Original Research Paper
Volume : 5 | Issue : 9 | September 2016 • ISSN No 2277 - 8179 | IF : 3.508 | IC Value : 69.48

Noise Reduction in Sparse Reconstructed Images

Engineering

KEYWORDS : Compressive sensing; GEM algorithm; Iterative Multiplier algorithm; quantization

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ABSTRACT

Compressive Sensing theory states that accurate reconstruction of the sparse signal is possible even from a sampling rate dramatically smaller than the Nyquist rate. For sparse signal reconstruction three techniques are commonly used: convex relaxation, greedy pursuit and probabilistic method. In Sparse signal reconstruction using GEM hard thresholding the method used for reconstruction is probabilistic model with ℓ0 – norm and an algorithm is developed known as GEM algorithm. The drawback of GEM method is that quality of reconstructed image is inadequate for further processing . To improve the quality of image, Iterative Multiplier noise reduction algorithm is tested on GEM reconstructed sparse image. Numerical evaluation results are reported.

I. INTRODUCTION

The Shannon/Nyquist sampling theorem states that for the accurate reconstruction of the signal one must choose sampling frequency atleast twice the modulating frequency. In some application like digital images and in video cameras the nyquist rate is very high. When the nyquist rate increases the number of samples per seconds also increases which will result in the compression of the samples instead of storage and transmission. In some other application including imaging systems and high speed analog-to-digital converters, increasing the sampling rate is expensive. i.e, it is computationally complex. So to overcome these disadvantages a new method is introduced known as compressive sensing. In Compressive sensing accurate reconstruction of the signal is possible even if the sampling frequency is less than twice the maximum frequency. The signals can be compressed using non adaptive linear projection methods and reconstructed using an optimization technique. The optimization technique include convex relaxation, greedy pursuit and probabilistic models [1],[2],[3]. The most important characteristics of modern systems is digital storage and processing for that

quantization is necessary. So the quantization effect take part in compressive sensing. Another technique for the reconstruction of the sparse signal is by using GEM hard thresholding [4]. The sparse signal and the noise variance are found out by the GEM algorithm and the signal is reconstructed by probabilistic model with ℓ_0 - norm**.** The reconstructed images by using GEM algorithm has low quality i.e, the PSNR values are low for higher quantization bins. The aim is to increase the PSNR values for higher quantization bins. For this purpose, noise reduction algorithms [5] are tested on GEM reconstructed images. In this paper, Iterative Multiplier algorithm is tested on GEM reconstructed sparse image. Rest of the article is organised as follows: section II describes the GEM algorithm and Iterative Multiplier algorithm and the section III describes Methodology. The section IV describes about Results and discussions.

II. BACKGROUND THEORY

Compressive sensing is a signal processing technique which is used for acquiring and reconstructing a signal. In this, first the signal is compressed by using non adaptive linear projection methods. The reconstruction of the sparse signal is done by optimization technique. The optimization technique include ℓ_0 - minimization, ℓ_1 -minimization and the ℓ_2 - minimization. By ℓ_2-minimization, get only the signal energy instead of sparse solution. ℓ_0 –minimization get the sparse signal solution and by ℓ_1 –minimization recover the signal exactly.

A. The GEM Algorithm

Model a N X 1 real-valued measurement vector y as

$$Y=Ax+b \qquad (1)$$

Where $y = [y_1, y_2 \ldots\ldots\ldots y_N]^T$, A is the N X m sensing matrix, x is the unknown sparse vector of

dimension m X 1 and it contain r non zero elements($r \le m$), b is the additive white Gaussian noise with zero mean and covariance matrix $\sigma^2 I_N$. Here the noise variance is assumed to be unknown. In this model there are two unknown parameters x and σ^2. Then the set of unknown parameters can be written as

$$\Theta = (x,\sigma^2) \in \Theta_r \qquad (2)$$

Where Θ_r is the parameter space. Parameter space means it is the set of all possible combinations of values for all the different parameters in the model. This can be written as

$$\Theta_r=S_r x(0,+\infty) \qquad (3)$$

Where S_r is the sparse signal parameter space and r is the sparsity level. In this assuming that the sparsity level is known and the noise variance σ^2 is positive. Then the elements of y are quantized into codeword b. so b can be represented as $[b_1, b_2 \ldots\ldots\ldots b_N]^T$, where b_i indicate the quantization interval or bin

$$y_i \in D(b_i) = [l(b_i),u(b_i)) = [l_i,u_i) , \; l_i< u_i, \; i=1,2\ldots\ldots\ldots\ldots N \qquad (4)$$

Where l_i and u_i are the lower and upper boundaries of the quantization interval. Our aim is to estimate the parameters θ from the quantized data b and the unobserved or the missing data y. For finding the missing data the concept of GEM algorithm is developed [4]. First find out the joint distribution of the observed data b and the missing data y given the unknown parameters θ by mathematical equations.

