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APPLICATION OF EXPONENTIAL SMOOTHING TECHNIQUE IN ESTIMATING CONDITIONAL EXPECTED SHORTFALL OF A FINANCIAL PORTFOLIO

Statistics

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ABSTRACT

Financial risk management is the practice of economic value in a firm by using financial instruments to manage exposure to risk. Expected shortfall is an important tool of risk measurement, and therefore, estimation of Expected Shortfall is important in financial risk management. In this paper, we explore exponential smoothing technique in estimating the conditional expected shortfall of the returns of a financial portfolio. We give the estimators of the conditional mean, conditional volatility and Value-at-Risk and show that they are consistent under some conditions. To test the practicability of the theories developed, an empirical study is undertaken using the share prices of Kenya Commercial Bank.

KEYWORDS

Value- at- Risk (VaR), Expected Shortfall (ES), Exponential Smoothing, Conditional mean, Conditional Volatility.

1. INTRODUCTION

The modern era of risk measurement began in 1973, when the Breton Woods system of fixed exchange rates collapsed. Since then, there has been a sharp increase of financial risks in financial institutions, see Thomas and Neil (1999). The recent financial disasters in tradable portfolios in Kenya, for example National bank of Kenya, have underlined the need for better financial risk measures in institutions such as banks and investment firms. The nature of financial risks in such tradable portfolios keeps on changing with time and therefore the methods to measure them should adapt accordingly. Furthermore, these methods should be easy to understand even in complex situations. It is in this context that quantitative risk measures have become vital in the management of risks.

There are many types of risks. These include market risks, credit risks and operational risks. In this study, we focus on the market risks. Market risks are risks that arise from the changes in the prices of financial assets and liabilities, and measured by changes in the value of open positions or in earnings.

Jorion (1997) defined VaR as a specified high quantile such as the 95%-quantile of negative financial returns and measures the worst expected loss over a given time interval under normal market conditions at a given confidence interval. VaR has become the standard measure used in financial risk management due to its conceptual simplicity, computational facility, and ready applicability. However, many authors claim that VaR has several conceptual problems. Artzner et *al.* (1997, 1999), for example, has cited the following shortcomings of VaR:

(i) VaR measures only percentiles of profit-loss distributions, and thus disregards any loss beyond the VaR level, a problem they referred to as "tail risk". In other words, VaR has problems measuring extreme price movements since it only measures the distribution quantile. Hence, VaR may ignore important information regarding the tails of the underlying distribution.

(ii) VaR is not a coherent risk measure since it is not sub-additive. This is supported by Acerbi and Tasche (2001) who also noted that, "To avoid confusion, if a measure is not coherent we just choose not to call it a risk measure at all".

Suppose a portfolio is made up of sub-portfolios. A risk measure ρ is sub-additive when the risk of the total position is less than or equal to the sum of the risk of individual portfolios. Acerbi and Tasche (2001) defined sub-additivity as follows: Let X and Y be random variables denoting the losses of two individual positions. A risk measure ρ is sub-additive if the following equation holds:

$$\rho(X+Y)\leq\rho(X)+\rho(Y)$$

To alleviate the problems inherent in VaR, Artzner et al (1997,1999) proposed the use of ES. ES is the conditional expectation of loss given that the VaR level has been exceeded, and measures how much one can lose on average in the states beyond the VaR level. Hence, it is clear from it's definition that ES considers loss beyond the VaR level. Furthermore, ES is in most relevant cases a coherent risk measure, see Acerbi and Tasche (2001). In addition, ES is also easy to understand and always more conservative than VaR. Yamai and Yoshiba (2002) have shown that expected shortfall has no tail risk under more lenient conditions than VaR.

Good risk measures may be used by financial risk managers and the shareholders for information reporting, resource and performance allocations. Due to globalization, which has resulted to a fast paced financial world, there is motivation to develop efficient and effective risk measures that will respond to news accordingly.

