cases}\sigma(v_i)\wedge\mu(e_j),\text{if }v_i\in V,e_j\in E\text{ and the node }v_i\text{ lies on the edge }e_j\\ 0,\qquad\qquad\qquad\qquad\qquad\qquad\text{otherwise}\end{cases}.$$

The size of middle fuzzy graph [2] is $S\big(M(G)\big)=2S(G)+\displaystyle\sum_{e_i,e_j\in E}\mu(e_i)\wedge\mu(e_j)$ and the order of middle fuzzy graph

is $O\big(M(G)\big)=O(G)+S(G)$.

A total fuzzy graph [2] $T(G):(\sigma_T,\mu_T)$ is defined as follows:

Let the node set of $T(G)$ be $V\cup E$.

The fuzzy subset σ_T is defined on $V\cup E$ as,

$$\sigma_T(v_i)=\begin{cases}\sigma(v_i),\text{ if }v_i\in V\\ \mu(e_j),\text{ if }e_j\in E\end{cases}.$$

The fuzzy relation μ_T is defined as,

$\mu_T(v_iv_j)=\mu(v_iv_j),\text{if }v_iv_j\in E$,

$$\mu_T(v_ie_j)=\begin{cases}\sigma(v_i)\wedge\mu(e_j),\text{if }v_i\in V,e_j\in E\text{ and the node }v_i\text{ lies on the edge }e_j\\ 0,\qquad\qquad\qquad\qquad\qquad\text{otherwise}\end{cases},$$

$$\mu_T(e_ie_j)=\begin{cases}\mu(e_i)\wedge\mu(e_j),\text{if the edges }e_i\text{ and }e_j\text{ have a node in common between them}\\ 0,\qquad\qquad\qquad\qquad\qquad\text{otherwise}\end{cases}.$$

The size of total fuzzy graph [2] is $S\big(T(G)\big)=3S(G)+\displaystyle\sum_{e_i,e_j\in E}\mu(e_i)\wedge\mu(e_j)$ and the order of total fuzzy graph is

$O\big(T(G)\big)=O(G)+S(G)$.

2.1. Theorem [10]:

2.2. Theorem [10]:

Let $G:(\sigma,\mu)$ be a fuzzy graph on an even cycle $G^*:(V,E)$ with n vertices and let $n \not\equiv 0 \pmod 4$. Then G is an edge regular fuzzy graph if and only if μ is a constant function.

2.3. Theorem [9]:

Let μ be a constant function in $G:(\sigma,\mu)$ on $G^*:(V,E)$. If G is regular, then G is edge regular.

3. EDGE REGULAR PROPERTY OF SUBDIVISION FUZZY GRAPH

In this section, we obtain the degree of an edge in subdivision fuzzy graph in terms of parameters of G and study its edge regular property.

3.1. Theorem:

Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. Let $e_a = uv \in E$ be any edge in G^*. Then the edge degree of the edge $ue_a \in E_{sd}$ is given by $d_{sd(G)}(ue_a) = d_G(u)$.

Proof:

Let $ue_a \in E_{sd}$ be any edge in $sd(G)$. Then $e_a = uv \in E$, for some $v \in V$. Therefore u and v are the only vertices adjacent to e_a in G.

By definition, $d_{sd(G)}(ue_a) = \displaystyle\sum_{ue_b \in E_{sd}, e_b \neq e_a} \mu_{sd}(ue_b) + \sum_{e_a v \in E_{sd}, v \neq u} \mu_{sd}(e_a v)$.

$\Rightarrow d_{sd(G)}(ue_a) = \displaystyle\sum_{e_b \in E, e_b \neq e_a} \left(\sigma(u) \wedge \mu(e_b)\right) + \mu_{sd}(e_a v)$.

$\qquad = \displaystyle\sum_{uw \in E, w \neq v} \left(\sigma(u) \wedge \mu(uw)\right) + \mu(e_a) \wedge \mu(v)$

$\Rightarrow d_{sd(G)}(ue_a) = \displaystyle\sum_{uw \in E, w \neq v} \mu(uw) + \mu(uv)$.

$\Rightarrow d_{sd(G)}(ue_a) = \displaystyle\sum_{uw \in E} \mu(uw)$.

$\Rightarrow d_{sd(G)}(ue_a) = d_G(u)$.