Then the marginal log likelihood function of θ is obtained by integrating y from the joint distribution of b and y.

$$L(\theta)=\ln (P_{b/\theta}(b/\theta)) \qquad (5)$$

For the computation of L(θ) the noise variance σ^2 is positive and set the parameter space accordingly in (3). The maximum likelihood (ML) estimate of θ is found out by maximizing L(θ). Then the exact ML estimate requires a combinatorial search and it is difficult task . To avoid these a new algorithm is developed known as GEM. Generalised expectation maximization (GEM) algorithm is derived for estimating the parameters θ. In this there are two steps: expectation(E) - step and maximization(M) -step. In the expectation step the missing datas are estimated and in maximization step the likelihood function is increased under the assumption that the missing data is known. In this assuming that the parameter estimate $\theta^{(p)} =(x^{(p)},(\sigma^2)^{(p)})$ is known , where p indicate the iteration index. Then the E-step can be found out by taking the expected complete data log likelihood function of (5).

$$Q(\theta/\theta^{(p)}) = E_{y/b,\theta}[\ln P_{y,b/\theta}(y, b/\theta)/b, \theta^{(p)}]$$

$$= -1/2\, N \ln(2\pi\sigma^2) - E_{y/b,\theta}[(y-Hs)^T(y-Hs)/b, \theta^{(p)}]/(2\sigma^2) \quad (6)$$

Expectation step which reduces to evaluate Bayesian minimum mean square error and variance using the mean and variance of the truncated pdf [6]. similarly the M-step increases the expected complete data log likelihood function. Here finding the new parameter estimate by the equation

$$x^{(p+1)} = Tr(s^{(p)} + (1/c^2)H^T(\hat{y}^{(p)} - Hs)) \quad (7)$$

c in (7)is the step size coefficient which should satisfy the inequality

$$c \geq \rho H;$$

where ρH denotes the largest singular value of H and Tr in (7) is the hard thresholding operator. So we get the new parameter $\theta^{(p+1)} = (s^{(p+1)}, (\sigma^2)^{(p+1)})$. Then the iteration between the E and M is repeated until two consecutive sparse signal estimates x^p and $x^{(p+1)}$ do not differ significantly. The signal is reconstructed by ℓ_0 - norm. ℓ_0 norm that counts the number of non-zero entries in s. This optimization can recover a sparse signal exactly.

B. Iterative Multiplier Algorithm

To remove noise in the GEM algorithm Iterative Multiplier algorithm is developed [5]. Consider z(x,y) is an image containing an unknown additive noise η . Then it can be written as

$$Z(x,y) = u(x,y) + \eta(x,y) \quad (8)$$

Our aim is to find u^* of the non linear equation with different lagrange multiplier λ, which will satisty the constrained condition. The non linear equation is

$$L_\lambda(u) = -\nabla\left(\frac{\nabla u}{\sqrt{\nabla u^2 + \epsilon}}\right) + \lambda(u - z) = 0 \quad (9)$$

and the constrained condition is given by

$$F(u) = \frac{1}{2}(\|u - z\|^2 - \sigma^2) = 0 \quad (10)$$

Where σ is the variance between the noisy image and the orginal image and λ is the regularization parameter.

Algorithm : Consider $\lambda_0 = 0$ and assume $\lambda_l = M > 0$. Let $F_l = F(u\lambda_0)$, $F_r = F(u\lambda_1)$, $\lambda_l = \lambda_0$ $\lambda_r = \lambda_1$ and k=2.

Step 1: Obtain λ_k by the equation

$$\lambda_k = \frac{\lambda_1|(F_r)| + \lambda_r|(F_1)|}{|(F_1) + (F_r)|} \quad (11)$$

Step 2 : Set $F_l' = F_l$ and $F_r' = F_r$, $\lambda_l' = \lambda_l$ and $\lambda_r' = \lambda_r$ .Then solve the linear equation (9) to obtain the unique solution $u\lambda_k$.

Case I : If $\|F(u\lambda_k)\| < \eta_0$,then $u\lambda_k$ solves the constrained problem (10) and get solution as $u^* = u\lambda_k$.

Case II: If $\|F(u\lambda_k)\| \leq -\eta_0$,then set $F_r = F(u\lambda_k)$, $\lambda_r = \lambda_k$.

Case III: If $\|F(u\lambda_k)\| >- \eta_0$,then set $F_l = F(u\lambda_k)$, $\lambda_l = \lambda_k$.

Step 3: Check $|\lambda_k - \lambda_{k-1}| < \eta_0$. Then we get the solution u*. otherwise repeat the step with k=k+1.

III. METHODOLOGY

Two dimensional signals were selected to test the algorithm. First an image of size 256^2 is taken as input and then the measurements are divided into B (B=3,4,8,16)bins. The two unknown parameters i.e; sparse signal and the noise variance is found out by the GEM algorithm and the signal is reconstructed using ℓ_0 - norm technique. Then the PSNR values for each bin is calculated. Normally, the PSNR values are low for the reconstructed images by GEM algorithm. The aim here is to increase the PSNR values by reducing the noise in the GEM algorithm . So here an Iterative Multiplier algorithm is tested on GEM reconstructed images. The GEM output is given as noisy image to the Iterative Multiplier algorithm and the noise is reduced in GEM algorithm.