Our underlying process of interest is of the form $X_t=\mu_t+\sigma_t e_t$, see Mwita (2003), where μ_t is the conditional mean function of X_t given the past information, F_{t-1}, σ_t is the conditional volatility function of X_t given the past information, F_{t-1} and e_t are the error terms, which we assume to be independent of F_{t-1} and are normally distributed with mean 0 and variance 1.

In section two, we propose the estimators for the conditional mean and conditional volatility, and derive their Consistency under some conditions. Section three gives the estimators for Value-at-Risk and Expected Shortfall and a proof that Value-at-Risk is consistent. In section four, we give empirical results and their interpretation. Finally, we give conclusion and our recommendations in section five.

2. Estimatorsfor the conditional mean and conditional volatility

Let $X_1,\ldots,X_H,X_{H+1},\ldots,X_n$ be a sequence of returns, where H is the size of the horizon we take at time t and n is the sample size large enough such that we can have a number of horizons that can give a vector of estimates. The estimator for the conditional mean is given by

$$\hat{\mu}_t = \sum_{j=t-H+1}^{t} \omega_J X_j \tag{2.1}$$

where

$\omega_j = (1-\lambda)\lambda^{t-j}$ are exponentially decreasing weights

$\lambda \in (0,1)$, is the smoothing constant

t is the current time period

H is the time horizon.

We define the estimator for conditional volatility under the following situations:

1. When the conditional mean, μ_t, is known, the estimator is given by

$$\hat{\sigma}_t^2 = \sum_{j=t-H+1}^{t} (1-\lambda)\lambda^{t-j} X_j^2 \tag{2.2}$$

2. When the conditional mean, μ_t, is unknown, the estimator is given by

$$\hat{\sigma}_t^2 = \sum_{j=t-H+1}^{t} (1-\lambda)\lambda^{t-j} \left(X_j - \hat{\mu}_t\right)^2 \tag{2.3}$$

Assumptions

(A1) $E(e_i) = 0$

(A2) $\sigma^2 = E(e_i^2) < \infty$

(A3) $\lim_{|i-j|\to\infty} Cov(X_i, X_j) = 0$ where i represents an observation in one horizon e.g $X_1, X_2, \ldots, X_H$ and j represents an observation in another horizon e.g $X_2, X_2, \ldots, X_{H+1}$

Theorem 21 (Consistency for the conditional mean estimator)

Let $X_1, X_2, \ldots, X_H, X_{H+1}, \ldots, X_n$ be a random sample of returns with mean μ_t and variance σ_t^2. Then, $\hat{\mu}_t = \sum_{j=t-H+1}^{t} (1-\lambda)\lambda^{t-j} X_j$ is a weakly consistent estimator for μ_t.

Proof

We show that $\lim_{H\to\infty} E(\hat{\mu}_t) = \mu_t$ and $\lim_{H\to\infty} Var(\hat{\mu}_t) = 0$. We proceed as follows:

$$\hat{\mu}_t = \sum_{j=t-H+1}^{t} (1-\lambda)\lambda^{t-j} X_j$$

$$= (1-\lambda)\lambda^t \sum_{j=t-H+1}^{t} \frac{1}{\lambda^j} X_j$$

$$E(\hat{\mu}_t) = (1-\lambda)\lambda^t \sum_{j=t-H+1}^{t} E\left(\frac{X_j}{\lambda^j}\right) \tag{2.4}$$

$$= (1-\lambda)\lambda^t \sum_{j=t-H+1}^{t} \frac{\mu_t}{\lambda^j} \tag{2.5}$$

$$= (1-\lambda)\lambda^t H \frac{\mu_t}{\lambda^j} \tag{2.6}$$

$$= (1-\lambda)\lambda^{t-j} H\mu_t$$

$$= (1-\lambda)\lambda^{H-1} H\mu_t \tag{2.7}$$

$$\lim_{H\to\infty} E(\hat{\mu}_t)=\mu_t \tag{2.8}$$

$$Var(\hat{\mu}_t)=(1-\lambda)^2\lambda^{2t}\sum_{j=t-H+1}^{t}\operatorname{var}\left(\frac{X_j}{\lambda^j}\right)-Cov\left(\frac{X_i}{\lambda^i},\frac{X_j}{\lambda^j}\right) \tag{2.9}$$

