VOLUME-6 | ISSUE-4 | APRIL - 2017 • ISSN No 2277 - 8179 | IF : 4.176 | IC Value : 78.46

OPTIMIZATION OF STOCHASTIC BI-CRITERIA TIME MINIMIZATION TRANSPORTATION PROBLEM

Science

KEYWORDS: Stochastic Programming, Bi-level Programming, Stochastic Transportation Problem, Exponential Distributed Demand.

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ABSTRACT

This paper provides a solution procedure to Bi-level Stochastic Transportation Problem (BSTP) under exponential distributed random variables. In this model, we suppose that the superior and the Assistant operate two separate groups of plants is a reputed distribution ship from the executive, who moves first, determines products shipped to customers and then, the assistant decides his own quantities rationally. Here both the superior's and the assistant's objective is to minimize the sum of the corresponding total transportation costs and the total expected shortage cost. Our proposed approach transformed into a single level non-linear programming by using its Karush – Kuhn – Tucker conditions. At the end a relevant numerical example is presented to illustrate the model.

1. INTRODUCTION

A Transportation problem refers to a class of Linear Programming Problems that involves selection most economical shipping routes for transfer of a uniform commodity from a number of destinations. A stochastic transportation model in which the constraints are stochastic in nature and the cost coefficients are multi-choice type is considered. In order to capture the impact of uncertainty the original stochastic programming problem is usually transformed into a non-linear deterministic equivalent problem by using probabilistic programming or two stage programming with recourse. Then the standard solution techniques for non-linear programming problems can be applied. To estimate unpredictable or uncertain problem parameters each source of randomness is necessarily represented by a probability distribution. The exponential distribution is usually to represent the inter arrival times of customers to a system (time between two independent events) that occurs at a constant rate and the time to the failure of a piece of equipment. In this paper, we consider a bi-level structured transportation planning type of problem involving demand uncertainty.

Several methods have been suggested to solve stochastic optimization problems with equilibrium constraints such as Smoothing implicit programming approach, Smoothing penalty method, Regularization method and Simple average approximation. Akdemir and Tiryaki(2011) proposed a bi-level stochastic transportation model for discrete customers demand cases. Katagiri et.al (2007) considered a hierarchical decision problem with two non-cooperative decision makers by constructing two level expectation optimization and two level variance minimization models.

In this paper, we arrange as follows: First, we present some preliminary essential concepts. Next, a descriptions of Karush – Kuhn – Tucker (KKT) conditions are provided. Finally, conclusions are given regarding the model.

2. BASIC CONCEPTS

2.1. PARAMETERS

Let N_i	Capacity of plant (i =1,2,......m)
D_j	Stochastic demand of customer (j = 1,2,.......n)
P_j	Shortage cost / unit (Penalty rate)
C_{ij}	Cost of transportation / unit from plant i
H_j	Holding cost / unit (Penalty rate for each unit in excess of quantity demanded) at customer zone j.
X_{ij}	Quantity shipped from plant i to customer j
Y_j	Total quantity shipped to customer j (both X_{ij}, Y_j are decision variables)
$\phi_j(t)$	Probability density function of customer demand j
$F_j(t)$	Cumulative density function of customer demand j

Where $\phi_j(t)$ and $F_j(t)$ are functions.

2.2 BI-LEVEL PROGRAMMING

Let the set of decision variables is partitioned between two vectors $\overline{X_1}$ and $\overline{X_2}$ The first level decision maker (Executive) controls over the vector $\overline{X_1}$ and second maker (Assistant) controls over the vector $\overline{X_2}$

3. MATHEMATICAL FORMULATION

3.1 Stochastic Transportation problem is formulated as,

$$\text{Min} \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij} + \sum_{j=1}^{n} H_j \int_0^{Y_j} (Y_j - t)\phi_j(t)dt + \sum_{j=1}^{n} P_i \int_{Y_j}^{x} (t - Y_j)\phi_j(t)dt \longrightarrow ①$$

Subject to

$$\sum_{j=1}^{n} X_{ij} \le N_i,\ i = 1,2,\ldots m$$

$$\sum_{i=1}^{m} X_{ij} \le Y_j,\ j = 1,2,\ldots n,$$

$$X_{ij} \ge 0, \text{ for all i,j}$$

Where the objective function of the problem is derived as,

$$\text{Min} \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij} + \sum_{j=1}^{n}\left(\{H_j + P_j\}\int_0^{Y_j} F_j(t)dt - P_jY_j \right)$$

by neglecting the constant term $\sum_{j=1}^{n} P_j E(D_j)$

3.2 BI-LEVEL TRANSPORTATION PROBLEM

The Bi-level programming problem can be formulated as,

$$\underset{X_j}{Min} Z(\overline{X_1}, \overline{X_2})$$

Subject to

$$Z_1(\overline{X_1}, \overline{X_2}) \le 0$$

Where $\overline{X_2}$ solves

$$\begin{cases} \underset{X_i}{Min} Z(x_1', x_2') \\ \text{Subject to } Z_1(\overrightarrow{x_1}, \overrightarrow{x_2}) \le 0 \end{cases} \longrightarrow ②$$