3.2. Remark:

If $G:(\sigma,\mu)$ is an edge regular fuzzy graph, then $sd(G):(\sigma_{sd},\mu_{sd})$ need not be edge regular fuzzy graph. For example, in figure 3.1 $G:(\sigma,\mu)$ is 1.8 – edge regular fuzzy graph, but $sd(G):(\sigma_{sd},\mu_{sd})$ is not an edge regular fuzzy graph.
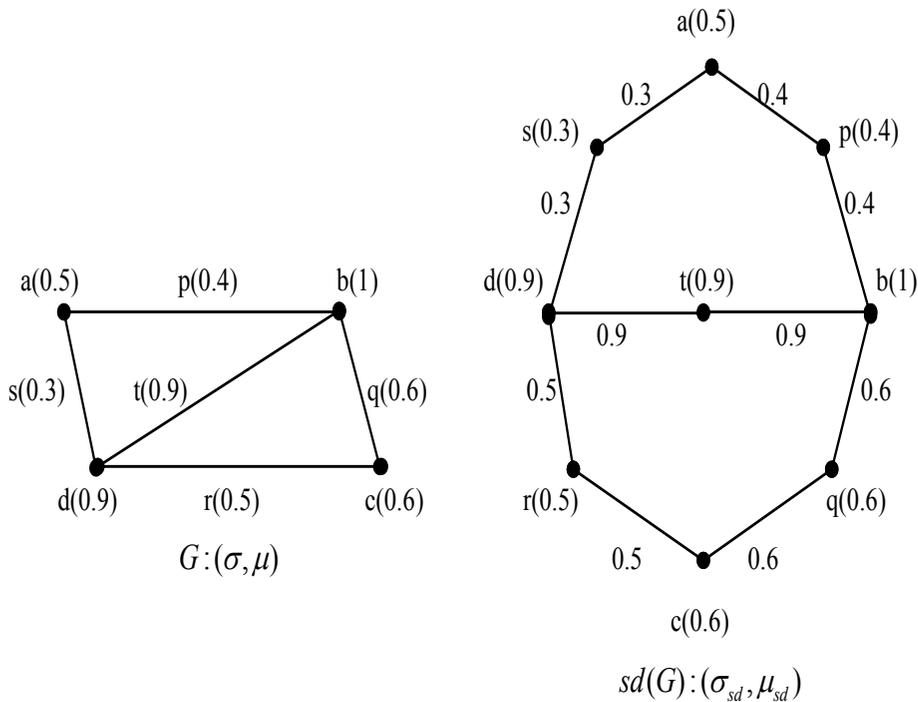
Fig.3.1.

3.3. Remark:

If $sd(G):(\sigma_{sd},\mu_{sd})$ is an edge regular fuzzy graph, then $G:(\sigma,\mu)$ need not be edge regular fuzzy graph.

For example, in figure 3.2 $sd(G):(\sigma_{sd},\mu_{sd})$ is 1.5 – edge regular fuzzy graph. But $G:(\sigma,\mu)$ is not an edge regular fuzzy graph.
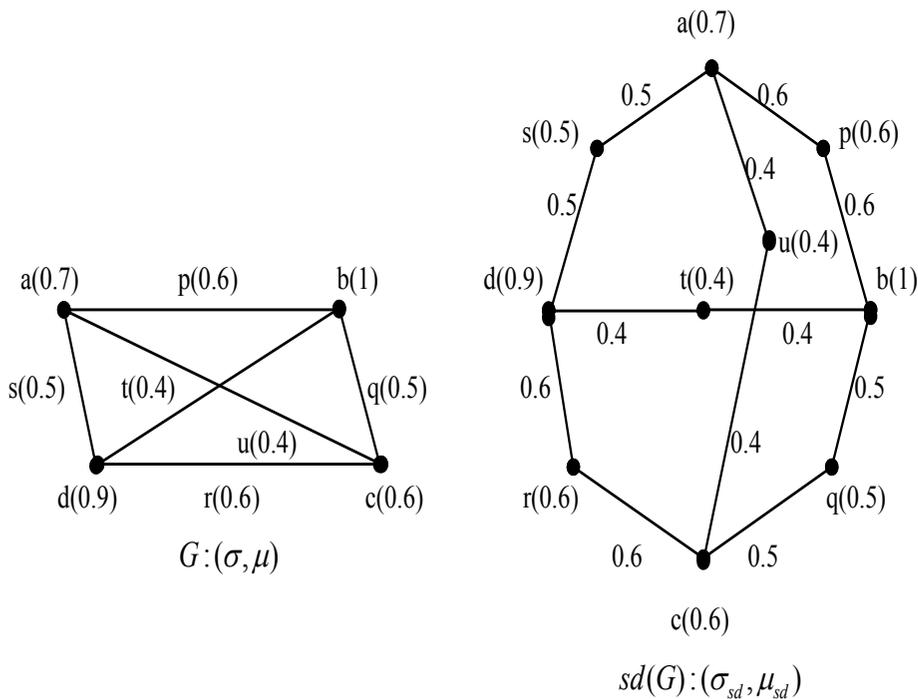

Fig.3.2.

3.4. Theorem:

Let $G:(\sigma,\mu)$ be a fuzzy graph. Then G_{sd} is an edge regular fuzzy graph if and only if G is a regular fuzzy graph.

Proof:

The result follows from theorem 3.1.