$$=(1-\lambda)^2\lambda^{2t}\sum_{j=t-H+1}^{t}\left(\frac{\sigma_t^2}{\lambda^{2j}}\right) \tag{2.10}$$

$$=(1-\lambda)^2\lambda^{2t}H\left(\frac{\sigma_t^2}{\lambda^{2j}}\right) \tag{2.11}$$

$$=(1-\lambda)^2\lambda^{2(t-j)}H\sigma_t^2$$

$$=(1-\lambda)^2\lambda^{2(H-1)}H\sigma_t^2 \tag{2.12}$$

$$\lim_{H\to\infty}(\operatorname{var}\hat{\mu}_t)=\lim_{H\to\infty}\left\{(1-\lambda)^2\lambda^{2(H-1)}H\sigma_t^2\right\}=0$$

which implies that $\hat{\mu}_t \xrightarrow{p} \mu_t$.

Remarks

a) The time horizon selected is large ($H>30$). Therefore, by the central limit theorem, the returns are normally distributed with mean μ_t and variance σ_t^2 i.e. $X_j \sim N(\mu_t,\sigma_t^2)$.It is in this respect that (2.4) leads to (2.5).

b) Equation (2.6) is obtained by summing equation (2.5) over the horizon, H .Similarly, equation (2.11) is obtained from equation (2.10).

c) Equations (2.7) and (2.12) are obtained by noting that $j=t-H+1$.

d) Equation (2.8) is obtained from equation (2.7) by noting that the weights sum up to 1 as $H\to\infty$.

e) By assumption *(A3)*, (2.10) is obtained from equation (2.9).

Theorem 2.2 (Consistency for $\hat{\sigma}_t$ when $\hat{\mu}_t$ is known)

Let $X_1,\ldots,X_H,X_{H+1},\ldots X_n$ be a sequence of returns with mean μ_t and variance, σ_t^2. Then, $\hat{\sigma}_t=\left[\sum_{j=t-H+1}^{t}(1-\lambda)\lambda^{t-j}X_j^2\right]^{1/2}$ is a simple consistent estimator for σ_t.

Proof of theorem.2

We show that $\lim_{H\to\infty}E(\hat{\sigma}_t^2)=\sigma_t^2$ and $\lim_{H\to\infty}Var(\hat{\sigma}_t^2)=0$. We proceed as follows:

$$\hat{\sigma}_t^2=\sum_{j=t-H+1}^{t}(1-\lambda)\lambda^{t-j}X_j^2$$

$$=(1-\lambda)\lambda^t\sum_{j=t-H+1}^{t}\frac{1}{\lambda^j}X_j^2$$

$$E(\hat{\sigma}_t^2)=(1-\lambda)\lambda^t\sum_{j=t-H+1}^{t}E\left(\frac{X_j^2}{\lambda^j}\right) \tag{2.13}$$

$$=(1-\lambda)\lambda^t\sum_{j=t-H+1}^{t}\frac{\sigma_t^2}{\lambda^j} \tag{2.14}$$

$$=(1-\lambda)\lambda^t H\frac{\sigma_t^2}{\lambda^j} \tag{2.15}$$

$$=(1-\lambda)\lambda^{t-j}H\sigma_t^2$$

$$=(1-\lambda)\lambda^{H-1}H\sigma_t^2 \tag{2.16}$$

$$\lim_{H\to\infty}E(\hat{\sigma}_t^2)=\sigma_t^2 \tag{2.17}$$

$$Var\left(\hat{\sigma}_t^2\right)=(1-\lambda)^2\lambda^{2t}\sum_{j=t-H+1}^{t}\mathrm{var}\left(\frac{X_j^2}{\lambda^j}\right)-Cov\left(\frac{X_i^2}{\lambda^i},\frac{X_j^2}{\lambda^j}\right) \tag{2.18}$$