Where upper level variables $\overline{X_1} \in \Re^p$, lower level variables $\overline{X_2} \in \Re^r$ and the upper level objective function $z : \Re^p \text{x} \Re^r \to \Re_p$ lower level objective function $z : \Re^p \text{x} \Re^r \to \Re_p$ and the upper level constraints $Z_1 : \Re^p \text{x} \Re^r \to \Re^r$ the lower level constraints $Z_1 : \Re^p \text{x} \Re^r \to \Re^r$.

The above problem ② transformed into the equivalent single level program of the bi-level programming problem follows as,

$$\underset{x_1, x_2, \mu}{Min} Z(x_1', x_2')$$

Subject to

$$Z_1(\overrightarrow{x_1}, \overrightarrow{x_2}) \le 0$$

$$Z(x_1, x_2) \le 0$$

$$\nabla x_2\, Z(x_1, x_2) + \mu^T \nabla x_2 Z_1(x_1, x_2) = 0$$

$$\mu_i Z_i(x_1, x_2) = 0,$$

$$\mu_i \ge 0,\ i = 1,2,\ldots s$$

Where $\mu \in \Re^s$ is the vector of lagrangian multipliers.

The KKT conditions are necessary optimality conditions for the second level problem. The KKT conditions are also sufficient, if the second level problem is a convex optimization problem in variables $x_2 \in \mathfrak{R}^r$ for fixed parameters $\vec{x_1} \in \mathfrak{R}^p$. Hence any local minimum will be global minimum for the second level.

In branch and bound algorithm the complementary constraints are removed to construct the relaxed program. Supposing that the solution of the relaxed program does not satisfy some complementary constraints.

$\mu_i Z_i(x_1, x_2) = 0$, branching is performed by separating two sub problems one with $\mu_i = 0$ as an additional constraint and the other with the constraint $Z_i(x_1, x_2) = 0$ selecting i for which $\mu_i Z_i(x_1, x_2) = 0$ is the largest. Branching is continuing until all complementary constraints are satisfied or an infeasible solution is obtained.

4. PROBLEM DESCRIPTION

4.1 ASSUMPTIONS

1. The set of plants is partitioned into two sets L_1 and L_2
L_1 is the set of level 1 plants which are by the superior. L_2 is the set of level 2 plants which are operated by the assistant.

2. The senior control variables $x_{ij}, i \in L_1$ and the assistant control variables $x_{ij}, i \in L_2$, for j = 1,2 ....n.

Where $X_1 = (x_{ij})_{i \in L_1}, X_2 = (x_{ij})_{i \in L_2}$

3. Customer demand amounts are stochastic variables with known continuous distribution functions.

In this paper, it is assumed that demands are exponentially

distributed random variables with mean $\frac{1}{\lambda_j}$

That is, the probability density functions are choosen to be the form

$\phi_j(t) = \lambda_j e^{-\lambda_j t}$, for j= 1.2 ....n

4. The assistant determines his quantities shipped to customers offer the superior does. Each decision maker has to describe before demands are realised.

4.2 THE PROBLEM FORMULATION

$$\underset{X_1}{Min} \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij} + \sum_{j=1}^{n} \left[H_j \int_0^{Y_j} (Y_j - t) \phi_j(t) dt \right]$$

Subject to
$$\sum_{j=1}^{n} X_{ij} \le N_i, i \in L_1$$
$$X_{ij} \ge 0, i \in L_1, j = 1,2,...n$$

Where X_2 solves

$$\underset{X_2}{Min} \sum_{j=1}^{n} \sum_{i \in L_2} C_{ij} X_{ij} + \sum_{j=1}^{n} \left[P_j \int_{Y_j}^{x} (t - Y_j) \phi_j(t) dt \right]$$

Subject to
$$\sum_{j=1}^{n} X_{ij} \le N_i, i \in L_2$$
$$\sum_{i=1}^{m} X_{ij} = Y_j, j = 1,2,.......n$$
$$X_{ij} \ge 0, i \in L_2, j = 1,2,...n$$

