

Repair Replacement Model for a Repairable System with two Different Repair Actions Using Alpha Series Process



Mathematics

KEYWORDS : Renewal theorem, Long-run average cost rate, alpha series process, Replacement N

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ABSTRACT

In this paper, the maintenance problem for a cold standby system consisting of two dissimilar components and one repairman with two repair actions is studied. Assume that the component 1 follow α -series process repair and is given priority in use while component 2 obeys a minimal repair. Under these assumptions, using an alpha series process repair model, we consider a replacement policy N under which the system is replaced when the number of failures of component 1 reaches N. The purpose of this paper is to determine an optimal replacement policy N^ such that the average cost rate (i.e. the long-run average cost per unit time) of the system is minimized. The explicit expression for the average cost rate of the system is derived and the corresponding optimal replacement policy N^* can be determined analytically or numerically. Finally, a numerical example is given to illustrate some theoretical results and the model applicability.*

1. Introduction

In the earlier study of maintenance problems, the stands by techniques are often adopted to improve the system reliability, availability and reduce the cost incurred by system failure. In the beginning, Lam [8,9] introduced a geometric process repair–replacement model which is extensively used to study a simple repairable system (i.e. a one-component repairable system with one repairman). Moreover, it is usually assumed that the system after repair is “as good as new”. This is a perfect repair model. However, this assumption is not always true. In practice, most repairable systems are deteriorative because of the ageing effect and the accumulative wear. Barlow and Hunter [1] first presented a minimal repair model in which a system after repair has same failure rate and same effective age as at the time of failure. Brown and Proschan [2], combined the perfect repair and the minimal repair, first reported an imperfect repair model in which the repair will be perfect with probability p or minimal with probability ‘1-p’. Research works on the minimal repair model and the imperfect repair models were studied by Park [11], Block et al. [4,5], Finkelstein[6], Kijima [7], while geometric process models were studied by Wang and Zhang[13,14]Zhang at.al[16] and Zhang[17,18].

However, for a deteriorating simple system, it seems is more reasonable to assume that the successive working times of the system after repair will become shorter and shorter, while the consecutive repair times of the system after failure will become longer and longer. Ultimately, it cannot work any longer, neither can it be repaired. For such a stochastic phenomenon, Lam [8,9] first introduced a geometric process repair model to approach it, and under this model, he studied two kinds of replacement policy for simple repairable systems, one based on the working age T of the systems and the other based on the failure number N of the system. The explicit expressions of the average cost rate under these two kinds of policy are respectively calculated, and the corresponding optimal replacement policies T^* and N^* can be found analytically or numerically. Under some mild conditions, he also proved that the optimal policy N^* is better than the optimal policy T^* .

Zhang and Wang [20] considered a replacement policy $M = (N_1, N_2, \dots, N_k)$ for a k-dissimilar-component series repairable system, where N_1, N_2, \dots and N_k are, respectively, the number of failures of component 1, component 2, ... and component k. An optimal replacement policy $M^*=(N^*_1, N^*_2, \dots, N^*_k)$ can be determined by minimizing the average cost rate of the system.

Zhang [18] analyzed a two-component cold standby repairable system with one repairman. It assumed that each component after repair is not “as good as new”, and obeys a geometric process repair. Under this assumption, he studied a replacement policy N based on the number of repairs of component 1. An optimal replacement policy N^* can be determined by maximizing the long-run expected reward per unit time. Zhang et al. [19] studied two kinds of replacement policy for a two-component cold standby repairable system with one repairman, one based on the working age T of component 1 under which the system is replaced when the working age of component 1 reaches T, and the other based on the failure number N of

component 1 under which the system is replaced when the failure number of component 1 reaches N . The optimal replacement policies T^* and N^* are, respectively, determined by minimizing the average cost rate of the system. They also proved that the optimal policy N^* is better than the optimal policy T^* for a cold standby repairable system under some mild conditions.

Similarly, in order to improve the reliability, raise the availability or reduce the cost of a system, the techniques for priority in use or repair are also used. For example, in the operating room of a hospital, an operation must be discontinued if only the power source is cut (i.e. the power station fails). Usually, there is a standby generator (e.g. a storage battery) which can provide electric power when the main power station fails. Thus, the power station (regarded as the main component, written as component 1) and the storage battery (as the cold standby component, written as component 2) form a repairable electricity-supply system. Obviously, it is reasonable to assume that the power station has use priority due to the operating cost of the power station is cheaper than the storage battery. Besides the electricity-supply system in a hospital, some similar examples can be found from wang and Zhang [15].