$$=(1-\lambda)^2\lambda^{2t}\sum_{j=t-H+1}^{t}\left(\frac{\sigma_t^2}{\lambda^{2j}}\right) \tag{2.19}$$

$$=(1-\lambda)^2\lambda^{2t}H\left(\frac{\sigma_t^2}{\lambda^{2j}}\right) \tag{2.20}$$

$$=(1-\lambda)^2\lambda^{2(t-j)}H\sigma_t^2$$

$$=(1-\lambda)^2\lambda^{2(H-1)}H\sigma_t^2 \tag{2.21}$$

$$\lim_{H\to\infty}\left(\mathrm{var}\,\hat{\sigma}_t^2\right)=\lim_{H\to\infty}\left\{(1-\lambda)^2\lambda^{2(H-1)}H\sigma_t^2\right\}=0$$

which implies that $\hat{\sigma}_t \xrightarrow{p} \sigma_t$.

Remarks

a) Equation (2.14) is obtained from equation (2.13) by noting that $H > 30$. That is, the time horizon we have selected is large. Therefore, by the central limit theorem, the returns are normally distributed with mean μ_t and variance σ_t^2 i.e. $X_j \sim N(\mu_t, \sigma_t^2)$. Hence, we have $E\left(\frac{X_j}{\lambda^j}\right)=\frac{\mu_t}{\lambda^j}$ and $Var\left(\frac{X_j}{\lambda^j}\right)=\frac{\sigma_t^2}{\lambda^{2j}}$. In a similar way, equation (2.19) is obtained from equation (2.20), under assumption *(A3)*.

b) Equation (2.15) is obtained by summing equation (2.14) over the horizon, H. Similarly, equation (2.20) is obtained from equation (2.19).

c) Equations (2.16) and (2.21) are obtained by noting that $j = t - H + 1$.

d) Equation (2.17) is obtained from equation (2.16) by noting that the weights sum up to 1 as $H \to \infty$.

Theorem 2.3 (Consistency for $\hat{\sigma}_t$ when $\hat{\mu}_t$ is unknown)

Let $X_1,\ldots,X_H, X_{H+1},\ldots X_n$ be a sequence of returns with mean μ_t and variance, σ_t^2. Then, $\hat{\sigma}_t = \left[\sum_{j=t-H+1}^{t}(1-\lambda)\lambda^{t-j}(X-\hat{\mu}_t)^2\right]^{1/2}$ is a simple consistent estimator for σ_t.

Proof of theorem.3

We show that $\lim_{H\to\infty} E\left(\hat{\sigma}_t^2\right)=\sigma_t^2$ and $\lim_{H\to\infty} Var\left(\hat{\sigma}_t^2\right)=0$. We proceed as follows:

$$\hat{\sigma}_t^2=\sum_{j=t-H+1}^{t}(1-\lambda)^{(t-j)}\left(X_j-\hat{\mu}_t\right)^2$$

$$=\sum_{j=t-H+1}^{t}(1-\lambda)^{(H-1)}\left(X_j-\hat{\mu}_t\right)^2$$

$$=(1-\lambda)^{(H-1)}\sum_{j=t-H+1}^{t}\left(X_j-\hat{\mu}_t\right)^2$$

$$\frac{\hat{\sigma}_t^2}{(1-\lambda)^{(H-1)}}=\sum_{j=t-H+1}^{t}\left(X_j-\hat{\mu}_t\right)^2$$

$$\frac{\hat{\sigma}_t^2}{(1-\lambda)^{(H-1)}\sigma_t^2}=\frac{\sum_{j=t-H+1}^{t}\left(X_j-\hat{\mu}_t\right)^2}{\sigma_t^2}$$

has a Chi-square distribution with $(H-1)$ degrees of freedom. Therefore,

$$E\left[\frac{\hat{\sigma}_t^2}{(1-\lambda)^{(H-1)}\sigma_t^2}\right] = (H-1)$$

and

$$Var\left[\frac{\hat{\sigma}_t^2}{(1-\lambda)^{(H-1)}\sigma_t^2}\right] = 2(H-1)$$

