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SKOLEM MEAN LABELING OF FOUR STAR GRAPHS $K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 + a_2 + a_3 + 2 \le b \le a_1 + a_2 + a_3 + 3$

Mathematics

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ABSTRACT

A graph G = (V, E) with p vertices and q edges is said to be a skolem mean graph if there exists a function f from the vertex set of G to{1, 2, . . . , p} such that the induced map f* from the edge set of G to {2, 3, . . . , p} defined by $f^*(e = uv) = \frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd, then the resulting edges get distinct labels from the set {2, 3, . . . , p}. In this paper, we prove that four star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \le a_2 \le a_3$ is a skolem mean graph if $|b - a_1 - a_2 - a_3| \le 3$.

KEYWORDS:

Skolem mean graph, skolem mean labeling, star graphs

Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [3]. The symbols V(G) and E(G) will denote the vertex set and edge set of the graph G. A graph with p vertices and q edges is called a (p, q) graph. In this paper, we prove that four star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \le a_2 \le a_3$ is a skolem mean graph if $a_1 + a_2 + a_3 + 2 \le b \le a_1 + a_2 + a_3 + 3$.

1. Skolem mean labeling

Definition 1.1 A graph G is a non empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. The vertex set and the edge set of G are denoted by V(G) and E(G) respectively. $|V(G)| = q$ is called the size of G, we say that u and v are adjacent and that u and v are incident with e.

Definition 1.2 A vertex labelling of a graph G is an assignment of labels to the vertices of G that induces for each edge xy a label depending on the vertex labels f(x) and f(y). Similarly, an edge labelling of a graph G is an assignment of labels to the edges of G that induces for each vertex v a label depending on the edge labels incident on it. Total labelling involves a function from the vertices and edges to some set of labels.

Definition 1.3 A graph G with p vertices and q edges is called a mean graph if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 0, 1, 2, …q in such a way that when each edge e = uv is labeled with $\frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd, then the resulting edge labels are distinct. The labeling f is called a mean labeling of G.

Definition 1.4 A graph G = (V,E) with p vertices and q edges is said to be skolem mean if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 1, 2, …q in such a way that when each edge e = uv is labeled with $\frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd, then the resulting edges get distinct labels from 2, 3, . . . , p. f is called a skolem mean labeling of G. A graph G = (V, E) with p vertices and q edges is said to be a **skolem mean graph** if there exists a function f from the vertex set of G to {1, 2, . . . , p} such that the induced map f* from the edge set of G to {2, 3, . . . , p} defined by

$$f^*(e=uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

the resulting edges get distinct labels from the set {2, 3, . . . , p}.

Theorem2.1 The four star $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \le a_2 \le a_3$ is a skolem mean graph if $a_1 + a_2 + a_3 + 2 \le b \le a_1 + a_2 + a_3 + 3$

Proof: Let $A_i = \sum_{k=1}^{i} a_k$. That is, $A_1 = a_1$; $A_2 = a_1 + a_2$ and $A_3 = a_1 + a_2 + a_3$.

Consider the graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$. Let $V = \bigcup_{k=1}^{4} V_k$ be the vertex set of G where $V_k = \{v_{k,i} : 0 \le i \le a_k\}$ for $1 \le k \le 3$ and $V_4 = \{v_{4,i} : 0 \le i \le b\}$. Let

$E = \bigcup_{k=1}^{4} E_k$ be the edge set of G where $E_k = \{v_{k,0}v_{k,i} : 0 \le i \le a_k\}$ for $1 \le k \le 3$ and

$E_4 = \{v_{4,0}v_{4,i} : 0 \le i \le b\}$.

The condition $a_1 + a_2 + a_3 + 2 \le b \le a_1 + a_2 + a_3 + 3 \Rightarrow A_3 + 2 \le b \le A_3 + 3$

That is, there are two cases viz. $b = A_3 + 3$ and $b = A_3 + 2$.

Let us prove in each of the two cases the graph G is a skolem mean graph.

Case 1: Let $b = A_3 + 3$. G has $A_3 + b + 4 = 2A_3 + 7$ vertices and $A_3 + b = 2A_3 + 3$ edges.

