Volume : 5 | Issue : 9 | September 2016 • ISSN No 2277 - 8179 | IF : 3.508 | IC Value : 69.48

Detour D-Distance of A Graph

Mathematics

KEYWORDS :

N.Arianayagam Assistant professor, Department of Mathematics, Government college of Engineering, Tirunelveli-627007, India

J.Vijaya Xavier Parthipan Associate professor, Department of Mathematics, St.John's College, Palayamkottai-627002,India

ABSTRACT

For vertices u and v in a connected graph G, the detour distance D (u,v) is the length of the longest u- v path in G. A u – v path of length D (u,v) is called a u – v detour. It is known that the detour distance is a metric on the vertex set V (G). Chart and et al introduced the concept of detour distance by considering the length of the longest path between u and v. Kathiresan et al introduced the concept of superior distance and signal distance. In some of these distances only the length of various paths were considered. By considering the degrees of dominating set vertices present in the path and in addition, subtract the length of the path. In this article we introduced the concept of domination D – distance. We study some properties of this new distance.

Introduction

For a graph $G = (V, E)$ we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. We consider connected graphs with atleast two vertices. For basic definitions and terminologies we refer to [1, 4].

For vertices u and v in a connected graph G, the detour distance $D\ (u, v)$ is the length of the longest $u - v$ path in G. A $u - v$ path of length $D\ (u, v)$ is called a $u - v$ detour. It is known that the detour distance is a metric on the vertex set $V(G)$. The detour eccentricity $e_D(v)$ of a vertex v in G is the maximum detour distance from v to vertex of G. The detour radius, $rad_D\ G$ of G is the minimum detour eccentricity among the vertices of G, while the detour diameter, $diam_D\ G$ of G is the maximum detour eccentricity among the vertices of G. These concept were studied by chartrand et al [2].

A set $S \subseteq V\ (G)$ is called a dominating set of G if every vertex in $V\ (G) - S$ is adjacent to some vertex in S. The domination number $\gamma(G)$ of G is the minimum order of its dominating sets and any dominating set of order $\gamma(G)$ is called γ-set of G. These concept were by Kathiresan and Sumathi introduced the concept of signal distance in G. Reddy Babu et al introduced a $D-$ distance as follows : The $D-$distance $d^D\ (u, v)$ between two vertices u, v of a connected graph G is denoted as $d^D\ (u, v) = \min\{l^D\ (s)\}$ where the minimum is taken over all $u - v$ paths s in G. In other words $d^D(u,v) = min\{d(u,v) + deg(u) + deg(v) + \sum deg(w)\}$,where the sum runs over all intermediate vertices w in S and minimum is taken over all $u - v$ path in G.

The Dominating D –distance $\gamma_D{}^D(u,v)$ between two vertices u, v of a connected graph G is defined as $\gamma_D{}^D(u,v) = \{l^D(p)\}$. In otherwords $\gamma_D{}^D(u,v) = \deg(u) + \deg(v) + \sum \deg(s) - D(u,v)$ where the sums runs over all dominating vertices in the set S in a connected graph G and the path p is taken over all $u - v$ paths in G & $u, v \notin S$. The dominating D – distance number $\gamma_D{}^D(G)$ of G is $l(p) - 1 = \Delta(G)$[7].

In this article we introduce a new distance, which we call as Detour D –distance between any two vertices of a graph G, and study some of its properties. This distance is significantly different from other distances. In some of the earlier distances, only path length was considered. Here we, consider the degree of beginning and end vertices of detour $u - v$ path and also in addition to find the degree of the intermediate vertices present in the detour path, and the detour distance. While we defining its length we will get the total degree of the graph,using the concept of Detour D –distance.

2000 Mathematics subject classification:05C12.

Definition 2.1.

For a connected graph G, $u \& v \in v(u)$ then the D- length of a detour $u - v$ path p is defined as $l^D(p) = D(u,v) + \deg(u) + \deg(v) + \sum deg\ (w)$ where sum runs over all intermediate vertices of the detour $u - v$ path p.

Definition 2.2

The Detour D-distance $D^D(u,v)$ between two vertices u, v of a connected G is defined as $D^D(u,v) = l^D(p)$. In other words, $D^D(u,v) = D(u,v) + \deg(u) + \deg(v) + \sum \deg(w)$ where the sums was over all intermediate vertices w in detour $u - v$ path p in G.