Hence,

$$E(\hat{\sigma}_t^2) = (1-\lambda)^{H-1}(H-1)\sigma_t^2$$

$$\lim_{H\to\infty} E(\hat{\sigma}_t^2) = \sigma_t^2$$

$$Var(\hat{\sigma}_t^2) = (1-\lambda)^{2(H-1)}2(H-1)\sigma_t^4$$

$$\lim_{H\to\infty} \mathrm{var}(\hat{\sigma}_t^2) = 0$$

which implies that $\hat{\sigma}_t^2 \xrightarrow{p} \sigma_t^2$ and so is $\hat{\sigma}_t$. Hence, proved.

3. ESTIMATORS FOR VALUE AT RISK AND EXPECTED SHORTFALL

3.1 Estimator for Value at Risk (VaR)

The Value-at-Risk summarizes the expected maximum loss (or worst loss) over a target horizon at a given confidence level θ. In our case, we use a target horizon of 250 trading days and a confidence level of 99%.This means that the probability to lose at the next day more than the calculated Value-at-Risk is less than 1%.

We obtain today's returns, X_t, as $X_t = -Log\left(\frac{P_t}{P_{t-1}}\right)$ where P_t and P_{t-1} are today's and yesterday's share prices respectively.

Refer to the econometric model in section 1. The conditional θ -quantile function for this model is given by

$$VaR_{t,\theta} = \mu_t + \sigma_t e_\theta$$

The estimator for the conditional θ -quantile is given by

$$V\hat{a}R_{t,\theta} = \hat{\mu}_t + \hat{\sigma}_t e_\theta$$

where $\hat{\mu}_t$ and $\hat{\sigma}_t$ are as defined in equations (2.1) and (2.3) respectively.

3.1.1 Consistency for the estimator for Value at Risk

We show that $\lim_{H\to\infty} E(V\hat{a}R_{t,\theta}) = VaR_{t,\theta}$ and $\lim_{H\to\infty} Var(V\hat{a}R_{t,\theta}) = 0$. We proceed as follows:

First, we check that;

$$E(V\hat{a}R_{t,\theta}) = E(\hat{\mu}_t) + e_\theta E(\hat{\sigma}_t) \quad (3.1)$$

$$= (1-\lambda)^{H-1}H\mu_t + e_\theta(1-\lambda)^{H-1}(H-1)\sigma_t^2 \quad (3.2)$$

$$\lim_{H\to\infty} E(V\hat{a}R_{t,\theta}) = \mu_t + \sigma_t e_\theta \quad (3.3)$$

$$= VaR_{t,\theta}$$

Next, we check the asymptotic variance.

$$\begin{aligned} Var(V\hat{a}R_{t,\theta}) &= Var(\hat{\mu}_t) + e_\theta^2 Var(\hat{\sigma}_t) - 2e_\theta Cov(\hat{\mu}_t, \hat{\sigma}_t) \\ &= \left((1-\lambda)^2\lambda^{H-1}H\sigma_t^2\right) + e_\theta^2\left((1-\lambda)^{2(H-1)}2(H-1)\sigma_t^4\right) \quad (3.4) \\ &= (1-\lambda)^2\lambda^{H-1}H\sigma_t^2\left(1 + e_\theta^2(1-\lambda)^{(H-1)}\sigma_t^2\right) \end{aligned}$$

$$\lim_{H\to\infty} Var(V\hat{a}R_{t,\theta}) = 0$$

Hence, proved.

Remarks

a) Equation (3.2) is obtained from equation (3.1) by using assumption *(A3)* and noting that $E(\hat{\mu}_t) = (1-\lambda)^{H-1}H\mu_t$ and $E(\hat{\sigma}_t) = (1-\lambda)^{H-1}(H-1)\sigma_t$.

b) Equation (3.3) is obtained from equation (3.2) by noting that the weights sum up to 1 as $H \to \infty$.

c) In equation (3.4), $Var(\hat{\mu}_t)$ and $Var(\hat{\sigma}_t)$ are obtained as in equations (2.12) and (2.21) respectively.