The vertex labelling $f : V \to \{1, 2, 3, \ldots, A_3 + b + 4 = 2A_3 + 7\}$ is defined as follows:

$f(v_{1,0}) = 1$; $f(v_{2,0}) = 2$; $f(v_{3,0}) = 3$;

$f(v_{4,0}) = A_3 + b + 3 = 2A_3 + 6$

$f(v_{1,i}) = 2i + 3 \qquad 1 \le i \le a_1$

$f(v_{2,i}) = 2A_1 + 2i + 3 \qquad 1 \le i \le a_2$

$$f(v_{3,i}) = 2A_2 + 2i + 3 \quad 1 \le i \le a_3$$

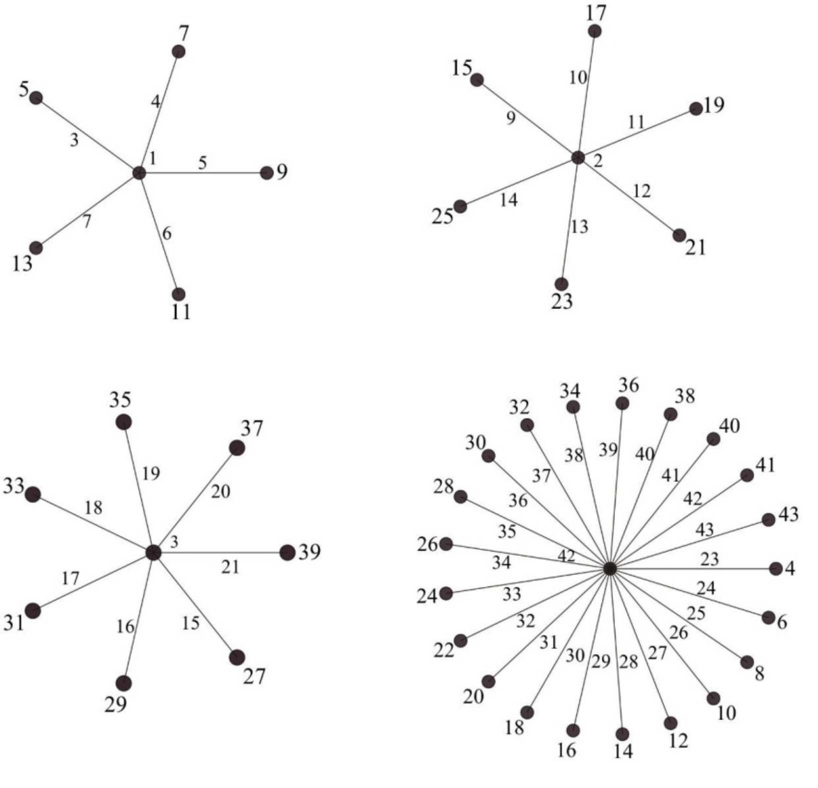
$$f(v_{4,i}) = 2i + 2 \quad 1 \le i \le b - 2 = A_5 + 1$$

$$f(v_{4,b-1}) = 2A_3 + 5$$

$$f(v_{4,b}) = 2A_3 + 7$$

The corresponding edge labels are as follows: The edge label of $v_{1,0}v_{1,i}$ is $2+i$ for $1 \le i \le a_1$ (edge labels are $3, 4, ..., a_1 + 2 = A_1 + 2$), $v_{2,0}v_{2,i}$ is $A_1 + 3 + i$ for $1 \le i \le a_2$ (edge labels are $A_1 + 4, A_1 + 5, ..., A_1 + 3 + a_2 = A_2 + 3$), $v_{3,0}v_{3,i}$ is $A_2 + 3 + i$ for $1 \le i \le a_3$ (edge labels are $A_2 + 4, A_2 + 5, ..., A_2 + 3 + a_3 = A_3 + 3$), $v_{4,0}v_{4,i}$ is $A_3 + 4 + i$ for $1 \le i \le b - 2 = A_3 + 1$ (edge labels are $A_3 + 5, A_3 + 6, ..., A_3 + 4 + A_3 + 1 = 2A_3 + 5$, $v_{4,0}v_{b-1}$ is $2A_3 + 6$ and $v_{4,0}v_b$ is $2A_3 + 7$.