4. EDGE REGULAR PROPERTY OF MIDDLE FUZZY GRAPH

In this section, we determine the degree of an edge in middle fuzzy graph in terms of parameters of G and study its edge regular property.

4.1. Theorem:

Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. Then the edge degree in its middle fuzzy graph is given by

$$d_{M(G)}(ue_a)=d_G(u)+\sum_{e_a e_c \in E_M}\left(\mu(e_a)\wedge\mu(e_c)\right) \text{ and}$$

$$d_{M(G)}(e_a e_b)=2\left(\mu(e_a)+\mu(e_b)\right)+\sum_{e_a e_c \in E_M, e_c \neq e_b}\left(\mu(e_a)\wedge\mu(e_c)\right)+\sum_{e_b e_c \in E_M, e_c \neq e_a}\left(\mu(e_b)\wedge\mu(e_c)\right), \text{ where } u \in V,$$

$e_a, e_b \in E$.

Proof:

Consider the edge ue_a in $M(G)$, where $u \in V$ and $e_a = uv \in E$ with $v \in V$. Then the edges adjacent to ue_a are of the form ue_b, where $e_b = uw \in E$ (or) $e_a v$ (or) $e_a e_c$, where the edges e_a and e_c are adjacent in G.

$$\therefore d_{M(G)}(ue_a)=\sum_{ue_b \in E_M, e_b \neq e_a}\mu_M(ue_b)+\sum_{e_a v \in E_M, v \neq u}\mu_M(e_a v)+\sum_{e_a e_c \in E_M}\mu_M(e_a e_c)$$

$$=\sum_{ue_b \in E_M, e_b \neq e_a}\mu_M(ue_b)+\mu_M(e_a v)+\sum_{e_a e_c \in E_M}\mu_M(e_a e_c)$$

$$=\sum_{ue_b \in E_M, e_b \neq e_a}\left(\sigma(u)\wedge\mu(e_b)\right)+\mu(e_a)\wedge\sigma(v)+\sum_{e_a e_c \in E_M}\left(\mu(e_a)\wedge\mu(e_c)\right)$$

$$=\sum_{ue_b \in E_M, e_b \neq e_a}\mu(e_b)+\mu(e_a)+\sum_{e_a e_c \in E_M}\left(\mu(e_a)\wedge\mu(e_c)\right)$$

$$=\sum_{uw \in E}\mu(uw)+\sum_{e_a e_c \in E_M}\left(\mu(e_a)\wedge\mu(e_c)\right)$$

$$=d_G(u)+\sum_{e_a e_c \in E_M}\left(\mu(e_a)\wedge\mu(e_c)\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.1)$$

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Now, consider the edge $e_a e_b$ in $M(G)$, where $e_a = uv$ and $e_b = vw \in E$ with u, v and $w \in V$. Then the edges adjacent to $e_a e_b$ are of the form $x e_a$, where $x = u$ or v (or) $e_c e_a$, where the edges e_c and e_a are adjacent in G (or) $e_b x$, where $x = v$ or w (or) $e_b e_c$, where the edges e_b and e_c are adjacent in G.

$$\therefore d_{M(G)}(e_a e_b) = \sum_{x e_a \in E_M} \mu_M(x e_a) + \sum_{e_a e_c \in E_M, e_c \neq e_b} \mu_M(e_a e_c) + \sum_{x e_b \in E_M} \mu_M(x e_b) + \sum_{e_b e_c \in E_M, e_c \neq e_a} \mu_M(e_b e_c)$$

$$= \mu_M(u e_a) + \mu_M(v e_a) + \sum_{e_a e_c \in E_M, e_c \neq e_b} \left(\mu(e_a) \wedge \mu(e_c) \right) + \mu_M(v e_b) + \mu_M(w e_b) + \sum_{e_b e_c \in E_M, e_c \neq e_a} \left(\mu(e_b) \wedge \mu(e_c) \right)$$

$$= 2\mu(e_a) + \sum_{e_a e_c \in E_M, e_c \neq e_b} \left(\mu(e_a) \wedge \mu(e_c) \right) + 2\mu(e_b) + \sum_{e_b e_c \in E_M, e_c \neq e_a} \left(\mu(e_b) \wedge \mu(e_c) \right)$$

$$= 2\left(\mu(e_a) + \mu(e_b) \right) + \sum_{e_a e_c \in E_M, e_c \neq e_b} \left(\mu(e_a) \wedge \mu(e_c) \right) + \sum_{e_b e_c \in E_M, e_c \neq e_a} \left(\mu(e_b) \wedge \mu(e_c) \right) \dots\dots\dots\dots\dots\dots (4.2)$$

4.2. Theorem:

Let $G : (\sigma, \mu)$ be a fuzzy graph such that μ is a constant function. Then

(i). For any $u e_a \in E_M$, $d_{M(G)}(u e_a) = d_G(u) + d_G(e_a)$.