3.2 Estimator for the Expected Shortfall

Expected shortfall is the conditional expectation of loss given that the loss is beyond the VaR level, and measures how much one can lose on average in the states beyond the VaR level. Suppose X_t is a random variable denoting the negative returns of a given portfolio and $VaR_{t,\theta}$ is the VaR at the θ - confidence level. The Expected Shortfall is given by

$$ES_\theta = E\left[X_t \mid X_t \geq VaR_{t,\theta}\right]$$

The function for expected shortfall based on our econometric model is given by

$$\begin{aligned} ES_\theta &= E\left[\mu_t + \sigma_t e_t \mid \mu_t + \sigma_t e_t \geq \mu_t + \sigma_t e_\theta\right] \\ &= \mu_t + \sigma_t E\left[e_t \geq e_\theta\right] \end{aligned}$$

and the estimator for the expected shortfall is given by

$$\begin{aligned} E\hat{S}_\theta &= \hat{\mu}_t + \hat{\sigma}_t E\left[\hat{e}_t \geq \hat{e}_\theta\right] \\ &= \hat{\mu}_t + \hat{\sigma}_t \frac{1}{k}\sum_{i=1}^{k} \hat{e}_t I_{\{\hat{e}_t \geq \hat{e}_\theta\}} \end{aligned}$$

where $\hat{\mu}_t$ and $\hat{\sigma}_t$ are as defined in equations (2.1) and (2.3) respectively. k is the number of negative returns that exceed $VaR_{t,\theta}$ and $I_{\{\hat{e}_t \geq \hat{e}_\theta\}}$ is an indicator function defined as

$$I_{\{e_t \geq e_\theta\}} = \begin{cases} 1 \ if \ \hat{e}_t \geq \hat{e}_\theta \\ 0 \ if \ \hat{e}_t < \hat{e}_\theta \end{cases}$$

4. EMPERICAL RESULTS AND THEIR INTERPRETATION

Here, we present analysis of returns on KCB share prices from January 2000 to April 2004 (trading days). This data was provided by the Nairobi Stock Exchange. The results are given in the figures below.

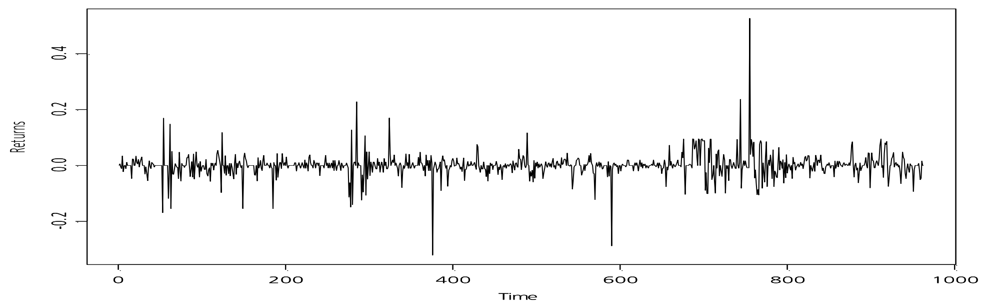

Figure 4.1: Plot of KCB returns from Jan 2000 to April 2004

In Figure 4.1, the upper (positive) side are the negative returns (losses) while the lower(negative) side are the positive returns (profits). This is because we have used the negative log returns. We observe that the returns tend to cluster as the threshold increases. This suggests that the returns are autocorrelated. The clustered returns over time represent clustering of volatilities. This is supported by Engle and Manganelli (2002) who noted that the distribution of returns tend to be autocorrelated. The long spikes on either side indicate extreme returns.