Nakagawa [10] assumed that both working time and repair time of the priority component follow general distributions while both working time and repair time of the non-priority component follow exponential distributions, and the repairs are perfect. Under these assumptions, they developed some interesting reliability indices for the system, using the Markov renewal theory. Brown et.al [3] studied some important properties of monotone processes and proved that alpha series processes is more appropriate to model the up times and proved that the second ordered moment does't exist for uptimes.

Further, Wang and Zhang [15] investigated a two component dissimilar-component cold standby system with different repair actions under the assumptions that the component 1 has priority in use and the successive working times component 1 form a decreasing geometric process while, the consecutive repair time forms a general repair process and component 2 obeys a minimal repair. Under these assumptions, using geometric process repair model, they consider a replacement policy N under which the system is replaced when the number of failures of component 1 reaches N . An optimal replacement policy N^* is determined such that the average cost rate (i.e. the long-run average cost per unit time) of the system is minimized. The explicit expression for the average cost rate of the system is derived and the corresponding optimal replacement policy N^* can be determined analytically or numerically. Finally, a numerical example is given to illustrate some theoretical results and the model applicability.

In this paper, we apply the α -series process repair model to a two-dissimilar component cold standby repairable system with one repairman by assuming that each component after repair is not "as good as new" and follows a α -series process repair, and component 1 has use priority and component 2 obeys minimal repair. A repair-replacement policy N based on the number of failures of component 1 under which the system is replaced when the failure number of component 1 reaches N is studied. The aim is to determine an optimal

replacement policy N^* such that the average cost rate of the system is minimized and derived an explicit expression for the average cost rate of the system. Finally, a numerical example is given to illustrate some theoretical results included the uniqueness of the optimal replacement policy N^* .

In modeling these deteriorating systems, the definitions according to Braun et al [3].are given below.

Definition 1: Given two random variables X and Y , if $P(X>t) > P(Y>t)$ for all real t , then X is called stochastically larger than Y or Y is stochastically less than X . This is denoted by $X >_{st} Y$ or $Y <_{st} X$ respectively.

$$Y_n <_{st} Y_{n+1}, n=1,2,;$$

Definition 2: Assume that $\{X_n, n=1,2,\dots\}$, is a sequence of independent non-negative random variables. If the distribution function of X_n is $F_n(t) = F(k^\alpha t)$ for some $\alpha > 0$ and all $n=1, 2, 3\dots$ then $\{X_n, n=1, 2\dots\}$ is called a α series process, α is called exponent of the process.

Obviously:

if $\alpha > 0$, then $\{X_n, n=1,2,\dots\}$ is stochastically decreasing, i.e, $X_n >_{st} X_{n+1}, n=1,2,\dots;$

if $\alpha < 0$, then $\{X_n, n=1,2,\dots\}$ is stochastically increasing, i.e., $X_n <_{st} X_{n+1}, n=1,2,;$

if $\alpha = 0$, then the α series process becomes a renewal process.

2. The Model

To study the maintenance problem for a two-dissimilar-component cold standby repairable system with different repair actions, the following assumptions are imposed.

Assumption 1. At the beginning, the two components are both new, and component 1 is in a working state while component 2 is in a cold standby state.

Assumption 2. Assume that the component 1 after repair is not “as good as new” and follow a α -series process repair. When both components are good, component 1 has use priority.

Assumption 3. The time interval between the completion of the $(n-1)^{th}$ repair and the completion of the n^{th} repair of the component 1 called n^{th} cycle (i.e., the n^{th} repair cycle) of component . Note that a component either begins to work or enters a cold standby state of the next cycle when its repair is completed. Because component 1 has use priority, the repair time of component 2 may be zero in some cycles.

Assumption 4. When component 2 fails, a minimal repair is performed that returns the component to the function state just before failure.

Assumption 5: Assume that the working time of the component 2 having the distribution function $H(Z)$ with failure $r(t)$.The repair time of component 2 is negligible.

Assumption 6. Let X_n and Y_n be respectively, the working time and the repair time of component ‘1’ in the n^{th} cycle. Then the distribution functions of X_n and Y_n are respectively

given by $F_n(x)$ and $G_n(y)$. Where
$$F_n(x) = F(k^\alpha x), \quad G_n(y) = G(k^\beta y)$$
 where $x, y \geq 0, \alpha > 0, \beta < 0$.