(ii). For any $e_a e_b \in E_M$, $d_{M(G)}(e_a e_b) = 2c + d_G(e_a) + d_G(e_b)$.

Proof:

Let $\mu(e_a) = c, \forall e_a \in E$.

(i). From (4.1), for any $u e_a \in E_M$, $d_{M(G)}(u e_a) = d_G(u) + \sum_{e_a e_c \in E_M} \left(\mu(e_a) \wedge \mu(e_c) \right)$

$$= d_G(u) + \sum_{e_b \in E} c \wedge c$$

$$= d_G(u) + c d_{G^*}(e_a)$$

$$= d_G(u) + d_G(e_a).$$

(ii). From (4.2), for any $e_a e_b \in E_M$,

$$d_{M(G)}(e_a e_b) = 2\left(\mu(e_a) + \mu(e_b) \right) + \sum_{e_a e_c \in E_M, e_c \neq e_b} \left(\mu(e_a) \wedge \mu(e_c) \right) + \sum_{e_b e_c \in E_M, e_c \neq e_a} \left(\mu(e_b) \wedge \mu(e_c) \right)$$

$$= 2(c + c) + \sum_{e_a e_c \in E_M, e_c \neq e_b} \left(c \wedge c \right) + \sum_{e_b e_c \in E_M, e_c \neq e_a} \left(c \wedge c \right)$$

$$= 4c + c\left(d_{G^*}(e_a) - 1 \right) + c\left(d_{G^*}(e_b) - 1 \right)$$

$$= 2c + c d_{G^*}(e_a) + c d_{G^*}(e_b)$$

$$= 2c + d_G(e_a) + d_G(e_b).$$

4.3. Remark:

If $G:(\sigma,\mu)$ is an edge regular fuzzy graph, then $M(G):(\sigma_M,\mu_M)$ need not be edge regular fuzzy graph.

For example, in figure 4.1 $G:(\sigma,\mu)$ is 1.8 – edge regular fuzzy graph, but $M(G):(\sigma_M,\mu_M)$ is not an edge
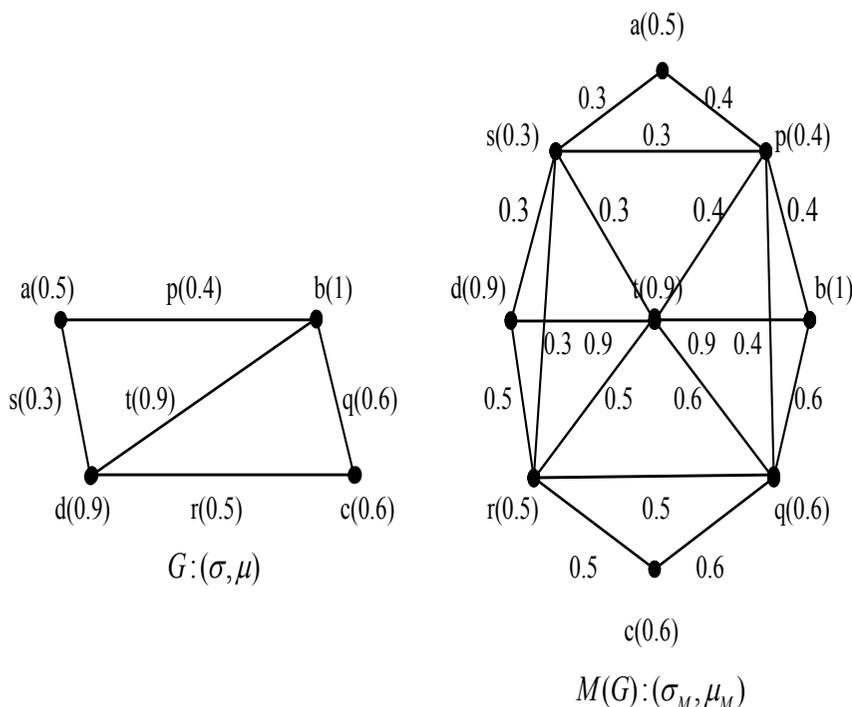
regular fuzzy graph.

Fig.4.1.

4.4. Biregular fuzzy graph:

If the degree of each edge in a fuzzy graph G is either k_1 or k_2, then G is a $(k_1$ or $k_2)$ edge biregular

fuzzy graph.

4.5. Theorem:

Let $G:(\sigma,\mu)$ be a fuzzy graph such that μ is a constant function. If $G:(\sigma,\mu)$ is a regular fuzzy graph,

then $M(G):(\sigma_M,\mu_M)$ is an edge biregular fuzzy graph.

Proof:

Let G be k – regular. Since μ is a constant function, by theorem 2.3, G is also edge regular, say, m –

edge regular.