IV. RESULT AND DISCUSSIONS

Here the Lena image with size $m=256^2$ are considered as example. After the reconstruction PSNR values are calculated. Figure 1 shows the reconstructed image by GEM for quantization bin B=16. In this the reconstructed image have a PSNR value 28.2604 dB.

Fig 1: Reconstructed image for B=16

Then the PSNR values are low for the reconstructed images by GEM algorithm. So Iterative Multiplier algorithm is developed. This algorithm along with GEM algorithm has better performance. Figure 2 shows the reconstructed image with Iterative Multiplier algorithm for the quantization bin B=16. In this the reconstructed image have a PSNR value 77.5532 dB.

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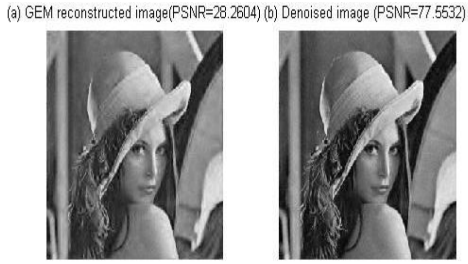

Fig 2: Reconstructed image for B=16

Figure 3 shows the reconstructed image with Iterative Multiplier algorithm for the quantization bin B=8. In this the reconstructed image have a PSNR value 76.4434 dB. Fig 4 shows the reconstructed image with Iterative Multiplier algorithm for the quantization bin B=4. In this the reconstructed image have a PSNR value 76.6415 dB.

Quantization bin (B)	PSNR for Cameraman image in dB	PSNR for lena image in dB	PSNR for bird image in dB
3	72.3984	72.7861	76.5265
4	73.8817	74.6415	76.7833
8	75.8713	76.4434	78.0281
16	77.0343	77.5532	79.3768

Fig 3: Reconstructed image for B=8

Fig 4: Reconstructed image for B=4

Figure 5 shows the reconstructed image with Iterative Multiplier algorithm for the quantization bin B=3. In this the reconstructed image have a PSNR value 72.7861dB.

Fig 5: Reconstructed image for B=3

Comparison of GEM and GEM with Iterative Multiplier algorithm for different Quantization bin is shown in Figure 6. From figure it is clear that when the Quantization bin increases the PSNR value also increases for both methods and it is also noted that the GEM with noise reduction algorithm has a better performance than GEM. The PSNR value maximum obtained by using the noise reduction algorithm is 77.5532 dB.

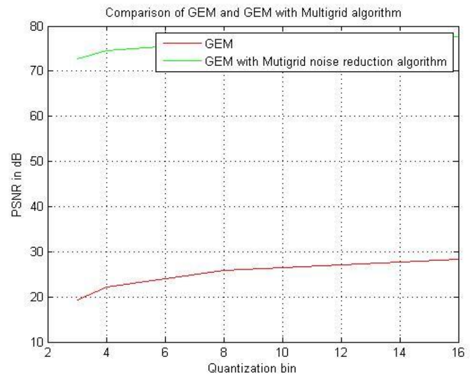

Fig 6. Comparison of GEM and GEM with Multigrid algorithm

TABLE I: PSNR values for different Images

TABLE I shows the PSNR values for Cameraman, Lena and Bird images. From the table it is clear that the Bird image have high PSNR value than other two images and when the quantization bin increases the PSNR value also increases.

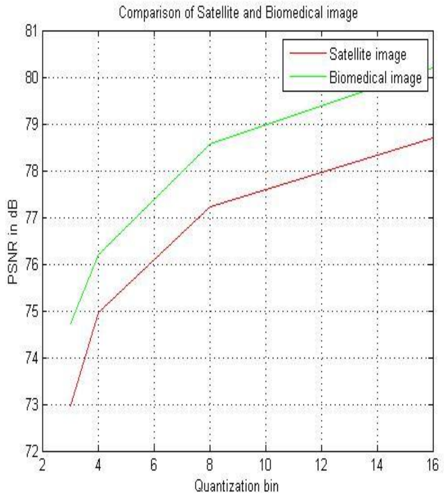

Fig 7. Comparison of Satellite and Biomedical image

Comparison of Satellite and Biomedical image for different Quantization bin is shown in Figure 7. From figure it is clear that when the Quantization bin increases the PSNR value also increases for both images and it is also noted that the Biomedical image has a better performance. The PSNR value maximum obtained by using the Biomedical image is 80.2086 dB.

V. CONCLUSIONS

In this paper, Iterative Multiplier algorithm is incorporated with the GEM algorithm for low noise reconstruction of sparse images . The numerical simulations shows that this method has better reconstruction performance than the sparse GEM algorithm. When the quantization bin increases the PSNR values are also increased upto 77.5532dB. Multiple noise reduction algorithms for better sparse reconstruction may be tested and applied.

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