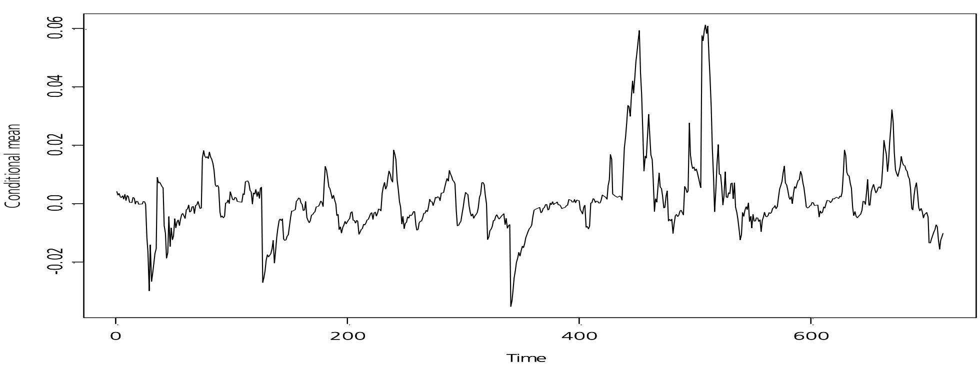

Figure 4.2: KCB conditional means plotted against time

In Figure 4.2, we can observe that the conditional mean tend to be stationary for a long period of time as it keeps on reverting to the long term value, that is, 0. There are also some few positive and negative “outliers” or extreme means that seem not be consistent with the rest of the means. The negative “outliers” suggests that the market conditions were very favorable and the positive outliers suggest that the market conditions were very unfavorable.

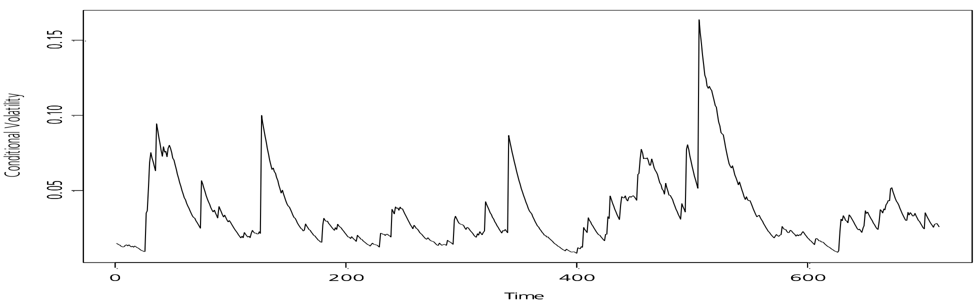

Figure 4.3: KCB Conditional volatilities plotted against time

In Figure 4.3, it is evident that volatility varies with time. We can also observe that large volatilities tend to be followed by large volatilities. Similarly, small volatilities tend to be followed by small volatilities. This is supported by Engle and Manganelli (2002) who noted that volatilities of stock market returns tend to cluster over time. We can also see that there was a shock as indicated by the extremely large volatilities that lasted for a short period (around trading day 500 to 550). A shock indicate either extreme gain or extreme loss in the market, depending on the market conditions.

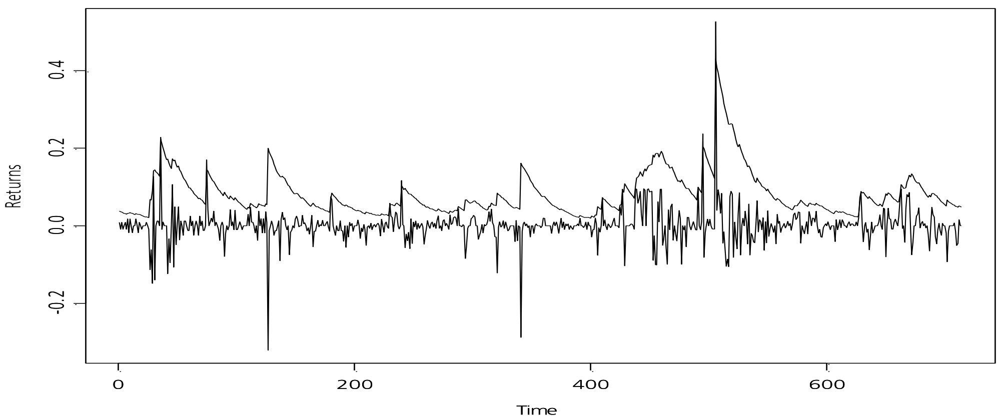

Figure 4.4: KCB Returns plotted together with $VaR_{t,0.99}$.