Then theorem 4.2 gives, $d_{M(G)}(ue_a) = k + m$ for each $ue_a \in E_{M(G)}$

$$\text{and } d_{M(G)}(e_a e_b) = 2c + 2m \text{ for each } e_a e_b \in E_{M(G)}.$$

Hence G is $(k + m, 2c + 2m)$ – edge biregular.

5. EDGE REGULAR PROPERTY OF TOTAL FUZZY GRAPH

In this section, we determine the degree of an edge in total fuzzy graph in terms of parameters of G and study its edge regular property.

5.1. Theorem:

Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. Then the edge degree in its total fuzzy graph is given by

$$d_{T(G)}(ue_a) = 2d_G(u) + \sum_{e_a e_c \in E_T} \left(\mu(e_a) \wedge \mu(e_c) \right),$$

$$d_{T(G)}(e_a e_b) = 2\left(\mu(e_a) + \mu(e_b) \right) + \sum_{e_a e_c \in E_T, e_c \neq e_b} \left(\mu(e_a) \wedge \mu(e_c) \right) + \sum_{e_b e_c \in E_T, e_c \neq e_a} \left(\mu(e_b) \wedge \mu(e_c) \right) \text{ and}$$

$$d_{T(G)}(uv) = 2\left(d_G(uv) + \mu(uv) \right), \text{ where } u, v \in V \text{ and } e_a, e_b \in E.$$

Proof:

Consider the edge ue_a in $T(G)$, where $u \in V$ and $e_a = uv \in E$ with $v \in V$. Then the edges adjacent to ue_a are of the form ux, where $_c$ (or) ue_b, where $e_b = uw \in E$ (or) $e_a v$ (or) $e_a e_c$, where the edges e_a and e_c are adjacent in G.

$$\therefore d_{T(G)}(ue_a) = \sum_{uv \in E} \mu(uv) + \sum_{ue_b \in E_T, e_b \neq e_a} \mu_T(ue_b) + \sum_{e_a v \in E_T, v \neq u} \mu_T(e_a v) + \sum_{e_a e_c \in E_T} \mu_T(e_a e_c)$$

$$= d_G(u) + \sum_{ue_b \in E_T, e_b \neq e_a} \mu_T(ue_b) + \mu_T(e_a v) + \sum_{e_a e_c \in E_T} \mu_T(e_a e_c)$$

$$= d_G(u) + \sum_{ue_b \in E_T, e_b \neq e_a} \left(\sigma(u) \wedge \mu(e_b) \right) + \left(\mu(e_a) \wedge \sigma(v) \right) + \sum_{e_a e_c \in E_T} \left(\mu(e_a) \wedge \mu(e_c) \right)$$

$$= d_G(u) + \sum_{ue_b \in E_T, e_b \neq e_a} \mu(e_b) + \mu(e_a) + \sum_{e_a e_c \in E_T} \left(\mu(e_a) \wedge \mu(e_c) \right)$$

$$= 2d_G(u) + \sum_{e_a e_c \in E_T} \left(\mu(e_a) \wedge \mu(e_c) \right) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (5.1)$$

Now, consider the edge $e_a e_b$ in $T(G)$, where $e_a = uv$ and $e_b = vw \in E$ with u, v and $w \in V$. Then the edges adjacent to $e_a e_b$ are of the form xe_a, where $x = u$ or v (or) $e_c e_a$, where the edges e_c and e_a are adjacent in G (or) $e_b x$, where $x = v$ or w (or) $e_b e_c$, where the edges e_b and e_c are adjacent in G.

$$\therefore d_{T(G)}(e_a e_b) = \sum_{xe_a \in E_T} \mu_T(xe_a) + \sum_{e_a e_c \in E_T, e_c \neq e_b} \mu_T(e_a e_c) + \sum_{xe_b \in E_T} \mu_T(xe_b) + \sum_{e_b e_c \in E_T, e_c \neq e_a} \mu_T(e_b e_c)$$

$$= \mu_T(ue_a) + \mu_T(ve_a) + \sum_{e_a e_c \in E_T, e_c \neq e_b} \left(\mu(e_a) \wedge \mu(e_c) \right) + \mu_T(ve_b) + \mu_T(we_b) + \sum_{e_b e_c \in E_T, e_c \neq e_a} \left(\mu(e_b) \wedge \mu(e_c) \right)$$

$$= 2\mu(e_a) + \sum_{e_a e_c \in E_T, e_c \neq e_b} \left(\mu(e_a) \wedge \mu(e_c) \right) + 2\mu(e_b) + \sum_{e_b e_c \in E_T, e_c \neq e_a} \left(\mu(e_b) \wedge \mu(e_c) \right)$$

$$= 2\left(\mu(e_a) + \mu(e_b) \right) + \sum_{e_a e_c \in E_T, e_c \neq e_b} \left(\mu(e_a) \wedge \mu(e_c) \right) + \sum_{e_b e_c \in E_T, e_c \neq e_a} \left(\mu(e_b) \wedge \mu(e_c) \right) \; \dots\dots\dots\dots\dots \; (5.2)$$