Assumption 7: A sequence $\{X_n, n=1,2,\dots\}$ and a sequence $\{Y_n, n=1,2,\dots\}$ and the life Z are mutually independent.

Assumption 8: The replacement policy N based on the number of failures of component 1 is used. The system is replaced by a new and identical one at the N^{th} failure of component 1, and the replacement time is negligible.

Assumption 9: The repair cost rate of component is $C_i, i=1,2$ while the working reward rate of two components is same C_w . And the replacement cost of the system is C

3. Average Cost Rate under Policy N

According to the assumptions of the model, the two components appear alternately in the system. When the failure number of component 1 reaches N, component 2 is in the cold standby state of the N^{th} cycle as the repair time of component 2 is negligible. Naturally, a reasonable replacement policy N should be that if component 2 is in the working state, it was not works until failure in the N^{th} cycle as component 1 has use priority. Thus, the renewal point under the policy N is efficiently established.

Let s_1 be the first replacement time of the system, and $s_n (n>2)$ be the time between the $(n-1)$ th replacement and the n th replacement of the system under policy N. Thus, $\{s_1, s_2, \dots\}$ forms a renewal process, and the inter arrival time between two consecutive replacements is called a renewal cycle.

Let $C(N)$ be the average cost rate of the system under policy N. According to renewal reward theorem (see, Ross [12]), then

$$C(N) = \frac{\text{The expected cost incurred in a renewal cycle}}{\text{The expected length in a renewal cycle}} \quad (3.1)$$

Because component 1 has use priority, component 1 only exist the working state. Therefore based on the renewal point under policy N we have the following length of renewal cycle. Let $W(N)$ and $R(N)$ be the total working time and total repair time of the system during a renewal cycle. That is

$$L = W(N) + R(N)$$

$$L = \sum_{k=1}^N X_k^{(i)} + \sum_{k=1}^{N-1} Y_k^{(i)} \quad (3.2)$$

Since Y_1, Y_2, \dots, Y_{N-1} are independent and identically distributed,

Let $R(N) = Y_1 + Y_2 + \dots + Y_{N-1}$ is follows $G^{(N-1)}(t)$.

Where $G^{(N-1)}(t)$ is the $(N-1)$ - fold convolution of the distribution G .

Denote $N(t)$, the failure number of component 2, in the time interval $(0,t]$ when it is operating in $(0,t]$ with the minimal repair. Then $\{N(t),t \geq 0\}$ will constitute a non-homogenous Poisson process with failure intensity $r(s)$. Thus, the mean of $N(t)$ is

$$E[N(R(N))] = \int_0^t r(s) ds \tag{3.3}$$

Because the total working time of component 2 in a renewal cycle is $R(N) = Y_1 + Y_2 + \dots + Y_{N-1}$, the failure number of component 2 in a renewal cycle is $N(R(N))$, and its expected value is

$$E[N(R(N))] = \int_0^\infty E[N(t)] dG^{(n-1)}(t) \tag{3.4}$$

$$= \int_0^\infty \left[\int_0^t r(s) ds \right] dG^{(n-1)}(t) \tag{3.5}$$

$$= \int_0^\infty r(s) \bar{G}^{(n-1)}(s) ds \tag{3.6}$$

Where $\bar{G}^{(N-1)}(s) = 1 - G^{(N-1)}(s)$ is the survival function of $R(N) = Y_1 + Y_2 + \dots + Y_{N-1}$.

Let $C(N)$ be the long-run average cost rate (ACR) of the system under policy N . According to the renewal reward theorem [see Ross(12)],

$$C(N) = \frac{C + C_1 E\left(\sum_{k=1}^{N-1} Y_k\right) + C_2 E[N(R(N))]}{E\left(\sum_{k=1}^N X_k\right) + E\left(\sum_{k=1}^{N-1} Y_k\right)} \tag{3.7}$$

Now, we evaluate the expected value of X_n and Y_n and $N(R(N))$ as follows:

According to the assumption of the model, we have

If $\{X_n, n=1,2,\dots\}$ is distributed according to an exponential failure law with distribution function $F_n(n^\alpha x)$, then we have

$$E(X_n) = \int_0^\infty x dF(n^\alpha x) \tag{3.8}$$

$$= \frac{\lambda}{k^\alpha}, \alpha > 1, \lambda > 0. \tag{3.9}$$

If $\{Y_n, n=1,2,\dots\}$ is distributed according to an exponential failure law with distribution function $F_n(n^\beta x)$, then we have

$$E(Y_n) = \int_0^{\infty} y dF(k^\beta y) \quad , \quad (3.10)$$

$$= \frac{\mu}{k^\beta}, \beta < 0, \mu > 0. \quad (3.11)$$

According to assumption (4) , the distribution function of Z is given by

$$H(z) = 1 - \exp(-z/\theta) \quad (3.12)$$

Then failure intensity is given by

$$r = \frac{1}{\theta} \quad (3.13)$$

If the life time of component 2 is exponentially distributed with constant rate r, then the failure number $\{N(t), t \geq 0\}$ of component 2 form a homogeneous Poisson process with intensity r. Thus the expected failure number of component 2 in a renewal cycle is given by

From equation (3.6) we have

$$E[N(R(N))] = \int_0^{\infty} r \bar{G}^{(n-1)}(s) ds \quad (3.14)$$

Since, Y_1, Y_2, \dots, Y_{N-1} are independent and identically exponentially distributed then the survival function

$\bar{G}^{(N-1)}(s) = 1 - G^{(N-1)}(s)$ is given by

$$P(Y_1 + Y_2 + \dots + Y_{N-1} > t) = \sum_{k=0}^{n-1} \left(\frac{\left(\frac{k^\beta t}{\mu} \right)^k e^{-\left(\frac{k^\beta t}{\mu} \right)}}{k!} \right) \quad , \quad (3.15)$$

From equation (3.14) and (3.15), we have

$$E[N(R(N))] = \int_0^{\infty} r \sum_{k=1}^{n-1} \left(\frac{\left(\frac{k^\beta t}{\mu} \right)^k e^{-\left(\frac{k^\beta t}{\mu} \right)}}{k!} \right) dt \quad , \quad (3.16)$$

From equation (3.13), we have

$$E[N(R(N))] = \int_0^{\infty} \frac{1}{\theta} \sum_{k=1}^{n-1} \left(\frac{\left(\frac{k^\beta t}{\mu} \right)^k e^{-\left(\frac{k^\beta t}{\mu} \right)}}{k!} \right) dt \quad ,$$

$$E[N(R(N))] = \frac{\mu}{\theta} \sum_{k=1}^{n-1} \frac{1}{k^\beta} \tag{3.17}$$

From equations (3.9), (3.11) and (3.17), the equation (3.7) becomes

$$C(N) = \frac{C + C_1 \sum_{k=1}^{N-1} \frac{\mu}{k^\beta} + C_2 \frac{\mu}{\theta} \sum_{k=1}^{N-1} \frac{1}{k^\beta}}{\sum_{k=1}^N \frac{\lambda}{k^\alpha} + \sum_{k=1}^{N-1} \frac{\mu}{k^\beta}} \tag{3.18}$$

4. Numerical results:

For the given hypothetical values of C=2500, C₁=60, C₂=15, α=0.05, β=-0.85, μ=10, λ=20, θ=5 using equation (3.18) the long-run average cost rate values are calculated at each failure number N.

Table: 4.1 The long average cost rate values against N

N	C(N)	N	C(N)
1	63.46473677	13	47.63506217
2	49.44175984	14	48.20435414
3	45.0110445	15	48.73888531
4	43.65348879	16	49.23993719
5	43.49271661	17	49.70938128
6	43.84064829	18	50.14932859
7	44.4071584	19	50.56193565
8	45.05900857	20	50.94929931
10	45.73191991	21	51.31340278
11	46.39431098	22	51.65609104
12	47.0309478	23	51.97906314
		24	52.283874