Volume : 5 | Issue : 9 | September 2016 • ISSN No 2277 - 8179 | IF : 3.508 | IC Value : 69.48

The Detour D-distance $D^D(u, v)$ between two vertices u, v of a connected G is defined as $D^D(u, v) = l^D(p)$. In other words, $D^D(u, v) = D(u, v) + \deg(u) + \deg(v) + \sum \deg(w)$ where the sums was over all intermediate vertices w in detour $u - v$ path p in G.

Example 2.3

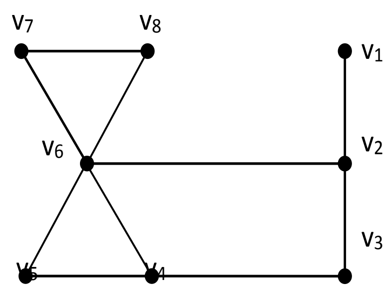

Figure 2.1

For a graph G in Figure 2.1 Let $u = v_1$, and $v = v_7$ then

$P = \{u = v_1, v_2, v_3, v_4, v_5, v_6, v_8, v_7 = v\}$ then Detour D – length of the path P is $l^D(p) = D(u, v) + \deg(u) + \deg(v) + \Sigma\deg(w)$== 7 + 1 + 2 + [3 + 2 + 3 + 2 + 5 + 2] = 27.

Therefore the Detour D – Distance $D^D(u, v) = 27$. The Detour Distance between u, v is $D(u, v) = 7$ from this $D^D(u, v) - D(u, v) = \Sigma\deg(w)$. ie 27 – 7 = 20. Since Σdeg (G) = deg (v_1) + deg (v_2) + deg (v_3) + deg (v_4) + deg (v_5) + deg (v_6) + deg (v_7) + deg (v_8) = 1 + 3 + 2 + 3 + 2 + 5 + 2 + 2 = 20.

Remark 2.4 There can be more than one detour D-Distance. Let $u = v_1$, and $v = v_8$ then $P = \{u = v_1\, v_2\, v_3\, v_4\, v_5\, v_6\, v_8 v_7 = v\}$ then Detour D – length of the path P is $l^D(p) = D(u, v) + \deg(u) + \deg(v) + \Sigma\deg(w)$== 7 + 1 + 2 + [3 + 2 + 3 + 2 + 5 + 2] = 27. So every graph have more than one $u - v$ path but the Detour D-Distance number is same for all $u - v$ path.

Remark 2.5

Observe that, for a connected graph G. We get $D^D\ (u,v) - D\ (u,v) = \Sigma\deg\ (G)$. Therefore $D^D\ (u,v) = \Sigma\deg\ (G) + D\ (u,v)$.

Example 2.6

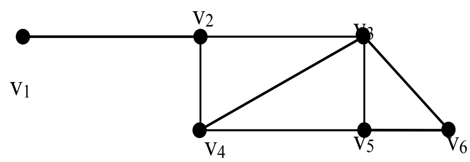

Figure 2.2

For the graph G given in Figure 2.2. By Remark 2.5 $D^D\ (u,v) = D\ (u,v) + \Sigma\deg\ (G)$.The detour $u - v$ path is $P = (u = v_1, v_2, v_3, v_4, v_5, v_6)$

$D^D\ (u,v) = D\ (u,v) + deg\ (v_1) + deg\ (v_6) + \Sigma deg\ (w)$.

$D^D\ (u,v) = 5 + 1 + 2 + 3 + 4 + 3 + 3$. $D^D\ (u,v) = 21$ and $D\ (u,v) + \Sigma deg\ (u) = 5 + 16 = 21$.From this $D^D\ (u,v) = \Sigma deg\ (u) + D\ (u,v)$.

Theorem 2.7

If G is any connected graph then the Detour D – Distance is a metric on the set of vertices of G.