Finally, consider the edge uv in $T(G)$, where $u, v \in V$. Then the edges adjacent to uv are of the form uw, where $w \in V$ (or) ue_a, where $e_a = uw \in E$ with $w \in V$ (or) vw, where $w \in V$ (or) ve_b, where $e_b = vw \in E$ with $w \in V$.

$$\therefore d_{T(G)}(uv) = \sum_{uw \in E, w \neq v} \mu(uw) + \sum_{ue_a \in E_T} \mu_T(ue_a) + \sum_{vw \in E, w \neq u} \mu(vw) + \sum_{ve_b \in E_T} \mu_T(ve_b)$$

$$= d_G(uv) + \sum_{uuw \in E_T} \left(\sigma(u) \wedge \mu(e_a) \right) + \sum_{vvw \in E_T} \left(\sigma(v) \wedge \mu(e_b) \right)$$

$$= d_G(uv) + \sum_{uw \in E} \mu(uw) + \sum_{vw \in E} \mu(vw)$$

$$= d_G(uv) + d_G(u) + d_G(v)$$

$$= 2\left(d_G(uv) + \mu(uv) \right) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \; (5.3)$$

5.2. Theorem:

Let $G : (\sigma, \mu)$ be a fuzzy graph such that μ is a constant function. Then

(i). For any $ue_a \in E_T$, $d_{T(G)}(ue_a) = 2d_G(u) + d_G(e_a)$.

(ii). For any $e_a e_b \in E_T$, $d_{T(G)}(e_a e_b) = 2c + d_G(e_a) + d_G(e_b)$.

(iii). For any $uv \in E_T$, $d_{T(G)}(uv) = 2\left(d_G(uv) + c \right)$.

Proof:

Let $\mu(e_a) = c, \forall e_a \in E$.

(i). From (5.1), for any $ue_a \in E_T$, $d_{T(G)}(ue_a) = 2d_G(u) + \sum_{e_a e_c \in E_T} \left(\mu(e_a) \wedge \mu(e_c) \right)$

$$= 2d_G(u) + \sum_{e_a e_c \in E_T} c$$

$$= 2d_G(u) + c d_{G^*}(e_a)$$

$$= 2d_G(u) + d_G(e_a).$$

(ii). From (5.2), for any $e_a e_b \in E_T$,

$$d_{T(G)}(e_a e_b) = 2\big(\mu(e_a) + \mu(e_b)\big) + \sum_{e_a e_c \in E_T, e_c \neq e_b} \big(\mu(e_a) \wedge \mu(e_c)\big) + \sum_{e_b e_c \in E_T, e_c \neq e_a} \big(\mu(e_b) \wedge \mu(e_c)\big).$$

$$= 2(c+c) + \sum_{e_a e_c \in E_T, e_c \neq e_b} (c \wedge c) + \sum_{e_b e_c \in E_T, e_c \neq e_a} (c \wedge c)$$

$$= 4c + c\big(d_{G^*}(e_a) - 1\big) + c\big(d_{G^*}(e_b) - 1\big)$$

$$= c\big(2 + d_{G^*}(e_a) + d_{G^*}(e_b)\big)$$

$$= 2c + d_G(e_a) + d_G(e_b).$$

(iii). From (5.3), for any $uv \in E_T$, $d_{T(G)}(uv) = 2\big(d_G(uv) + \mu(uv)\big)$

$$= 2\big(d_G(uv) + c\big).$$

5.3. Remark:

If $G:(\sigma, \mu)$ is an edge regular fuzzy graph, then $T(G):(\sigma_T, \mu_T)$ need not be edge regular fuzzy graph. For example, from the figure 5.1 $G:(\sigma, \mu)$ is 1.8 – edge regular fuzzy graph, but $T(G):(\sigma_T, \mu_T)$ is not an edge regular fuzzy graph.

5.4. Theorem

Let $G:(\sigma, \mu)$ be a fuzzy graph such that $\mu(e_i) = c, \forall e_i \in E$, where c is a constant. If G^* is a k – regular fuzzy graph, then $T(G):(\sigma_T, \mu_T)$ is a $2c(2k-1)$ – edge regular fuzzy graph.