4.3 BI-LEVEL STOCHASTIC TRANSPORTATION PROBLEM UNDER KKT CONDITION

The objective functions are obtained by calculating integrals in model ①

$$\underset{X_1}{Min} Z(X_1, X_2) = \sum_{j=1}^{n} \sum_{i=1}^{m} C_{ij} X_{ij} + \sum_{j=1}^{n} \left[H_j Y_j + \frac{H_j}{\lambda_j} e^{-\lambda_j Y_j} \right]$$

and
$$\underset{X_2}{Min} Z(x_1, x_2) = \sum_{j=1}^{n} \sum_{i=1}^{m} C_{ij} X_{ij} + \sum_{j=1}^{n} \frac{P_j}{\lambda_j} e^{-\lambda_j Y_j}$$

Objective function of the second level problem Z is a convex function in X_2 with partial derivatives,

$$\frac{\partial Z}{\partial x_{ij}} = C_{ij} - P_j e^{-(\lambda_j \sum_{k=1}^{m} X_{kj})}$$

and
$$\frac{\partial^2 Z}{\partial x_{ab} \partial x_{ij}} = \begin{cases} P_j \lambda_j e^{-(\lambda_j \sum_{k=1}^{m} X_{kj})}, & \text{if j=b} \\ 0, & \text{if j≠b} \end{cases}$$

The Lagrangian function of the second level problem is

$$L(x_1, x_2, u, v) = \sum_{j=1}^{n} \sum_{i \in L_2} C_{ij} X_{ij} + \sum_{j=1}^{n} \frac{P_j}{\lambda_j} e^{-(\lambda_j \sum_{k=1}^{m} X_{kj})} + \sum_{i \in L_2} u_i (\sum_{j=1}^{n} X_{ij} - N_i) - \sum_{j=1}^{n} \sum_{i \in L_2} v_{ij} X_{ij}$$

Where the variables $u_i, v_{ij}, i \in L_2, j = 1,2,......n$ are the lagrangian multipliers.

The KKT conditions for the second level problem becomes,

$$u_i, v_{ij} \ge 0, i \in L_2, \text{ j= 1,2,...............n}$$
$$\frac{\partial L}{\partial x_{ij}} = C_{ij} - P_j e^{-(\lambda_j \sum_{k=1}^{m} X_{kj})} + u_i - v_{ij} = 0, i \in L_2, \text{ j=1,2.................n}$$
$$u_i (\sum_{j=1}^{n} X_{ij} - N_i) = 0, i \in L_2$$
$$v_{ij} X_{ij} = 0, i \in L_2, \text{ j= 1,2...........................n}$$
$$\sum_{j=1}^{n} X_{ij} \le N_i, i \in L_2$$
$$X_{ij} \ge 0, i \in L_2, j = 1,2,...n$$

Variables v_{ij} can be eliminated so, the equivalent single level program ② is derived as,

$$\underset{X_1, X_2}{Min} Z(x_1, x_2) = \sum_{j=1}^{n} \sum_{i=1}^{m} C_{ij} X_{ij} + \sum_{j=1}^{n} \left[H_j \sum_{i=1}^{m} X_{ij} + \frac{H_j}{\lambda_j} e^{-\left(\lambda_j \sum_{i=1}^{m} X_{ij}\right)} \right]$$

Subject to
$$\sum_{j=1}^{n} X_{ij} \le N_i, \text{ for all i}$$
$$C_{ij} - P_j e^{-(\lambda_j \sum_{k=1}^{m} X_{kj})} + u_i \ge 0, i \in L_2, \forall j$$
$$x_{ij} \left[C_{ij} - P_j e^{-\left(\lambda_j \sum_{k=1}^{m} X_{kj}\right)} + u_i \right] = 0, i \in L_2, \forall j$$
$$u_i \left(\sum_{j=1}^{n} X_{ij} - N_i \right) = 0, i \in L_2$$
$$u_i \ge 0, i \in L_2$$
$$X_{ij} \ge 0, \text{ for all i,j}$$

5. NUMERICAL EXAMPLE

A fertilizer company has three plants and four distribution centres. Production capacity of those fertilizers are 150,200 and 100 units of products respectively. The customer demand varies from centre to centre, the demand of customers are exponentially distributed with

$$\lambda = \begin{pmatrix} 0.012 \\ 0.007 \\ 0.008 \\ 0.006 \end{pmatrix}$$

Where the parameters λ_i are chosen from the interval $[0.005, 0.02]$ which yields expected demands in the interval $[50, 200]$

Transportation, holding and shortage costs are given by

$$c = \begin{pmatrix} 8 & 2 & 5 & 4 \\ 2 & 4 & 6 & 7 \\ 6 & 5 & 3 & 4 \end{pmatrix}, \; h = - \begin{pmatrix} 16 \\ 18 \\ -5 \\ -6 \end{pmatrix} \text{ and } p_{sc} = \begin{pmatrix} 60 \\ 28 \\ 20 \\ 30 \end{pmatrix}$$