Proof :

Let G be a connected graph and $u, v \in V\ (G)$. Then it is clear by definition that $D^D\ (u,v) > 0\ and\ D^D\ (u,v) = 0 \Rightarrow u = v$. Also we have $D^D\ (u,v) = D^D\ (v,u)$. Next to prove that D^D satieties the triangle inequality. Let $u, v, w \in V(G)$. Let P and Q be $u - w$ and $w - v$ Path in G respectively. Such that $D^D\ (u,w) = l^D\ (P)\ and\ D^D\ (w,v) = l^D\ (q)$. Let $R = P\ U\ Q$ be the detour $u - v$ path obtaining by joining p and q at W. Then

$D^D\ (u,w) + D^D\ (w,v) = D\ (u,w) + D\ (w,v) + \Sigma deg\ (G)$

$= D\ (u,v) + \Sigma deg\ (G)$

$= D^D (u, v).$

This the triangle inequality holds and hence D^D is a metric on the vertex set $V(G)$.

Theorem 2.8

For a connected graph G, two district vertices u, v are adjacent and $D (u, v) = 1$ its $D^D (u, v) = deg (u) + deg (v) + 1$.

Proof :

If u, v be the two vertices of G and they are adjacent & $D (u, v) = 1$ and hence. $D^D (u, v) = D (u, v) + deg (u) + deg (v) + \Sigma deg (w) = 1 + deg (u) + deg (v) + 0 = deg (u) + deg (v) + 1$.

Conversely, if $D^D (u, v) = deg (u) + deg (v) + 1$, then by definition of Detour D – distance are get $D (u, v) + \Sigma deg (w) = 1$. Hence $D (u, v) = 1$ and $\Sigma deg (w) = 0$. This implies that u and v are adjacent. Hence the theorem .

3. Real time Application of Domination D- distance and Detour D- Distance of a graph

Let v_1, v_2, v_3........ be the vertices of the graph and the connected line between the vertices are edges of the graph. The number of edges incident to the vertices are called the degree of a vertex. Highest degree of vertex is can be identified by using Domination D- distance of a graph .The edges are considered as the different roads and this application of Domination D - Distance of a graph is implemented to identify a place for providing a most populated area and the vertex which has highest degree can be identified which helps in providing a traffic police booth to regulate the traffic. Generally traffic police booth is placed in the place where we have roads in all the direction .Heavily populated road can be identified when there are number of roads intersect each other. Traffic can to be regulated by using this theorem. This application can be utilized in different areas .One major example is briefed as we are marching towards the Smart city . The same can be applied by a business man to incorporate his business by identifying the people polling places. Where the shopping complex can be constructed using this method . Same way banking sector can use this method to install its ATM machine and Airtel ,Aircel ,Bsnl ,Vodafone tower etc .The ultimate result of this method is to identify the heavily dominating point. But in this paper the lowest degree is the end vertex of the $u-v$ path .In these end vertex is providing a low populated area and the vertex which has lowest degree can be identify which helps in providing the Car show room ,pesticide shop, manure shop ,tasmac, school , colleges ,and gowdon purpose .The ultimate result of this method is to find the total number of a roads using the Detour D-Distance number minus distance divide by 2 or $[D^D\,(u,v) - D\,(u,v)] \div 2$ .And also find the total number of roads intersects or vertex using Detour D-Distance number divide by 2 and then minus 2 or $[(D^D\,(u,v) - D\,(u,v)) \div 2] - 2$.

References

1. F.Buckley and F. Harary, Distance in Graph, Addition – Wesley, reading M.A. 1990.
2. G. Chartrand, H. Escuadro and B. Zang. The Detour Distance in Graph, J. Combin-math. Combin computer., 53 (2005), 75-94.
3. G. Chartrand, N. Johns, and P. Zang, The Detour number of Graph, Utill, Math.64 (2003), 97-113.
4. Haynes TW, Hedetniemi S.T and slater P.J, Fundamentals of Domination in graphs, Marces Decker, Inc., New York 1998.
5. K.M. Kathiresan and G. Marimuthu, Superior distance in graphs, J. Combin, Com pvt., 61(2007) 73-80.
6. K.M. Kathiresan and R. Sumathi, A study on signal distance in graphs, Algebia graph theory and their applications, Norasa publishing house Pvt. Ltd., (2010) 50-54.
7. N.Arianayagam and J.John , Domination D-Distance of a graph , Global journal for research Analysis,SARA publishing Academy(2016)(Accepted).