Proof:

Since G^* is a k – regular fuzzy graph, it is $2(k-1)$ – edge regular.

$$G:(\sigma,\mu) \qquad\qquad 0.5 \qquad 0.6$$

$$c(0.6)$$
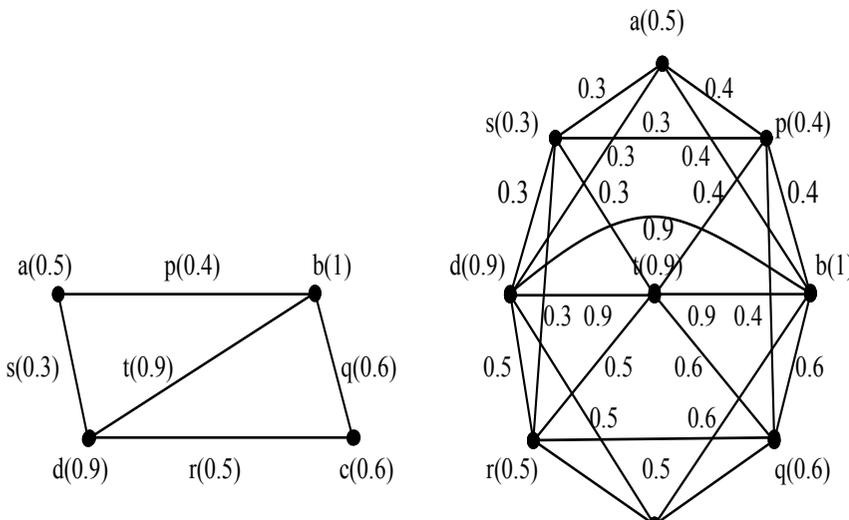
$$T(G):(\sigma_T,\mu_T)$$

Fig.5.1.

From theorem 5.2, for any $ue_a \in E_T$, $d_{T(G)}(ue_a) = c\left(2d_{G^*}(u) + d_{G^*}(e_a)\right).$

$$= c\left(2k + 2(k-1)\right).$$

$$= 2c\left(2k-1\right) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(5.4)}$$

From theorem 5.2, for any $e_a e_b \in E_T$, $d_{T(G)}(e_a e_b) = c\left(2 + d_{G^*}(e_a) + d_{G^*}(e_b)\right)$

$$= c\left(2 + 2(k-1) + 2(k-1)\right)$$

$$= 2c\left(2k-1\right) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(5.5)}$$

From theorem 5.2, for any $uv \in E_T$, $d_{T(G)}(uv) = 2c\left(d_{G^*}(u) + d_{G^*}(v) - 1\right)$

$$= 2c\left(k + k - 1\right)$$

$$= 2c\left(2k-1\right) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(5.6)}$$

From (5.4), (5.5) & (5.6), $T(G):(\sigma_T,\mu_T)$ is an $2c\left(2k-1\right)$ – edge regular fuzzy graph.

5.5. Theorem

Let $G:(\sigma,\mu)$ be a fuzzy graph on an odd cycle $G^*:(V,E)$. If $G:(\sigma,\mu)$ is an edge regular fuzzy graph, then $T(G):(\sigma_T,\mu_T)$ is an edge regular fuzzy graph.

Proof:

From the hypothesis of this theorem, G^* is 2 – regular and using theorem 2.1, μ is a constant function.

Using theorem 5.4, $T(G):(\sigma_T,\mu_T)$ is an edge regular fuzzy graph.

5.6. Theorem

Let $G:(\sigma,\mu)$ be a fuzzy graph on an even cycle $G^*:(V,E)$ with n vertices and let $n \not\equiv 0 \pmod{4}$. If G is an edge regular fuzzy graph, then $T(G):(\sigma_T,\mu_T)$ is an edge regular fuzzy graph.

Proof:

Proof is similar to proof of theorem 5.5.

5.7. Remark

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Let $G : (\sigma, \mu)$ be a fuzzy graph on an even cycle $G^* : (V, E)$ with n vertices and let $n \equiv 0 \ (\text{mod } 4)$. If G

is an edge regular fuzzy graph, then $T(G) : (\sigma_T, \mu_T)$ need not be an edge regular fuzzy graph.