By using KKT conditions, reformulation of bi-level problem becomes,

$$\left.\begin{array}{ll}
 & Min\, z(x_1, x_2) = -16x_{11} - 18x_{12} + 5x_{13} + 6x_{14} - \\
 & 16x_{21} - 18x_{22} + 5x_{23} + 6x_{24} - \\
 & 10x_{31} - 13x_{32} + 8x_{33} + 10x_{34} - \\
 & 1333.33^{-0.012(x_{11}+x_{21}+x_{31})} - \\
 & 2571.42e^{-0.007(x_{12}+x_{22}+x_{32})} + \\
 & 625e^{-0.008(x_{13}+x_{23}+x_{33})} + \\
 & 1000e^{-0.006(x_{14}+x_{24}+x_{34})} \\
\text{Subject to} & x_{11} + x_{12} + x_{13} + x_{14} \le 150 \\
 & x_{21} + x_{22} + x_{23} + x_{24} \le 200 \\
 & x_{31} + x_{32} + x_{33} + x_{34} \le 250 \\
 & 8 - 60e^{-0.012(x_{11}+x_{21}+x_{31})+u_1} \ge 0 \\
 & 2 - 28e^{-0.007(x_{12}+x_{22}+x_{32})+u_1} \ge 0 \\
 & 5 - 20e^{-0.008(x_{13}+x_{23}+x_{33})+u_1} \ge 0 \\
 & 4 - 30e^{-0.006(x_{14}+x_{24}+x_{34})+u_1} \ge 0 \\
 & 2 - 60e^{-0.012(x_{11}+x_{21}+x_{31})+u_2} \ge 0 \\
 & 4 - 28e^{-0.007(x_{12}+x_{22}+x_{32})+u_2} \ge 0 \\
 & 6 - 20e^{-0.008(x_{13}+x_{23}+x_{33})+u_2} \ge 0 \\
 & 7 - 30e^{-0.006(x_{14}+x_{24}+x_{34})+u_2} \ge 0 \\
 & x_{11}\left(8 - 60e^{-0.012(x_{11}+x_{21}+x_{31})+u_1}\right) = 0 \\
 & x_{12}\left(2 - 28e^{-0.007(x_{12}+x_{22}+x_{32})+u_1}\right) = 0 \\
 & x_{13}\left(5 - 20e^{-0.008(x_{13}+x_{23}+x_{33})+u_1}\right) = 0 \\
 & x_{14}\left(4 - 30e^{-0.006(x_{14}+x_{24}+x_{34})+u_1}\right) = 0 \\
 & x_{21}\left(2 - 60e^{-0.012(x_{11}+x_{21}+x_{31})+u_2}\right) = 0 \\
 & x_{22}\left(4 - 28e^{-0.007(x_{12}+x_{22}+x_{32})+u_2}\right) = 0 \\
 & x_{23}\left(6 - 20e^{-0.008(x_{13}+x_{23}+x_{33})+u_2}\right) = 0 \\
 & x_{24}\left(7 - 30e^{-0.006(x_{14}+x_{24}+x_{34})+u_2}\right) = 0 \\
 & u_1(x_{11} + x_{12} + x_{13} + x_{14} - 150) = 0 \\
 & u_2(x_{21} + x_{22} + x_{23} + x_{24} - 200) = 0 \\
 & u_1, u_2 \ge 0
\end{array}\right\} \longrightarrow ③$$

By using the algorithm of Branch and Bound method is applied to the above problem ③ and obtain the results are given by in Table –I.

Table – I - Non-Cooperative Solution

The optimal solution are
$x_1^* = \left(x_{3j}^*\right) = \begin{pmatrix} 0 & 0 & 46.33 & 53.67 \end{pmatrix}$
$x_2^* = \begin{pmatrix} x_{1j}^* \\ x_{2j}^* \end{pmatrix}$
$= \begin{pmatrix} 0 & 74.32 & 0 & 75.69 \\ 150.95 & 49.05 & 0 & 0 \end{pmatrix}$

Hence,

$$u_1^* = 9.80, u_2^* = 7.80, Z\left(x_1^*, x_2^*\right) = -3684.93$$

CONCLUSION

In this paper we proposed BSTP and its KKT conditions. The superior tries to optimize in Total transfer cost + Total holding (Shortage) cost. Assuming that, the customer demands are exponentially distributed random variables. Here we have applied the bi-level non-linear programming problem by using KKT conditions.

We convert the bi-level problem equivalent to single level problem involves complementary constraints which are obtained from Branch and Bound algorithm.

The number of constraints is 35 and 59 sub problems are solved to obtain non cooperative solutions.

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