Grapg:4.1

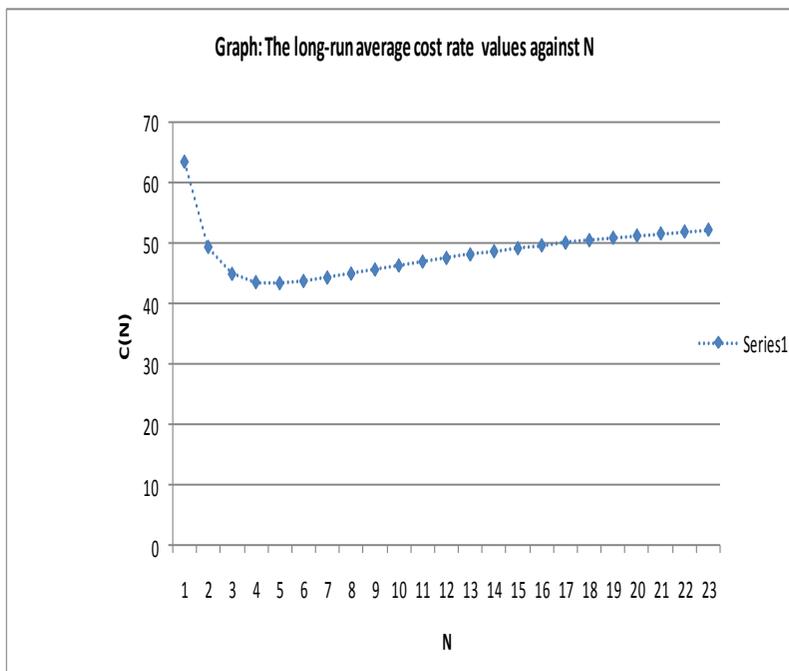
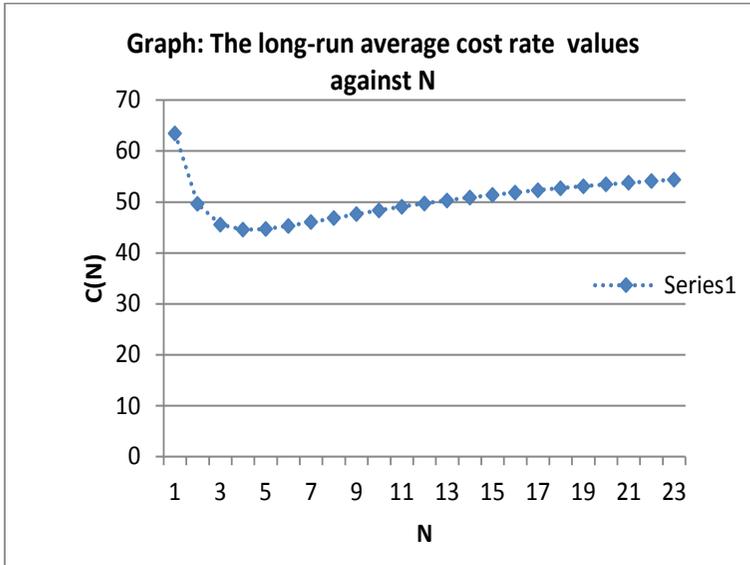


Table: 4.2 The long average cost rate values against N

For the given hypothetical values of $C=2500$, $C_1=60$, $C_2=15$, $\alpha=0.05$, $\beta=-0.95$, $\mu=10$, $\lambda=20$, $\theta=5$ using equation (3.18) the long-run average cost rate values are calculated at each failure number N.

N	C(N)	N	C(N)
1	63.46473677	14	50.87874048
2	49.6420663	15	51.39348132
3	45.57868906	16	51.86996289
4	44.57603806	17	52.31160997
5	44.71279072	18	52.72162679
6	45.29804463	19	53.10295021
7	46.04984264	20	53.45824027
8	46.84469335	21	53.78988983
10	47.62700212	22	54.10004342
11	48.37228272	23	54.39061949
12	49.07091388	24	50.87874048
13	49.72051603		

Graph:4.2



CONCLUSIONS

1. From the table 4.1 and graph 4.1, it is examined that the long-run average cost per unit time $C(5) = 43.49271661$ is minimum for the given $\beta = 0.85$ $\alpha = 0.05$. Thus, we should replace the system at the time of 5th failure.
2. From the table 4.2 and graph 4.2, it is observed that the long-run average cost per unit time $C(4) = 44.57603806$ is minimum for the given $\beta = 0.95$ $\alpha = 0.05$. We should replace the system at the time of 5th failure. Thus, from above conclusions in (1) and (2), it can be concluded that the value of β is negatively related with the long-run average cost per unit time.
3. Similar conclusions may be drawn as ' α ' decreases an increase in the number of failure, which coincides with the practical analogy and helps the decision maker for making an appropriate decision.

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