These induced edge labels of graph G are distinct.
Hence G is a skolem mean graph.



Figure$\left(K_{1,5} \cup K_{1,6} \cup K_{1,7} \cup K_{1,21}\right)$

Case 2:Let $b = A_3 + 2$. G has $A_3 + b + 4 = 2A_3 + 6$ vertices and $A_3 + b = 2A_3 + 2$ edges.

The vertex labelling $f : V \rightarrow \{1,2,3,\ldots,A_3+b+4 = 2A_3+6\}$ is defined as follows:

$f(v_{1,0}) = 1; \quad f(v_{2,0}) = 2; \quad f(v_{3,0}) = 3;$

$f(v_{4,0}) = A_3 + b + 3 = 2A_3 + 5$

$f(v_{1,i}) = 2i + 3 \qquad 1 \le i \le a_1$

$f(v_{2,i}) = 2A_1 + 2i + 3 \qquad 1 \le i \le a_2$

$f(v_{3,i}) = 2A_2 + 2i + 3 \qquad 1 \le i \le a_3$

$f(v_{4,i}) = 2i + 2 \qquad 1 \le i \le b$

The corresponding edge labels are as follows:

The edge label of $v_{1,0}v_{1,i}$ is $2+i$ for $1 \le i \le a_1$ (edge labels are $3,4,\ldots,a_1+2 = A_1+2$), $v_{2,0}v_{2,i}$ is A_1+3+i for $1 \le i \le a_2$ (edge labels are $A_1+4, A_1+5,\ldots,A_1+a_2+3 = A_2+3$), $v_{3,0}v_{3,i}$ is A_2+3+i for $1 \le i \le a_3$ (edge labels are $A_2+4, A_2+5,\ldots,A_2+a_3+3 = A_3+3$), $v_{4,0}v_{4,i}$ is A_3+4+i for $1 \le i \le b = A_3+2$ (edge labels are $A_3+5, A_3+6,\ldots,2A_3+6$.

These induced edge labels of graph G are distinct.

Hence G is a skolem mean graph.

Example:

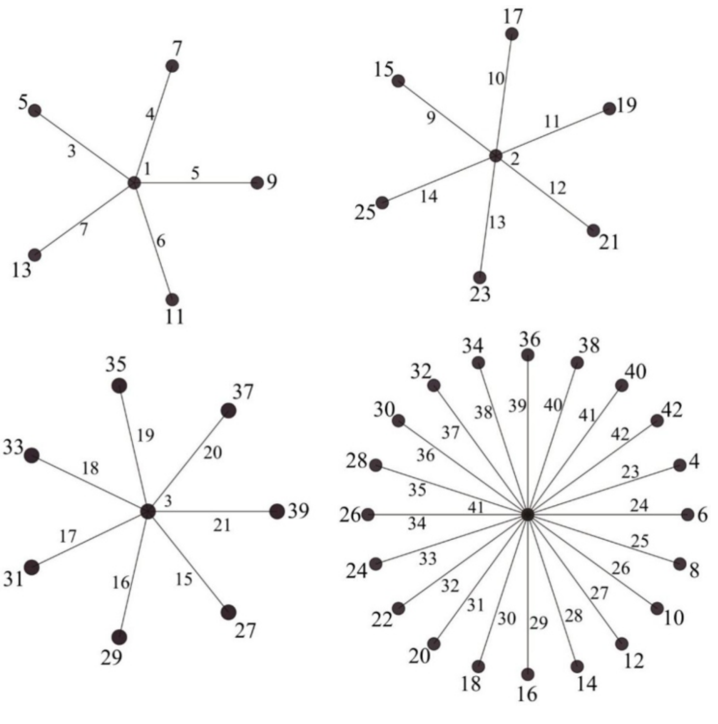


Figure $K_{1,5} \cup K_{1,6} \cup K_{1,7} \cup K_{1,20}$

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