In Figure 4.4, we have superimposed 0.99-conditional quantile on the returns. VaR is always based on negative returns hence the plot of $VaR_{t,0.99}$ on the upper side. From the definition of VaR, 0.99-conditional quantile means that there is one chance in a hundred chances, under normal market conditions, for a loss greater than the VaR set by a given portfolio’s management, to occur in a given day. Therefore, the 0.99-conditional quantile measures the maximum loss a portfolio can incur at 99% confidence level. It is clear from figure 4 that the 0.99-conditional quantile responds well to the distribution of the returns. We can also see from the figure that there are some few returns that exceed the VaR.

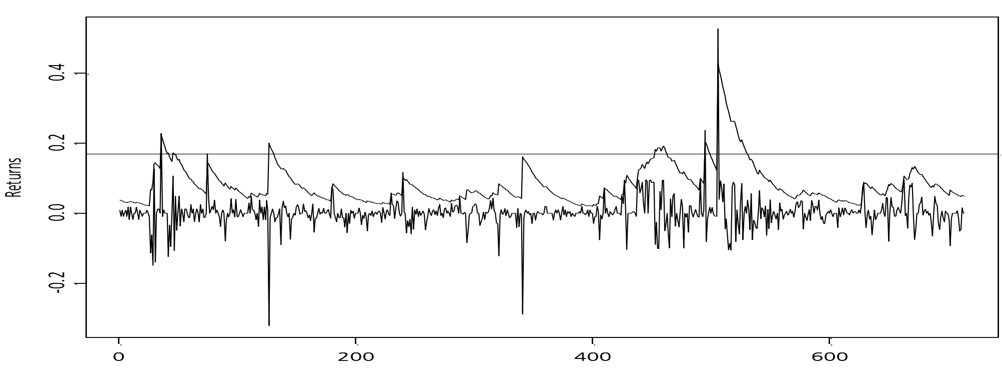

Figure 4.5: Plot of KCB returns with $VaR_{t,0.99}$ *and Expected Shortfall.*

In Figure 4.5, we have superimposed the $VaR_{t,0.99}$ and expected shortfall on the returns. The straight line represents the expected shortfall. We can observe from the figure that the straight line representing the expected shortfall is slightly above the $VaR_{t,0.99}$. Therefore, it is clear from the figure that VaR tells us the most we can expect to lose if a tail (or worst) event does not occur while the expected shortfall tells us what we can expect to lose if a tail event does occur.

In our study, we have analyzed 963 returns on KCB share prices. As per the definition of VaR at 99% confidence level, we expect, under normal market conditions, about 10 returns to exceed the $VaR_{t,0.99}$. In our results, we have obtained 9 exceedences.

5. CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

In our study, we have estimated the Expected Shortfall of returns on share prices of Kenya Commercial Bank. In estimating the conditional mean and conditional volatility of these returns, we explored the exponential smoothing technique, whereby we assigned exponentially decreasing weights to the returns. Exponential smoothing technique is easy to understand and apply. We proved that the estimators for the conditional mean and conditional volatility are consistent. Further, we have given the estimators for the Value-at-Risk and Expected Shortfall and have shown that the estimator for Value-at-Risk is consistent.

5.2 Recommendations

Though the exponential smoothing technique has a number of merits as we have seen, we also note the following shortcomings:

a) Exponential smoothing lags. In other words, the forecast will be behind as the trend increases over time.
b) Exponential smoothing may fail to account for the dynamic changes at work in the real world, and the forecast may constantly require updating to respond to new information.

In this respect, other techniques, like general autoregressive conditional heteroscedastic (GARCH) estimation, may be explored, see Thorsten et al (1999/2000).

We also suggest that the following work may be done in future as a continuation of this research:

a) The choice of the optimal smoothing parameter.
b) The proof of the asymptotic normality for the estimators.

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