For example, in the following figure 5.2, G is an 0.7 – edge regular fuzzy graph, but $T(G) : (\sigma_T, \mu_T)$ is not an edge regular fuzzy graph.
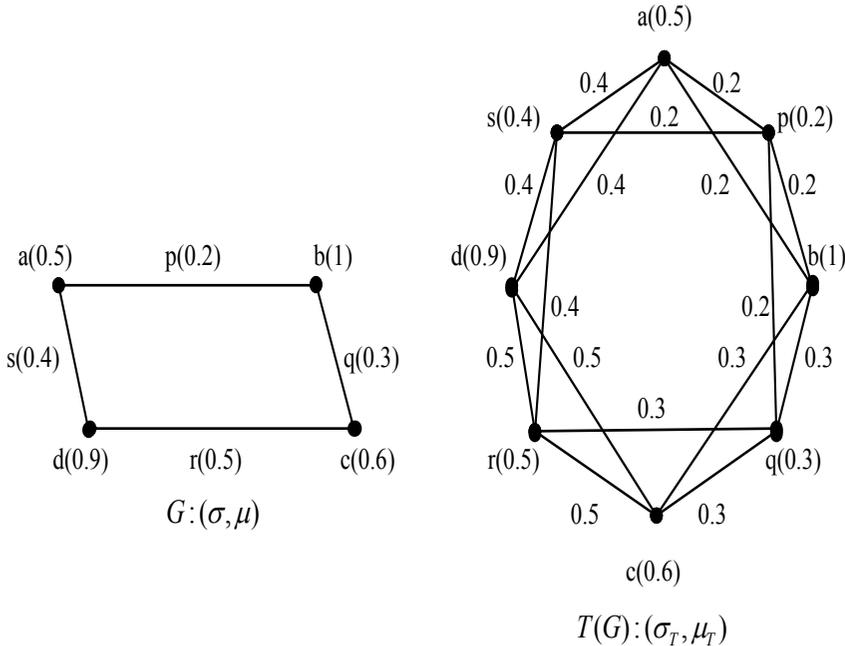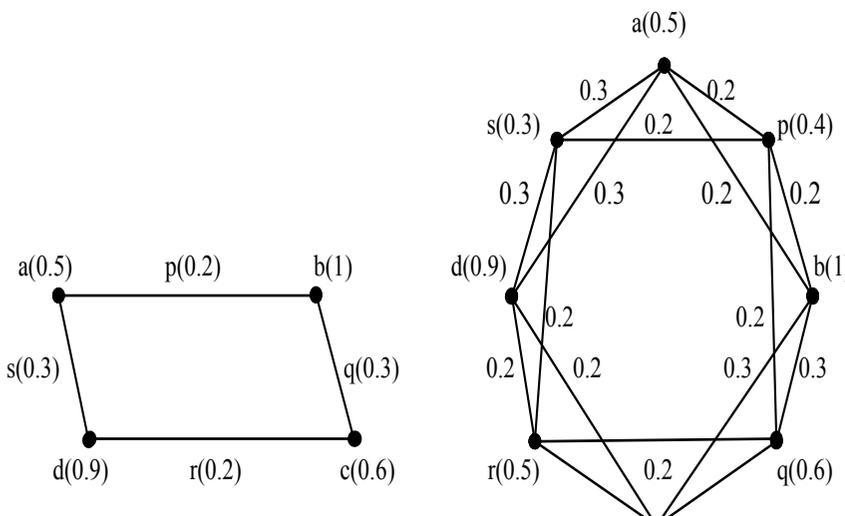

Fig.5.2.

5.8. Remark

If G is a regular fuzzy graph, then $T(G) : (\sigma_T, \mu_T)$ need not be an edge regular fuzzy graph.

For example, in the following figure 5.3, G is an 0.5 – regular fuzzy graph, but $T(G) : (\sigma_T, \mu_T)$ is not an edge regular fuzzy graph.

$$G:(\sigma,\mu) \qquad\qquad 0.2 \qquad\qquad 0.3$$

$$c(0.6)$$

$$T(G):(\sigma_T,\mu_T)$$

Fig.5.3.

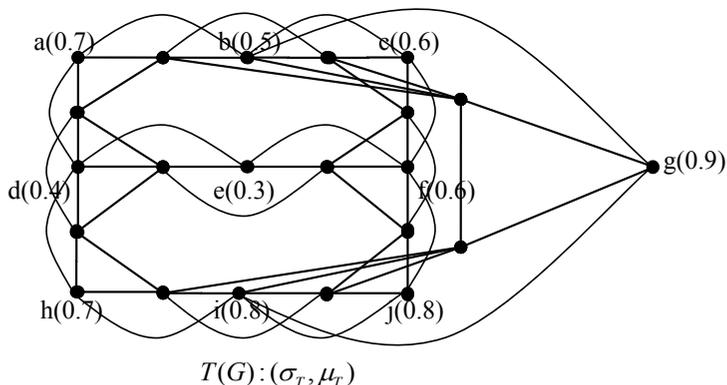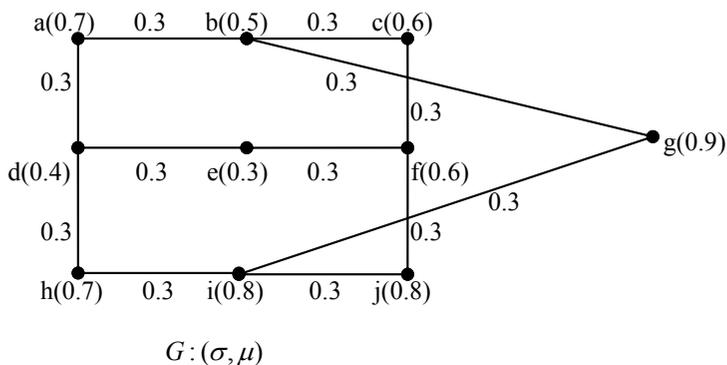
$$G:(\sigma,\mu)$$

$$T(G):(\sigma_T,\mu_T)$$
Fig.5.4.

5.9. Remark

Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$ with μ is a constant function. If $G:(\sigma,\mu)$ is an edge

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